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ABSTRACT

This guide represents the final experimental version of a pilot project which was conducted in the United States between 1973 and 1976. The ideas and the manner of presentation are based on the works of Georges and Frederique Papy. They are recognized for having introduced colored arrow drawings ("papygrams") and models of our numeration system (the Papy "minicomputer") into the teaching of mathematics at the elementary and secondary level in Belgium. This program follows the "spiral approach." Six workbooks are included which provide students with opportunities to review ideas they have met before, apply their knowledge to new situations, and learn to read by using mathematics workbooks and story-workbooks. The workbooks are written at three levels of difficulty and students are encouraged to work at the easiest level. Questions most frequently asked by teachers about the program are answered in this guide. (Author/SA)

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Comprehensive School Mathematics Program

CEMREL, Inc.

Experimental Version

5-9531

CSMP Mathematics for the Intermediate Grades Part III

Teacher's Guide

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

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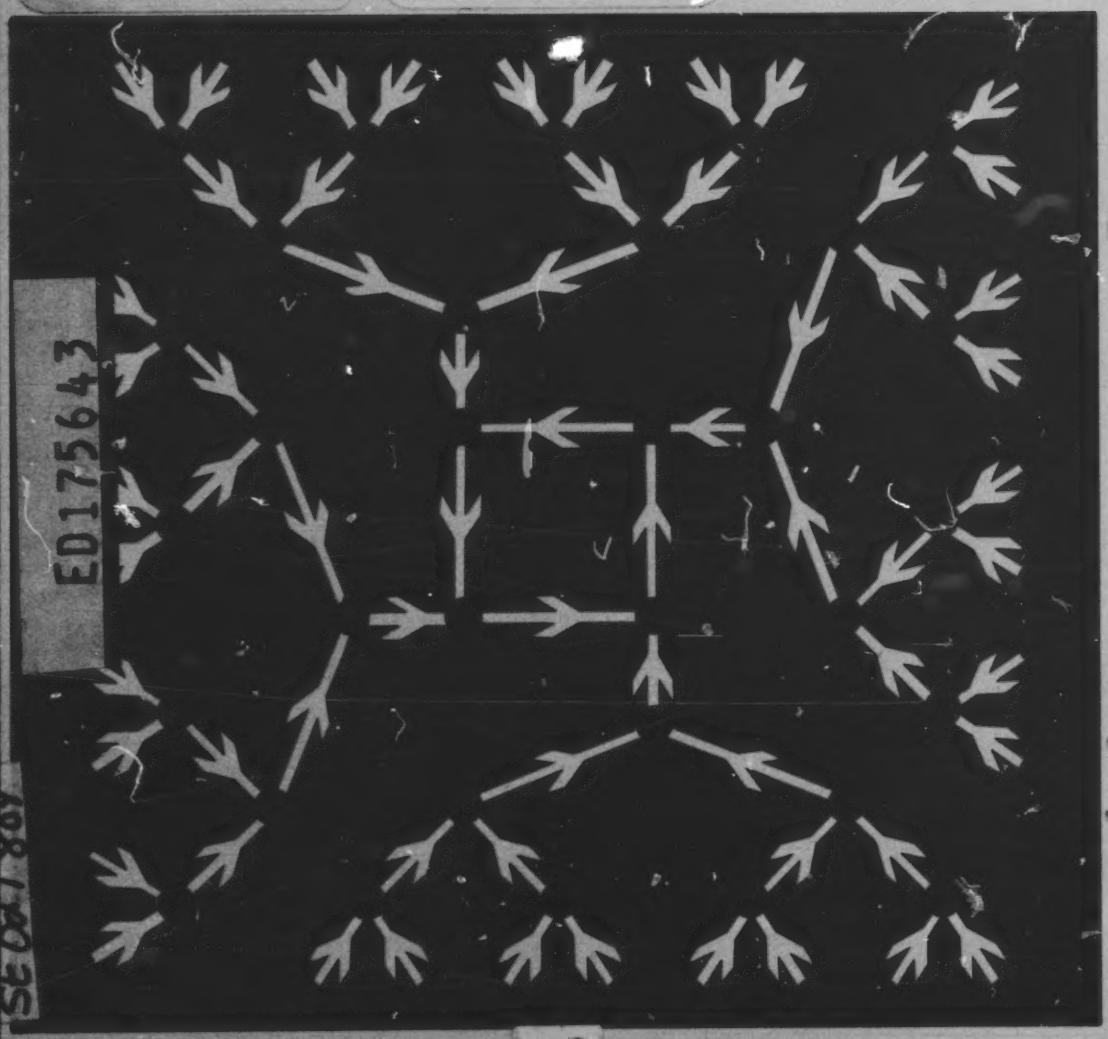
General Introduction
Workbooks



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PREFACE

CSMP is firmly committed to the proposition that the study of mathematics should be an interesting, challenging, intellectual and aesthetic experience for all children. The teachers who will teach its curriculum should be similarly committed to that proposition. The CSMP curriculum expects to truly engage children in the study of mathematics. It expects to call upon their curiosity, their intuition, their powers of reasoning, their imagination, their perseverance, and their organizational skills. We believe that the study of mathematics not only calls upon these faculties of children, but will help to develop them. But the curriculum cannot do the job by itself--there must be teachers who share the basic intellectual values built into the curriculum.

There is an almost infinite variety of uses to which mathematics can be and has been put. Certainly, a reasonable mastery of arithmetic skills is part of the mathematical equipment of a competent, modern adult. Our job as educators is to provide children with a set of mathematical experiences far broader than the acquisition of simple arithmetic skills. It is just as important to expose children to a wide variety of ways in which mathematics can help them to formulate and think about problems. We believe further that the arithmetic skills themselves can be much more successfully taught in the context of a rich and challenging mathematical environment. It is especially important to expose children to a wide variety of mathematical activities in view of the fact that no one can possibly know what career and lifestyle an elementary school child will ultimately want to choose.

It is CSMP's conviction that mathematics is a unified whole and should be learned as such. The organizational basis of the CSMP curriculum is the spiral approach. Thus, the program is sequenced so that students come into contact with each area of content continuously throughout the program. This approach consciously precludes atomizing the content and mastering each bit before continuing to the next. Rather, students encounter repeatedly the ideas, building interlocking experiences of ever increasing sophistication.

CSMP believes that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation and independent individual study.

The CSMP materials are constructed to maximize teacher-student and student-student interaction. Children begin immediately to investigate mathematical concepts in concrete situations appealing and meaningful to them at their level of experience.

Individualization at this level is achieved through graded series of workbooks, integrated into the basic sequence of lessons. Due to the spiral nature of the curriculum, each student can engage in a workbook for a period of two (possibly more) weeks without losing touch with the progress of the class and, at the same time, working his or her way up through the series to the limits of his or her capacity.

Unlike most modern programs, the content is not presented as an artificial structure external to the experience of children and into which they are thrust but rather as an extension of experiences children have encountered in their development, both at the real life and fantasy levels. Children are led through sequences of problem solving experiences presented in game-like as well as story settings. Nonverbal languages give children immediate access to the mathematical ideas and methods necessary not only for solving these problems but also for continually expanding the students' understanding of the mathematical concepts themselves. These languages include:

1. <u>The language of strings.</u> This language mimics the basic method of organizing and collecting data related to problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of sets.

2. <u>The language of arrows</u>. This graphical language models the fundamental process involved in comparing and analyzing sets and operations on them. That is, it deals with the useful and important mathematical notions of

relations and functions including, but not restricted to, permutations of sets, ordering relations and the numerical processes of adding to, subtracting from, multiplying by and sharing equally among.

3. The language of the Papy Minicomputer. The Papy Minicomputer, an extremely effective abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to very sophisticated algorithmic processes, including the standard algorithms for the basic numerical operations.

Through these languages the children are able to apply the basic mathematical ideas effectively and creatively to explore the content directly without wading through the cumbersome, sterile terminology and notation which, in most other programs, pervade the early stages of the curriculum and stand as barriers between the child and the content.

The content is learned in an atmosphere of constant connection with applications, from simple story situations in the early grades to challenging applications to nontrivial simulations of real world problems in the upper grades. The emphasis is at all times on a two level approach to learning: understanding the content and its applications, but equally important, developing the techniques and processes of learning the content. It is the latter form of knowledge which gives the power to apply the former.

The preface above also serves as the Preamble to the Memorandum of Agreement between CEMREL, Inc., and each school district involved in the extended pilot test stage of the CSMP Elementary School curriculum.

NOTE ON AUTHORSHIP

"Teacher's Guide to CSMP Mathematics for the Intermediate Grades, Part III" is the product of the concerted efforts of a number of people over several years' time. This edition was produced during the 1976-77 academic year by the following members of the CSMP staff and includes substantial contributions from the staff associates.

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The lessons in this guide were developed and tried out with two fifth grade classes at the Jackson Part Elementary School, University City, Missouri. The students, representing the full range of abilities, had been introduced to the CSMP curriculum at the beginning of fourth grade in a one semester program using "CSMP Mathematics for the Intermediate Grades, Entry Module I". The entry program prepared these students to begin the first part of the CSMP Intermediate Grades Program for veteran students during the second semester of the fourth grade.

We are extremely grateful for the daily cooperation and high level of constructive criticism received from Mr. Robert Graham and Ms. Patricia Lee, the regular teachers of the two classes used for development purposes. Our greatest motivation for this work was the enthusiasm and steady progress of the students themselves.

GENERAL INTRODUCTION

Comprehensive School Mathematics Program

Mathematics for the Intermediate Grades Part III

General Introduction

CEMREL, Inc.

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1. The CSMP Viewpoint

The CSMP curriculum is based on the belief that mathematics belongs to that part of the school curriculum which deals with ideas rather than that part which stresses techniques. This is not to say that there are no techniques involved in the learning of mathematics. There are, just as there are in the learning of English. But these techniques, just like spelling and grammar in the learning of English, are not ends in themselves; they are important only in so far as they help children to understand and use some area of mathematics.

What are the objects of mathematical study? For us, they include numbers of various kinds, operations on and relationships between those numbers; geometrical figures and their properties; relations and functions (e.g., the 2 x function, the "less than" relation, etc.) and operations on them.

Among all objects of mathematical study, from the simplest to the most complex, there is one common property: if we separate these objects of study from the real life situations that they model, they are all abstract; that is, they are things of the mind. For this reason, mathematics is, at heart, an intellectual activity. For example, the number 2 represents the phenomenon of twoness, a quality which is present in an endless variety of contexts in life. We can picture 2 by two dots, or by a stick two centimeters long, and these representations are certainly useful in suggesting to our minds what may or may not be true about the number 2. But ultimately the number 2 is accessible to us only through our minds. In the same sense, a geometrical line is neither a pencil line nor a stretched string nor a ray of light, but is an abstraction which may be pictured in many ways.

This is surely an adult point of view. But it is one that should be understood and taken into consideration by those who develop mathematical materials for children and those who teach the materials to children.

Since mathematical objects are ideas, one of the important ways we explore the properties of and relationships among such objects is to think about them. It is for this reason that the growth of the ability to reason plays such an important role is mathematics. We strongly argue, however, that in mathematics the development of the art of reasoning goes hand in hand with the

growth of imagination, ingenuity, and intuition. The really important thing at the elementary level is that mathematical objects, their properties and their interrelationships be thought about and discussed in various suggestive and interesting situations.

Seen in this perspective, elementary arithmetic then takes the form of "adventures in the world of numbers". Individual numbers such as 0, 1, 2, 100, $\frac{1}{5}$, $^{-}7$, $\frac{^{-}4}{9}$, 3.72, $^{-}1.006$, and so on can assume definite personalities of their own in this world. The teaching of arithmetic shifts its emphasis from an obstacle race for mastery of the standard algorithms to the stimulation of exploring the world of numbers. Skills such as balancing a checkbook have their place in real life but are of absolutely no interest to a fourth or fifth grader. It is unrealistic to view such activities as goals of an elementary school mathematics education. Far more genuinely rewarding are the stretching of the powers of imagination and the challenging of the mind. Facility then grows naturally, along with knowledge and understanding. Necessary skills become by-products of such an educational experience.

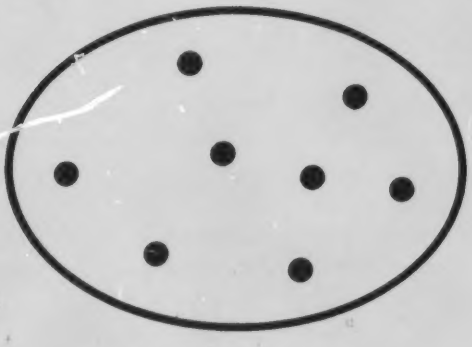
Such a curriculum requires a special kind of pedagogy. The pedagogy employed should include teaching strategies which encourage creativity and allow freedom to explore but still foster sound intellectual habits. Attitudes toward student achievement that allow for intellectual and emotional growth and respect the individuality of each student are essential to the successful attainment of the goals of this type of curriculum.

The pedagogy which CSMP has developed in this spirit can be described as a "pedagogy of situations".* The introductions to the various strands of the CSMP Intermediate Grades Program and those to the chapters within them contain many examples which illustrate aspects of this pedagogy. All these examples are taken from the lessons themselves, and a prime source of "pedagogical situations" is the lessons of the program. At the risk of oversimplification, a pedagogy of situations may be described as one which is

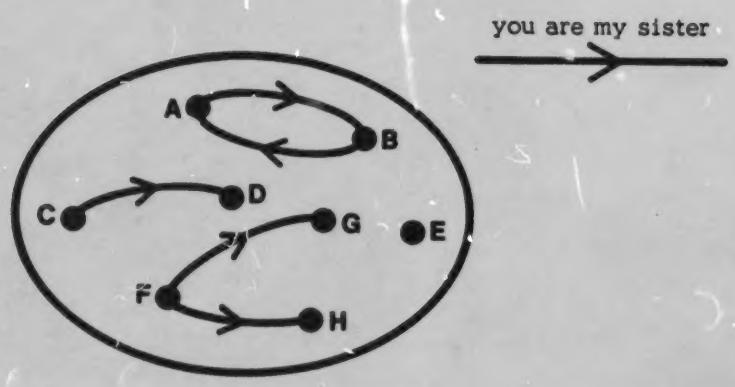
*This phrase is due to the mathematician and psychologist, Caleb Gattegno.

based on the belief that learning occurs in reaction to the experience of confronting a situation (real, simulated, or imagined) that is rich in consequences, worthy of confrontation, and has genuine intellectual content. For example, we can tell a story about some children in a playground.

As we tell the story , we draw this picture.



Each child in the playground is represented by exactly one dot, and each dot represents exactly one child. (There is no need to draw pictures of the children; indeed, such pictures would not serve our purposes at all. Even young children are quite able and willing to accept a dot as a symbolic representation for a child. On other days and in different situations they will accept the dot as representing a duck in a pond, or a marble in Johnny's collection, or a number, or whatever else you or they decide.) The children in the playground are playing a game: each child points to his or her sister(s) and says, "You are my sister." As we continue the story we draw arrows as indicated. (We label each dot here with a letter for easy reference as we discuss the situation.) So, for example, C is pointing to D and saying, "You are my sister."



What kind of comments will young children make when involved in this situation? Here are a few typical responses.

1. There are eight children in the playground.
2. D is C's sister.
3. D is a girl (Why? Because D is someone's sister.)
4. E has no sisters--at least no sister in the playground.
5. A and B are sisters and therefore both are girls.
6. There are two arrows missing. (Which? G and H are sisters of F and so they should be pointing to each other.)

These two arrows are added to the diagram and the children are now told that all the children that can say "You are my sister" have done so. That is, all the red arrows that belong have been drawn.

7. At this point some child in every group will almost always suggest "C is a boy!" If we ask "How do you know that?", we get an answer that can be summarized in adult language as: Suppose C were a girl. Then C would be D's sister. But there is no arrow from D to C. So the diagram tells us that C is not D's sister, and hence C is not a girl. So C is a boy.

8. There are children from four different families in the playground.

9. C is D's brother.

The students are now asked to draw arrows for "You are my brother" in a different color. They draw "You are my brother" arrows from D to C, G to F, and H to F.

In this example, the children react to a situation that intrigues them. They grapple with problems of symbolization and the relation of a symbol to the thing it represents. Arguments are given that are close to those which are the stuff of mathematics. Although adults may protest that it is not a "real" situation, it is real to children, who can often involve themselves in and feel at home with "pretend" situations more readily than with many "real" situations. On the basis of several years' experience in a wide variety of classes, we can say with confidence that such situations

- involve children at a personal level (they are interested in the situation presented, and sometimes want to dramatize it or devise similar or related situations);
- present an intellectual challenge that is accessible to a broad range of abilities (even the slowest students understand the challenge and can learn something from their experience in confronting it; some may even be able to realize the full learning potential of that challenge);
- suggest opportunities for creativity (some children are motivated by such situations to generalize from their experiences or to make conjectures to be tested in similar situations);
- provide experiences that have mathematical content. In the example of the "brother-sister situation" above there are two important mathematical activities at work. One is the classification of objects according to certain attributes (brother-sister, boy-girl, same family). More importantly, the experience concerns, at a very elementary level but in a natural setting, the process of deducing certain conclusions from given data and at the same time realizing the limits of deduction in the situation at hand.

The CSMP curriculum fulfills what we consider to be the fundamental role of mathematics in the elementary school by presenting a large number of varied yet interrelated situations that provide the experiences out of which mathematics grows. Two questions immediately arise. How do students learn necessary "number facts" via a pedagogy of situations? And how can we "grade the achievement" of a child as he or she reacts to a situation?

Believe it or not, the "number facts" are among the things which students learn almost inadvertently under a pedagogy of situations. Numbers take an almost living form and have personalities of their own. Their interrelations are the subject of many stories in which numbers talk to each other and react to each other. In confronting some of these situations it is necessary to make various calculations, and calculation which serves some higher, more motivating purpose presents nowhere near as many difficulties as does calculation for its own sake. There are further remarks concerning calculation scattered through the latter part of this introduction, and also in the introductions that appear in the World of Numbers strand.

As for "grading of achievement", CSMP takes the position that learning often stops when the purpose of a lesson or sequence of lessons is the mere mastery of skills. Every child should learn something from each situation; otherwise it is not a fruitful situation and should be revised or eliminated. But to measure what or how much has been learned in any one situation is probably not possible. Learning is not a linear process. It occurs in quantum jumps that take place at times when you least expect them. It is possible to monitor the progress of students, and you all have your own ways of doing this. But who is to say at what precise time everyone in a class is supposed to have gained a particular skill or concept from his or her experiences? We comment further on this matter in the next section of this General Introduction.

A more substantive question is: How can we present situations rich in mathematical content before children have the verbal equipment and the computational skills to deal with them? It is presupposed by many people that children must be able to read and to calculate before they can tackle any mathematical situation other than trivial ones. Yet if one analyzes the types of thinking and the few basic concepts underlying elementary school mathematics, one finds that

there is nothing esoteric or even difficult about these thinking modes or these concepts. What has been difficult for students in the past is the language in which these modes of thinking and these concepts are traditionally couched. For some reason, in our natural languages these modes of thinking are difficult to communicate precisely. That is why mathematicians devise languages of their own, languages that minimize the ambiguities of natural languages. Part of the problem in the past has been that elementary school children have found it difficult to deal with such tricky uses of language at a time when they do not yet have a firm command of their natural language.

On the other hand, since the underlying ideas and modes of thinking are inherently simple, there is no reason why very young children should not have the pleasure of such thinking and of exploring mathematical ideas. All that is needed are languages with which to express such ideas and to communicate the thinking that is generated by the ideas. These languages must be very simple, essentially nonverbal, and precise.

CSMP uses three such languages that fit the criteria and that have been highly successful in allowing students to do the kind of thinking that we described above. We call them the language of strings, the language of arrows, and the language of the Papy Minicomputer.

The first two, as their names imply, are picture languages: brightly colored strings and dots enable precise descriptions of and the accurate recording of thinking about classifications of objects; colored arrows between pairs of dots stimulate thinking about relations between objects. For detailed descriptions and discussions of these two languages you are referred to the introduction to the Languages of Strings and Arrows strand.

The third language is based on a simple abacus called the "Papy Minicomputer". Using this language, one may represent all decimal numbers (positive or negative) in such a way as to encourage creative thinking about the nature and properties of those numbers. With the support provided by the Minicomputer, the children, even at an early age, have the opportunity to develop excellent mental arithmetic skills and to gain sophisticated insights into the world of numbers. The ability to perform mechanical computation is one of the incidental by-products of such an approach to work with numbers.

2. Program Description

a) Description of Materials

The CSMP Intermediate Grades curriculum is designed as a program for students in grades four, five, and six. There are six parts to the curriculum, each corresponding to one semester's work. This is Part III of the six. It follows Part II of the CSMP Intermediate Grades Program.

In addition there are three Entry Modules, one for each grade level. As their names imply, each Entry Module is designed to facilitate transition into the CSMP Elementary School curriculum.

The materials of Part III of the CSMP Intermediate Grades Program require some explanation. This collection of three volumes (six books) which are directed specifically to you, the teacher, is known as the "Teacher's Guide to CSMP Mathematics for the Intermediate Grades, Part III". (By way of abbreviation, in what follows we shall refer to this collection simply as the "Teacher's Guide".) With the exception of this book, "General Introduction", each book of the Teacher's Guide addresses itself to a single area of the curriculum, and each of these areas is known as a "strand". The four content strands are:

- the World of Numbers. The abbreviation for this strand is "N".
- the Languages of Strings and Arrows. The abbreviation for this strand is "L".
- Geometry and Measurement. The abbreviation for this strand is "G".
- Probability and Statistics. The abbreviation for this strand is "P".

The six books comprising the "Teacher's Guide" are bound into three volumes, with a separation page inserted between the various books of a given volume. This volume contains both this General Introduction as well as the Workbook strand (which is abbreviated "W"). One of the other two volumes contains the World of Numbers strand, and the other contains the Languages of Strings and Arrows strand, the Geometry and Measurement strand, and the Probability and Statistics strand. Each of these six books has its own table of contents and is paginated individually.

The content and Workbook strands have the following arrangement of material.

- A table of contents. This enables you to locate whatever you need in that particular strand.
- A suggested schedule for the teaching of the lessons in that strand. This provides you with one among the many possible ways of organizing the material in the strand. It also includes for each suggested lesson a list of the materials you will need when you teach that lesson. See Section 3 of this General Introduction for a discussion of the way in which you should view this suggested schedule.
- A suggested chronological order of the lessons in that strand. This provides any easy, quick reference uncluttered by extraneous information. It shows you in which order the lessons described in the strand will be taught if you follow the suggested schedule.
- An introduction to the strand. This provides you with information explaining why certain material finds its place in that strand, what are the overall goals of study in that strand, and how this material fits in with that which is described in other strands.
- Suggestions for the planning and conducting of lessons in that strand. These include lesson descriptions, suggestions on the use of worksheets and manipulative materials, general remarks relating to certain sequences of lessons, etc.

The suggested lessons are grouped in chapters, all the lessons described in the same chapter having some particular feature(s) in common. Because of this commonality of purpose, it is possible to precede each chapter by an introduction which indicates where the lessons in that chapter are going and on occasion gives a few more details concerning the mathematical underpinnings of the lessons described in the chapter than are contained in the lesson descriptions themselves.

Many of the suggested lessons require student worksheets for individual work. Classroom sets of these worksheets are printed and packaged separately from this guide. However, an answer key for each of the worksheets is included in the guide. Worksheets are numbered consecutively within each strand and their answer keys can be found at the end of the lessons for which the worksheets are needed. For example, an answer key for Worksheet L22 is at the end of Section 2.3 of the Languages of Strings and Arrows strand since it is suggested that this worksheet be used with the lesson described in that section.

In order to accommodate the various ability levels in a given class, the worksheets generally are written at several levels of difficulty denoted by stars. "*" represents an easy worksheet, "**" represents an average level worksheet, and "***" and "****" represent difficult level worksheets. All students should begin with the "*" level worksheets for a given lesson or sequence of lessons and proceed as far as possible through the other levels. In classes containing the usual range of abilities, most of the students should succeed with the * worksheets; the ** worksheets should be appropriate for about two-thirds of the class; and approximately one-third of the students will find the *** and **** worksheets challenging and enjoyable to do. No one should ever be <u>required</u> to attempt <u>all</u> the worksheets provided for a given lesson. Each student should progress as far as his or her abilities allow.

There are various other items which are (or could be) used in the program. Among them are the following: various manipulative materials such as translators, rulers, grid boards, and so on (these are described in more detail in the lessons in which they are used); several workbooks, storybooks, and story-workbooks (these are described in more detail in Sections 2 (b) and (e) of this General Introduction, and in the introduction to the Workbook strand); a book entitled "Math Play Therapy", which includes a collection of suggested small group activities that could be instrumental in stimulating an open and positive attitude toward mathematics on the part of those of your students who find the subject difficult. See the introduction to "Math Play Therapy" for suggestions as to its use in the program.

b) Role and Use of Workbooks

It is suggested that approximately one day each week be scheduled for students to do individual work in designated workbooks. The workbooks are designed to review ideas encountered in the content strands and to apply these ideas to new situations posed as stories. Many of the workbooks are written with problems of varying levels of difficulty in order to accommodate differences in students' abilities and interests.

More detailed descriptions of the workbooks, their purposes, and answer keys are given in the Workbook strand. There you will also find suggestions for keeping records of student progress.

c) The Spiral Development

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson is an end in itself. It is neither intended nor expected that every student will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations and strands from day to day rather than to continue with one type of situation or piece of content until so-called mastery has occurred. Thus, we suggest that the lessons be scheduled so that a different topic is studied each day of the week. In a typical week, for example, your class might work with numerical games one day, probability the next day, workbooks the third day, numerical relations the fourth day, and geometry the fifth day.

This spiral development, in which a student experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging, reflects the fact that learning is not a linear process. Not only do different children learn at different times and different rates, but even on an individual level learning can occur in quantum leaps that take place at times when you least expect them. The spiral development gives each student a new chance to catch on to an idea at each turn of the spiral.

Of course, acceptance of the CSMP pedagogy requires a certain act of faith on your part. If you have been accustomed to staying with a given topic until everybody (or nearly everybody) in the class knows it "cold", then you may be initially very reluctant to leave a lesson at the end of the time suggested, believing that several students "haven't got it yet". By keeping the spiral idea in mind, you may become more comfortable with these time schedules. Although it may appear to you that little learning is taking place in a sequence of lessons, you may be surprised that suddenly even your slowest students are understanding the ideas when they encounter them later in the spiral. Whereas with a mastery approach, presented in a purely individualized mode, slow students often spend most of the school year trying to "master" low level skills (with generally poor results), the spiral approach gives these students the opportunity to benefit from the richness of the situations presented throughout the curriculum. In this manner students are allowed to become involved in interesting and enjoyable situations which are modeled by important mathematical notions. They are not deprived of a variety of experiences just because they haven't mastered every bit and piece along the way. Therefore, in the long run, their interest in, understanding of, and even facility with mathematics has a much better chance to develop.

d) Evaluation

On the basis of the preceding discussion, it should come as no surprise, as stated earlier, that CSMP takes the position that learning often stops when the purpose of a lesson or sequence of lessons is the mere mastery of skills. Furthermore, we do not believe it is desirable or reasonable to attempt to assess student mastery of skills from day to day or even from week to week. As we said before, learning has a habit of occurring when you least expect it, so how does one determine when everyone in a class should have mastered a specific skill or attained understanding of a particular concept as a result of his or her experiences? We know of no method for measuring what or how much has been learned from any one situation except in the most trivial cases. On the other hand, teachers, parents, and even the students themselves can benefit from some continuous monitoring process that records involvement in activities and assesses progress through the program. Indeed, it is possible to

monitor your students' progress without contradicting the statements made above; all of you have your own ways of doing this. Such a process is suggested in the introduction to the Workbook strand. Of course, you should also continue to use any monitoring devices with which you feel comfortable.

As far as traditional computational skills are concerned, the standardized test data collected during the extended pilot test stage of the development of this curriculum indicate that CSMP classes generally do better than average on the items which test "basic" skills.

About 75% of the lessons for the CSMP Intermediate Grades Program involve numerical concepts. The situations do not involve just the integers (as is generally the case at this level), but introduce the students to many other citizens of the world of numbers at as early a stage as possible. You should not infer from this that your students are expected to have mastered the arithmetic of the whole numbers, the integers, the decimal numbers, and the rational numbers before they are ready for Part IV of the CSMP Intermediate Grades Program. On the contrary, it means that the students will become more and more friendly and at ease with the various types of numbers and will develop their own methods for calculating with them long before some standard mechanical algorithms are introduced.

It is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying, and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of natural synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

We therefore strongly urge you to follow the suggested schedule of lessons, teaching the algorithms as they come, but not giving them more emphasis than they deserve. Skill in mental arithmetic and estimation, however, is extremely important in our opinion and should be promoted at every opportunity.

While CSMP is opposed to the philosophy underlying standardized testing, we realize that students as well as teachers are under extreme pressure to perform in certain ways with regard to such tests. If your students or their parents express concern about the apparent lack of emphasis on mastery of standard algorithms (such as long division by three or four digit numbers), they should be reassured that CSMP students have a definite advantage when it comes to applying mathematics to novel situations, strategic thinking, organizational ability, and understanding of the methods of mathematics. Because CSMP feels it is wrong for such tests to dictate emphasis on the mastery of standard algorithms and that drill for algorithmic mastery for its own sake defeats CSMP's purposes, you will find that even after algorithms have been introduced into the CSMP curriculum the role they play will be as tools for doing other interesting things. Therefore, if you feel pressured by the tests or otherwise to put more emphasis on drill activities, then we suggest you use any supplementary materials that have proved successful in the past to provide this extra practice. We leave scheduling choices for these kinds of activities to your judgement.

e) Role and Use of Storybooks and Story-Workbooks

Among the instructional materials for the CSMP Elementary School curriculum is a collection of mathematical storybooks, STORIES BY FREDERIQUE. This collection consists of 24 stories for young people, each of which is presented in the format of a newspaper for children. Nearly every story is printed in color on tabloid size ($17\frac{1}{2} \times 11\frac{1}{2}$)* newsprint and is intended to appeal to youngsters in the 5-14 age range. The stories are completely independent of each other and need no external explanation. They may be used for individual or collective reading, at home or at school.

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*Six of the stories are printed on $8\frac{1}{2} \times 11$ newsprint rather than the tabloid size.

Each of the stories is portrayed by means of the mathematical languages of arrows, dots and strings which reinforce the ideas of the story. Three of the stories (TWO BY TWO, THE WEIRD STORY OF 24, and SUMMER SCHOOL IN THE OLD DAYS) also use the language of the Minicomputer. The amazing versatility of these nonverbal languages is exploited to the full.

Young readers are personally involved in each story. They have to answer questions and use their imaginations to invent additional adventures for the characters in the story. They learn that there is room in the make-believe world of mathematics for exercising the aesthetic sense and for expressing emotions.

Twelve of the stories are intended for very young children (5 to 8 years of age). Here the focus is on the anecdotal aspect of the story. One of the goals of these twelve books is to make the children familiar with nonverbal mathematical languages and to leave them with the impression that these languages are as natural and useful as the usual language.

Nine of the remaining stories concern numbers and their relationship with the world. These particular stories are about a boy and some of his friends who just happen to be numbers. The personification of numbers introduces the "games" of mathematics into one of the most important domains of childhood -- the world of fantasy. Free from the usual constraints so often placed upon them, the numbers invent marvelous games and dance to the tunes of vividly colored arrows. These games are never frivolous. There are rules to be followed and as a result, orderly patterns emerge as the numbers play or dance.

Of the remaining three books, two are detective stories introducing some interesting mathematical problems, and one emphasizes the various and very different aspects of a simple straight line.

The titles of the books in the collection and the intended age range for each story are:

Ages 5 to 8

THE PLAYFUL NUMBERS
81 ROSES
I AM A VERY HAPPY BOY
ONE OUT OF SEVEN
THE HAPPY PUPPET
TWO BY TWO
THE LITTLE DREAMER
THE LITTLE DONKEY
THE MAGIC BOX
THE BABY IS BORN
THE WEIRD STORY OF 24
THE OLD SHOEMAKER

Ages 8 to 12

WHERE'S MY NOSE?
SINGING FRIENDS
DANCING FRIENDS
THE LIVING LINES
I AM NOT MY NAME
NABU WINS AN AWARD
THE SQUARE TRAP
SUMMER SCHOOL IN THE OLD DAYS

Ages 10 to 14

THE HIDDEN TREASURE
A VERY STRANGE NEIGHBORHOOD
ELECTION IN THE NUMBER WORLD
A VALENTINE MYSTERY

In the classroom set of materials for Part III of the Intermediate Grades Program, two copies each of the following storybooks are provided: TWO BY TWO, WHERE'S MY NOSE?, SINGING FRIENDS, DANCING FRIENDS, THE LIVING LINES, I AM NOT MY NAME, THE SQUARE TRAP, and NABU WINS AN AWARD. They should be put in the class library for students to read whenever they like. These are books that were in the third grade program and in previous parts of the Intermediate Grades Program and that students may enjoy reading from time to time. If Part III is being used in the second semester, following Part II in the first semester, these storybooks are not included with the Part III materials since they were provided with Part II materials.

Besides the collection, STORIES BY FREDERIQUE, another collection, THE CSMP LIBRARY, has been developed as part of the instructional materials for the CSMP Elementary School Program. This is a series of story-workbooks which combines the spirit of the storybooks and the workbooks into a single integrated experience for the children, engaging them in reading about and

doing mathematics in stimulating, challenging story situations. Two of these, NUMBER 1,000's DREAM and SEVEN SECRET NUMBERS are incorporated into the Workbook strand. Another, SHUNDA'S NEWSSTAND, is the basis for four lessons in the Probability and Statistics strand.

f) Role and Use of Hand-Calculators

The electronic hand-calculator, if used creatively, can have a remarkably beneficial effect on the students' interest in and understanding of arithmetic, and its use can be a contributing factor in the improvement of their capabilities in that area. The reasons for this are many and varied, but among them are certainly the following:

> By removing the burden of mechanical computation from the children's shoulders, the calculator frees them to concentrate on the thinking that is a necessary part of the solution to any meaningful problem.
>
> The instant feedback provided by the calculator enables the children to indulge in making and testing conjectures, an area into which they would not otherwise venture because the quantity of computation involved would be too forbidding.

This pedagogical tool is particularly useful in the introduction of new numerical notions such as negative integers, decimal numbers, multiples of a given whole number and so on, because through suitably chosen, rich situations, these concepts can be made to arise in a natural way.

Two battery powered hand-calculators are recommended materials for Part III* and many different kinds are available from CSMP as a distributor for National Semi-Conductor and Texas Instruments. Any of the products of these two companies can be obtained at discounted prices.

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* Some Part III lessons require two hand-calculators in the class. If you already have calculators, be sure to check your calculators for algebraic logic and constant mode (as explained on the following page).

While the two hand-calculators are adequate for the lessons in Part II, the ideal situation would be for each student to have one. Encourage the students to bring calculators from home. The hand calculators that are available from CSMP have algebraic logic and function keys which are permanently in a constant mode. You should check to be sure that any calculators brought in by students also have these characteristics.

- Algebraic logic. This means that the calculator responds to instructions given in the order in which they are usually written. To check that this is the case, press [2] [+] [3] [=] . If you have "5" on the display, then all is well.

- Constant mode. Some calculators have a switch so that the calculator can be set either in a constant mode or in a chain mode. If your calculator lacks a switch, it is quite likely that the calculator has an automatic constant. For calculators with an automatic constant, the first number entered during multiplication is usually the constant, while the second number entered during the other three operations (addition, subtraction, division) is usually the constant.

  To check your calculator for the constant mode operation of [+] is a simple procedure. Press [0] [+] [2] [=] [=] [=] and so on. If the number on the display is increasing by two at each depression of [=] , then [+] is operating in a constant mode. Check the constant mode operation of [-] in a similar fashion.

  To check the constant mode operation of [x] (where the first number entered will be the constant) press [2] [x] [1] [=] [=] [=] and so on. If the number on the display doubles at each depression of [=] , then [x] is operating in a constant mode.

While it would be possible to use calculators without these characteristics in the lessons, the adjustments that would be required in the instructions make the calculators unsuitable.

In several lessons the hand-calculator relations [+] [n] [=] ... and [+] [=] ... are explored quite extensively. Even if your calculators have algebraic logic and the constant mode, it may be necessary for you to check to see if your calculators operate in the way that is described in the lessons.

If you find that your calculators do not operate in this manner, then try letting

[+] [n] [=] ... mean

[+] [n] [=] [+] [n] [=] [+] [n] [=] ...

and

[+] [=] ... mean

[n] [+] [0] [=] [+] [n] [=] [+] [n] [=] ...

where n is some starting number.

Futhermore, with some calculators there is a correspondence between a multiple of a number and the number of times [=] is pressed in the [+] [=] ... relation. For example,

Starting with "4" on the display and pressing [+] [=] yields 4 (= 1 x 4). Pressing [=] again yields 8 (= 2 x 4); pressing [=] again yields 12 (= 3 x 4); pressing [=] again yields 16 (= 4 x 4); pressing [=] again yields 20 (= 5 x 4); and so on .

Notice that when [=] has been pressed 5 times, the fifth positive multiple of 4 (i.e., 20) is displayed.

The ability to make predictions about numbers based on the number of times [=] is pressed is an activity that occurs several times in the World of Numbers strand. For this reason, it is convenient if all of the calculators that you use for the lessons work in this way. Frequently, calculators that do not behave in the manner described above will differ by one less (i.e., the fifth positive multiple of 4 appears after pressing [=] four times). This discrepancy can be overcome by asking students to count the pressing of [+] as the first press, then the pressing of [=] as the second press, and so on.

3. Guidelines for Organizing the Program

The content of the lessons in Part III is designed to afford you the utmost flexibility in teaching the program successfully. It is a truism to say that your students this year are very different, both individually and as a class, from the students you taught last year, and indeed from other students in any other CSMP class. And your own teaching style and interests are probably very different from those of your colleagues. It is difficult for those of us who develop curricula to guess correctly as to which topics will be very easy for your students and which lessons will require more instructional time. You are the only one who will be able to judge that as you begin using the materials. This program is not meant to be a static sequence of exercises to be followed rididly; the schedule and design of lessons in this guide should be regarded as suggestions; they simply reflect our own experience teaching the materials. You and your students will make the program dynamic as together you explore the mathematical ideas and follow your own interests and inclinations.

a) Use of Schedules

If this is the first time you are teaching a CSMP class and you would like specific suggestions for organizing the lessons, we offer some guidelines and various suggested schedules of lessons. Please remember that our suggested sequence is but one of many that could have been assembled. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate.

The master schedule which is included at the end of this section was developed for a hypothetical fifth grade class of average ability, and it illustrates some important ideas about organizing the program. You will notice that several different strands are scheduled each week and that even within a strand the topics are interspersed. This was done to carry out the idea of a spiral curriculum. If you organize your own schedule it is best to follow a similar pattern rather than to spend two or three full weeks, for example, "covering" one particular topic such as composition of relations or decimal numbers. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Workbook strand, and the other two days should be shared by the P, G and L strands.

Within each strand (or part of a strand) we provide a more detailed schedule aimed specifically at that strand (or part of a strand). In the case of the P strand which has only four lessons, the schedule shows how these lessons are included with the L lessons. The schedule which appears in each of the L and P strand books has the lessons of the other strand shaded over so that only the lessons of the strand itself stand out. In each schedule, a block represents one full period of approximately 45-50 minutes duration. It may very well be, however, that you and your class will spend more than one period on some lessons. Exercise your own professional judgment in deciding when to stop one particular lesson and when to take it up again. Just be sure that you do not insist that every single child understands every single aspect of a lesson before you go on. It is better to stop a lesson at a point when student interest is high and a problem remains unsolved than to teach so methodically that everyone in the class understands everything, but is completely bored.

Inside each block of the detailed strand schedules, the lesson of the day is designated in several ways. First, there is listed a number which indicates its suggested chronological order among the lessons in that strand. Hence, "5N" indicates that the lesson in question is suggested as the fifth lesson in the N (World of Numbers) strand. This identifying number also appears in the upper outside corner of each page in the description of that lesson, thus enabling a lesson to be located simply by flipping through the pages of the strand book. Second, you are told on which page of the strand book the description of the lesson in question begins. Third, as a double-check, you are given the section number and title of the lesson. Next to each block in each schedule there is a list of the materials you and the students will need in order for you to be able to teach the corresponding lesson.

Please bear in mind that the model schedule was designed for a hypothetical class with the usual range of abilities. In order to be successful with the program, you should adjust the schedule to the ability, maturity, and interests of your class. For example, you may wish to extend a lesson or a workbook for an additional period beyond those scheduled.

b) Use of Adjustment Days

From time to time an adjustment day appears in the schedule. Since we have intended that Part III would be the mathematics curriculum for one semester of the Intermediate Grades and a usual school semester is 18 weeks, the schedule has been designed to indicate 18 weeks of lessons. However, it is also usual for there to be days when no mathematics lesson is taught; e.g., parent-teacher conference days, field trip days, snow days, and so on. These adjustment days should account for such non-class days, and should also give you opportunities to do many kinds of things. You could use an adjustment day to review a lesson you feel needs review, or to complete an unfinished lesson, or to extend some interesting ideas from completed lessons. You could also use an adjustment day to give your students time to become more familiar with the manipulative materials in the program, such as the translators or the coordinate grids or storybooks. The students may especially enjoy playing games such as Minicomputer Golf, Minicomputer Tug-of-War, Guess My Rule, or the String Game, and an adjustment day could give them another opportunity to play a game. You may wish to use an adjustment day to do some mental arithmetic exercises, or to teach a topic in mathematics you think would be especially interesting or useful to your class.

SUGGESTED SCHEDULE OF LESSONS

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART III

WEEK	The World of Numbers	Languages of Strings and Arrows Probability and Statistics
1	N1.1 Arrow Roads #1 Np. 3	L1.1 Composition of Relations Lp. 5
2	N3.1 Hand-Calculator Sentences Np. 65	L2.1 String Game Analysis #1 Lp. 90
3	N5.1 Hand-Calculator Relations with Decimals Np. 170	L1.2 Addition and Multiplication with Ten Friends Lp. 21
4	N3.2 Spies and Bridges #1 Np. 76	L1.3 Powers of ⊗2 Lp. 32
5	N1.2 Arrow Roads #2 Np. 11	L2.2 String Game Analysis #2 Lp. 95
6	N7.2 Fractions #2 Np. 250	L1.4 Arrow Messages Lp. 43
7	N5.3 Arrow Roads with Decimals Np. 192	L1.5 The Kite Lp. 52
8	N6.2 Multiplication #2 Np. 227	ADJUSTMENT DAY
9	N4.2 Minicomputer Golf Preparation #2 Np. 154	L2.3 String Game Analysis #3 Lp. 105

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART III

Workbooks	The World of Numbers	Geometry and Measurement
A Variety of Problems 1 Wp. 1	N2.1 String Pictures Np. 23	ADJUSTMENT DAY
A Variety of Problems 1 Wp. 2	N4.1 Minicomputer Golf Preparation #1 Np. 144	G1.1 Length #1 Gp. 5
A Variety of Problems 2 Wp. 13	N6.1 Multiplication #1 Np. 214	G2.1 Projection #1 Gp. 50
A Variety of Problems 2 Wp. 14	N7.1 Fractions #1 Np. 241	G1.2 Area #1 Gp. 12
Arrow Game #1 Wp. 25	N5.2 Decimals on the Number Line Np. 183	ADJUSTMENT DAY
A Variety of Problems 3 Wp. 61	N8.1 Decimal Distance #1 Np. 312	G2.2 Projection #2 Gp. 60
A Variety of Problems 3 Wp. 62	N3.3 Spies and Bridges #2 Np. 90	G2.3 Couples and Equipollence Gp. 68
Number 1,000's Dream Wp. 73	N9.1 Division #1 Np. 332	G1.3 Length #2 Gp. 16
Number 1,000's Dream Wp. 83	N8.2 Decimal Distance #2 Np. 322	ADJUSTMENT DAY

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART III

WEEK	The World of Numbers	Languages of Strings and Arrows Probability and Statistics
10	N7.3 A New Name for One-Third Np. 258	L1.6 Addition and Multiplication with Eighty Friends Lp. 60
11	N7.4 Dangerous Roads Np. 273	ADJUSTMENT DAY
12	N2.2 Arrow Pictures for "Is a Positive Divisor of" Np. 42	L2.4 String Game Analysis #4 Lp. 111
13	N4.3 Minicomputer Golf Np. 163	L1.7 The Kite with Four Tails #1 Lp. 72
14	N9.3 Division #3 Np. 351	L1.8 The Kite with Four Tails #2 Lp. 78
15	N10.2 Who is Pif? Np. 370	P1.1 Shunda's Newsstand #1 Pp. 2
16	N3.4 How Many Permutations? Np. 104	P1.2 Shunda's Newsstand #2 Pp. 7
17	N3.5 Random Art #1 Np. 118	P1.3 Shunda's Newsstand #3 Pp. 12
18	N3.6 Random Art #2 Np. 132	P1.4 Shunda's Newsstand #4 Pp. 18

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART III

Workbooks	The World of Numbers	Geometry and Measurement
A Variety of Problems 4 Wp. 91	N9.2 Division #2 Np. 342	G2.4 Translation #1 Gp. 72
A Variety of Problems 4 Wp. 92	N10.1 Who are Zig and Zag? Np. 360	G2.5 Translation #2 Gp. 79
Seven Secret Numbers Wp. 105	N7.5 Guess My Rule Np. 281	G2.6 Translation #3 Gp. 82
A Variety of Problems 5 Wp. 119	N2.3 Hand-Calculator Relations Np. 51	ADJUSTMENT DAY
A Variety of Problems 5 Wp. 120	N7.6 A New Name for One-Half Np. 291	G1.4 Area #2 Gp. 19
Arrow Game #2 Wp. 41	N7.7 Fractions #3 Np. 305	G1.5 Area #3 Gp. 32
A Variety of Problems 6 Wp. 131	N5.4 Decimals on the Minicomputer Np. 199	G1.6 Area #4 Gp. 40
A Variety of Problems 6 Wp. 132	N10.3 Who is Jack? Np. 378	ADJUSTMENT DAY
Arrow Game #3 Wp. 53	ADJUSTMENT DAY	G2.7 Lattice Construction Gp. 85

4. Commonly Asked Questions about the CSMP Program - and Suggested Answers.

As you teach the CSMP curriculum (or even as you prepare to teach it) there will surely be some basic questions that come to your mind. They are the kind of questions that not only you as a teacher, but also parents and supervisors, will ask about any curriculum that appears to be different from the usual program.

The following questions and answers are listed as an aid to you in understanding our curriculum and in responding to those who request information about it.

1. Q: For what sort of student is the CSMP elementary curriculum designed?

   A: It is designed for all children in the schools. Each learning situation is designed to involve all students at their own levels in such a way that the slowest can succeed in dealing with some aspects of the situation that challenge them. In fact, it has been found that such a "pedagogy of situations" (see Question 2) is particularly effective with the so-called slow learners, and recent work suggests it might be effective also with educable mentally handicapped children.

2. Q: Briefly, how does CSMP differ from traditional mathematics program?

   A: The core content of CSMP is not revolutionary. The program aims to inculcate notions and facts which have always been considered important in mathematics. The major differences are in method. Traditional teaching takes pride in disciplining the student to be submitted to the usual algorithms. With CSMP the student is involved in real mathematical activity, with understanding at his or her level, so as to master the situation.

   For that purpose CSMP uses new teaching aids which are essential nonverbal languages and which are themselves both pure and applied mathematics. These mathematical objects (graphs, arrows, etc.) are used as aids rather than as ends in themselves.

   A pedagogy of situations is employed to permit the pupil to participate

with creative freedom in the learning process. The student is faced with situations under which, by spontaneous reaction, he or she is led to an important idea in mathematics. Thereby, the child learns to mathematize situations, which is of fundamental importance whenever mathematics is applied.

CSMP also makes use of a spiral approach rather than the mastery approach found in most mathematics curricula. (See Question 13)

3. Q: Isn't this the "new math"?

A: There is a current popular belief that all school mathematics today is either "new math" or "old math". Neither of these labels is well defined. "New math" usually denotes the programs that heavily emphasize the teaching of sets; introduce technical mathematical language for concepts such as "commutativity", "associativity", "distributivity", and so on; and that teach "concepts" before "skills". The "old math" refers to programs of the "good old days": memorization of the multiplication tables and complete emphasis on paper and pencil drill in calculations.

The CSMP curriculum is definitely not the "new math" nor the "old" by any definition. It uses no technical language, yet achieves the understandings sought by the "new math". It uses no repetitive drills, yet elicits the ability to calculate stressed by the "old". But these achievements are just by-products of a more pervasive goal: to help children use their curiosity, their intuition, their powers of reasoning to formulate and think about interesting problems and situations.

4. Q: Is the student really studying mathematics in the CSMP curriculum?

A: Mathematics has a subject matter which involves much more than calculation. CSMP believes that the number system is very important and that it is essential to introduce the pupil to the world of numbers. However, the intelligibility of that world is not essentially a matter of computation. The teaching aids of CSMP give concrete support to some important internal relations in the world of numbers

as well as some logical reasoning. The deep understanding of mathematics is not always verbal but is very often pictorial and CSMP tries to respect that aspect.

5. Q: Why use the languages of strings, arrows, and the Papy Minicomputer?

A: It is surprising that the language of strings was not used earlier and more extensively in the teaching of mathematics. Mathematicians have used strings for a very long time and they support notions so fundamental and natural that they go back to Aristotle. The scope of the string diagrams is broader than mathematics, the string diagram being a marvelous instrument for logical thinking. String diagrams permit the understanding of a situation at a glance.

Arrow diagrams are probably the most important teaching aid in CSMP. They are used everywhere in our daily lives; signals, advertisements, directions, and so on. Why not use them in teaching mathematics? Arrows are a pictorial way to represent relations. Relations have always been important in mathematics, but in the past seemed to be something rather abstract, elusive, and mysterious. With arrows they become vivid, concrete, and easy to understand.

The Minicomputer allows very young children to become familiar with numbers and operations on them (especially large numbers and the more difficult calculations) before they are able to work with them on paper. Later the Minicomputer is used as a vehicle for posing situations involving mental arithmetic and for examining the workings of the arithmetic operations. It provides the experience necessary so that rather than memorizing an algorithm and applying it to situations, the situations give rise to the algorithm.

6. Q: If the students use the Minicomputer, will they be able to calculate without it?

A: This question suggests that the only purpose of the Minicomputer in the CSMP curriculum is to perform calculations. To be sure, the Minicomputer does enable very young children to deal with "large"

numbers and perform relatively difficult calculations long before they can write the paper and pencil algorithms for the calculations. But very soon these experiences with the Minicomputer suggest how the paper and pencil algorithms come about, and from that moment the Minicomputer is no longer regarded as a calculating tool. Instead, for the remainder of the curriculum, the Minicomputer assumes its primary roles: as a source of very interesting problems and games, as an effective vehicle for stimulating mental arithmetic, as a teaching device for introducing more advanced topics such as the negative numbers and the decimal numbers, and, in general, as an alternate representation of numbers.

7. Q: What is the CSMP position on hand-calculators and what role do they play in the program?

A: Hand-calculators should be a part of any mathematics curriculum. As more and more children have hand-calculators, these tools will become incorporated in the elementary and secondary curricula in a natural way with uses very similar to those of the Minicomputer. In some cases they may be used as a tool for mechanical computation where the difficulty of the calculation might turn an otherwise interesting problem into an exercise in drudgery. In other cases they may be used as the context for posing situations leading to the discovery of important mathematical properties or providing opportunities for interesting exercises in mental arithmetic.

Hand-calculators are of particular benefit to "slow" students. Many slow learners have mechanical difficulties rather than problems in thinking about the situation. The hand-calculator alleviates this problem.

Some lessons in the CSMP curriculum require only two calculators, while others suggest that each student have one. Lessons of this latter sort, however, also include suggestions on how they may be modified since not every student may have a hand-calculator. In either situation, students with access to hand-calculators should always be encouraged to bring them to class.

8. Q: Why is there a lack of drill and little emphasis on "skills"?

A: CSMP holds that training for rote skill should never precede understanding. The avowed purpose of drill in arithmetic is to acquire the "skills", that is, the skills of calculation. Certain numerical skills <u>are</u> essential, but, unfortunately for school children, the usual drill techniques for acquiring them are dull, stultifying and sometimes even counterproductive. The CSMP curriculum <u>appears</u> to place little emphasis on "skills" because it does not resort to drill. But as a matter of fact, the essential skills of both mental and written calculation are learned almost inadvertantly in response to the many situations where the ability to calculate is needed in order to respond to the situation: to win the number game, to identify the secret number of the detective story, or to build a number road. Always, calculation is considered as a tool for doing interesting things, rather than a chore done for its own sake. See also the comments on Evaluation, Section 2 (d) of the General Introduction.

9. Q: How do students learn to use mathematics in practical situations?

A: For students of elementary school age, a situation that intrigues them and catches their imagination or their curiosity will certainly motivate them to learn whatever is necessary in order to deal with the situation. But situations that adults consider "practical" very often are outside the realm of interest of school children and do not provide motivation for learning. CSMP has found that students will indeed learn to use mathematics in situations they consider worthy of their attention. As they approach adulthood they will soon consider "practical" situations to be most worthy of their attention, but in the meanwhile, they are more interested in games, fantasy, puzzles, stories, and so on.

10. Q: Why are we teaching probability?

A: Next to arithmetic, probability has the most direct application to life in such areas as economics, meteorology, data processing,

insurance, most of the natural sciences, and most of the social sciences. Probability differs from many areas of mathematics in that it is applied directly. One does not need to combine knowledge of probability with any deep theoretical knowledge in order to apply it to "real world" problems. One is exposed to statistics nearly every day of one's life, and the better developed one's intuitive sense of probabilistic situations is, the less likely one is to be "duped" by statistics.

11. Q: How are telling time, money, and the metric system treated?

A: Telling time is not included in the CSMP program. It is our belief that telling time is not a part of mathematics, although it is frequently included in the mathematics curriculum. We do not feel that we could improve on the methods currently used, and, for this reason, we urge you to teach this topic using those methods and materials that have proved successful for you in the past.

The topic of money is first introduced in the CSMP curriculum in lessons involving combinatorics. It is treated again in the Upper Primary and Intermediate Grades as a support for the teaching of the decimal numbers. Should the students need more exposure to this topic, teachers should feel free to develop supplementary materials in order to provide extra practice.

The metric system is introduced in the second and third grades, and provides all standardized units of measure for the Geometry and Measurement strand. We recommend that you do not ask the students to convert from the English to the metric system or vice versa (inches to centimeters, yards to meters, Celsius to Fahrenheit, grams to ounces, and so on). Since the United States has not yet "gone metric", your students will still need to be familiar with the English system. We would suggest that you use whatever materials and methods you have found successful in the past in teaching this topic, but we repeat our recommendation that you not ask the students to convert from one system to the other.

12. Q: What is the role of storybooks and story-workbooks in the program?

A: The storybooks, "Stories by Frederique" are a collection of 24 stories for young people, each of which is presented in the format of a newspaper for children. They are intended to appeal to children in the 5-14 age bracket. The stories are completely independent of each other and need no external explanation. They may be used for individual or collective reading, at home or at school.

The story-workbooks, as their name implies, are a cross between storybook and workbook. They present more problems for the student to solve than do the storybooks, but within a story setting. The story-workbooks comprise the collection called "The CSMP Library".

Each of the stories is portrayed by means of the mathematical languages of arrows, dots, and strings which reinforce the ideas of the stories. The young readers are personally involved in each story. They must answer questions and use their imaginations to invent additional adventures for the characters in the story. They learn that there is room in the make-believe world of mathematics for exercising the aesthetic sense and for expressing emotions.

Among the goals of the various storybooks and story-workbooks are the following: to make the children familiar with nonverbal mathematical languages and to leave them with the impression that these languages are as natural and useful as the usual language; to introduce some mathematical games; to present detective stories which introduce some interesting mathematical problems; to provide reinforcement for all students and especially for the slower learners.

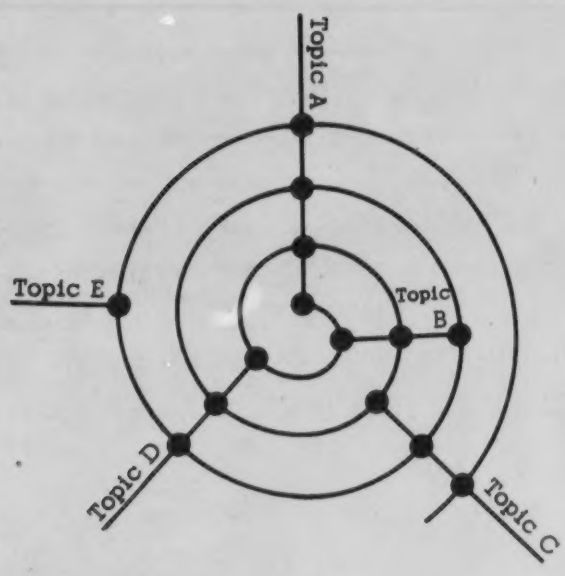
13. Q: What should I do if some of the students don't fully understand a lesson?

A: Much of the following appeared in Section 2 (c) but is repeated here for emphasis and your convenience.

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson

is an end in itself. It is neither intended nor expected that each child will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations and topics from day to day rather than to continue with one type of situation or piece of content until so-called mastery has occured. Students at this age do not seem to enjoy staying on the same topic for a very long time, but prefer to be involved in a variety of situations. Thus, the lessons are scheduled so that different topics will be studied during each week and similar situations appear again and again at different times and levels in a spiral development.

If a spiral curriculum seems strange at first, consider the following illustration: Imagine an academic subject that could be divided into five subtopics. Then imagine a child's progress being represented by a spiral, starting at the center and traveling outwards in the direction of increasing degrees of maturity and sophistication. In the diagram, an intersection point between the spiral and a topic line indicates a lesson on that topic.



The child is first introduced to Topic A, then to B, then to D, followed by a slightly more sophisticated lesson on A, and so on. Topics C and E are not introduced until a little later in the spiral, while Topic B may be of a more terminal nature and is considered completed after a few lessons.

This spiral development, in which a child experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging, is wholly consistent with CSMP's pedagogy of situations.

Acceptance of the CSMP pedagogy requires a certain act of faith on your part. If you have been accustomed to staying with a given lesson until everybody (or nearly everybody) in the class knows it "cold", then you may be initially very reluctant to leave a lesson at the end of the time suggested, believing that several children "haven't got it yet". By keeping the spiral idea in mind, you may become more comfortable with these time schedules. Although it may appear to you that no learning is taking place in a sequence of lessons, you may be surprised that suddenly even your slower students are understanding the ideas when they encounter them later in the spiral. Whereas with a mastery approach, presented in a purely individualized mode, slow students often linger most of the school year trying to "master" low level skills (with generally poor results), the spiral approach gives these students the opportunity to benefit from the richness of the situations presented throughout the curriculum. In this manner children are allowed to become involved in interesting and enjoyable situations which are modeled by important mathematical notions. They are not deprived of a variety of experiences just because they haven't mastered every bit and piece along the way.

14. Q: Should I stick to this approach even if some students are visibly upset over not having comprehended a lesson?

A: If a child is upset, that child has been made to feel upset, probably by a reversion to the mastery approach. In the spiral approach each situation offers something for each child. The child will only be upset if your attitude indicates that there was more to the situation than the child was able to understand. Every experience in life may be understood at different levels. This is naturally accepted by people except when it is indicated to them that they should have a deeper understanding.

Rather than abandoning the spiral approach, if a child seems upset, stick more closely to it. Find a way for the child to express what understanding he or she does have and accept that expression in a positive manner.

15. Q: What provisions are made for the slow learner?

A: The spiral approach provides for the slow learner by introducing situations that each child can experience and learn from at his or her own level. The next time the spiral reaches a similar situation each child's level of understanding will be somewhat higher but again the experience and learning take place at each child's own level. This is as true for the slow learner as it is for any other student.

The slow learner should participate in math with the other students in the class but might benefit from additional small group work. The additional time is not to rehash or drill on the situations presented in class, but rather to provide additional experiences for these students. See the book "Math Play Therapy" Volumes I and II for suggestions for working with slow learners.

16. Q: While providing for the slow learner, how do I challenge the brighter students?

A: The spiral approach is very good for addressing the needs of all levels of students and gives students a chance to excel in mathematics. The situations presented are often open-ended and allow the student to experience and understand them at as high a level as the child is capable.

In order to accommodate the various ability levels in a given class the worksheets generally are written at several levels of difficulty denoted by stars. "*" represents an easy worksheet, "**" represents an average level worksheet, and "***" and "****" represent difficult level worksheets. All students should begin with the "*" level worksheets for a given lesson or sequence of lessons and proceed as far as possible through the other levels. In classes containing the usual range of abilities, most of the students should succeed with the one-star worksheets; the two-star worksheets should be appropriate for about two-thirds of the class; and approximately one-third of the students will find the three- or four-star worksheets challenging and enjoyable to do. The workbooks are written with problems at different levels of difficulty in the same booklet, but arranged so that the first pages of the book are appropriate for all students. Students should work through as much of a workbook as they are able during the lessons devoted to the workbook.

If your school is flexible, there is also the possibility of letting your students take math with a higher or lower grade level class. For example, your very bright second graders could go to a third or fourth grade class for math instruction.

17. Q: Is it necessary to follow the suggested schedule exactly?

A: This program is not meant to be a static sequence of exercises to be followed rigidly. The lessons in this guide are suggestions and the time allotments are merely approximations based on our own experience teaching the materials. If this will be the first time you are teaching the program and you would like specific suggestions for organizing the lessons, we offer some guidelines and a suggested schedule of lessons. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate.

It is probably best to follow a one-day, one-strand pattern when you organize your own schedule, rather than to spend two or three full weeks "covering" one particular topic. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Workbook strand, and the other two days should be shared by the P,G, and L strands.

Exercise your own professional judgment in deciding when to stop a particular lesson. It is better to stop a lesson when student interest is high than to teach so methodically that everyone in the class understands everything, but is completely bored. In order to be successful with this curriculum, you should adjust the schedule to the ability, maturity, and interests of your class.

18. Q: How do I handle individual students who are new to CSMP?

A: In most cases, the differences between new and veteran students tend to fade rather quickly. One exception might be in lessons involving the Minicomputer. Veteran students are usually adept with the Minicomputer and often are very good tutors for the new students in the use of this abacus. In addition, the storybook "Two By Two" is suggested as an aid for new students who are unfamiliar with the Minicomputer. Some of the storybooks and story-workbooks written for younger children can be used to introduce new students to the languages of strings and arrows.

19. Q: How do I handle whole classes who are new to CSMP?

A: CSMP does not assume that Kindergarten and first grade students have any prior experience with CSMP. The Kindergarten and First Grade Programs will need no adaptation for classes who are new to CSMP.

Second grade classes which are new to CSMP will need to be introduced to the CSMP languages. A suggested adaptation of the second grade program is included in the CSMP Mathematics for the Upper Primary Grades (UPG) Part I teacher's guide.

Third grade classes which are new to CSMP will need to become fluent in the CSMP languages before beginning the lessons in UPG Part III. A special schedule has been prepared for these classes which consists of prerequisite lessons and workbooks from UPG Part I and II followed by a modified UPG Part III and IV program.

Fourth grade classes which are new to CSMP will begin with CSMP Mathematics for the Intermediate Grades, Entry Module I which lasts one semester, before the class begins the regular fourth grade program. Future development plans provide for the creation of Entry Modules II and III for entry into the CSMP curriculum at the fifth and sixth grades, respectively.

20. Q: What should I do with my class if they did not finish the previous year's schedule?

A: No special measures are required. Each level of the CSMP curriculum begins with introductory lessons. In addition, the spiral approach to the lessons permits each student to participate at his or her own level of understanding, even if that level is not quite as high as it might have been if the previous year's schedule had been completed.

21. Q: How do I handle a student who has an extended illness?

A: The spiral approach used by CSMP permits a student to miss one or two weeks of school without needing much catch up work. When the student returns to school either you or one of his or her classmates can assist him or her with any material for which some help is deemed necessary.

If a student misses several months of classes and experiences difficulty with any of the CSMP languages upon return, he or she can be given the same help as you would give a transfer student new to CSMP.

Because of the nature of the materials and the parents' possible unfamiliarity with the languages, it is probably best not to have the worksheets and/or workbooks sent home to a sick child. In fact, without the lesson or collective introduction which often precedes the individual work on worksheets and workbooks, it would be practically impossible for the student to proceed at all.

22. Q: How can I assess student progress and assign grades?

A: CSMP does not believe that it is either desirable or reasonable for teachers to attempt to evaluate student mastery of skills from day to day or even from week to week. On the other hand, teachers, parents and even the children themselves, benefit by some monitoring procedure that records involvement in activities and assesses progress through the program.

Most teachers have their own monitoring devices with which they are comfortable. It is recommended that such devices be continued.

23. Q: How do CSMP students do on standardized tests?

A: While CSMP is opposed to the philosophy underlying standardized testing, test results are available to us from several sources. The results indicate that CSMP classes generally do as well or better on tests than do non-CSMP classes. It should be pointed out that standardized testing and traditional programs are correlated and that the CSMP student has been exposed to a great deal of mathematics that is not taught in more traditional programs or considered in the construction of standardized tests. There are also many immeasurable or unmeasured areas such as the enjoyment of mathematics, strategic thinking, organizational ability and depth of understanding of the methods, power and limits of mathematics where we feel CSMP students have a definite advantage. See also the comments in Evaluation, Section 2 (d) of the General Introduction.

24. Q: What problems would CSMP students have, if they return to a more traditional program in the next school year?

A: There should be no problems for CSMP students who return to a traditional program. Test results indicate that CSMP students are at or above the level of non-CSMP students in material covered in traditional courses. In addition, CSMP students have encountered many situations that are not introduced in traditional programs nor tested for on standardized tests. (See Question 23) In particular, their reasoning ability should be much sharper than their non-CSMP counterparts.

There are, of course, differences in timing as to when different programs introduce the standard algorithms and often CSMP's introduction will occur later than in some of the other programs. The reason for this is our belief that it is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

CSMP students can usually solve a problem before having learned the standard algorithm. If they enter a program that expects an algorithm they have not learned, they are still more than adequately prepared for the algorithm and should have no trouble quickly learning to use it.

CSMP introduces the usual addition algorithm in the second grade, one of the two standard subtraction algorithms in the third grade, the multiplication algorithm in the fourth grade, and the division algorithm in the fifth grade.

25. Q: How can I explain CSMP to parents?

A: You will find a letter to parents at the end of this section which may be reproduced and sent home with the students.

The best way would be for your coordinator to organize a workshop for interested parents. The parents can then see the languages of the program in use and have some of the same experiences their children have had. Workshops can be of almost any length but anything short of two hours does not allow for much familiarity with the material. In such a workshop we would suggest that the language of the Minicomputer be explained with particular emphasis on its use in presenting situations where mental arithmetic is called upon and in the development of algorithms (for example, physically carrying a checker from the ones' to the tens' board in the "8 + 2 = 10" play, paralleling "carrying" in the paper and pencil operation). A lesson using the storybook "Where's My Nose?" provides a good example of the power of the language of strings. A nonnumerical example, such as the brother-sister graph found at various grade levels, and a numerical example, such as building a road from 5 to 55 using +3 and 5 x arrows, will provide an introduction to the language of arrows.

26. Q: What are some ways to organize materials and manage my class?

A: It is very important to organize your CSMP materials before school begins. Decide where you will store the materials once they are organized. Filing drawers, shelves, boxes, or closets are all satisfactory as long as you can easily get to the materials. Take them all out of their shipping boxes and spread them on several tables and/or desk tops in your classroom so they can be organized into the order you will be using them.

Workbooks: The teacher's guide (or guides) are needed from second grade onward to find the order in which the workbooks are used. Find the table of contents in the Workbook strand. Collect all the workbooks in the order which they are listed.

After organizing all the workbooks, place them in an upright position in your file drawer, box, shelf, etc., so that the first workbook to be used is at the front of the stack.

Worksheets: There is a separate worksheet booklet for each student and you should decide whether you will store these booklets and pass them out when a particular lesson calls for a worksheet, or simply let the students keep their own worksheet booklets in their desks.

Each worksheet can be removed from the booklet or left in depending on how you wish to manage worksheets. However, once a worksheet is removed from the booklet it cannot be replaced, so make sure students are careful about tearing off the correct sheet.

Manipulatives: Storage of each manipulative in a container, properly marked, is the best way to make everything accessible; a shoe box for the rulers and translators, an envelope or small box for the magnetic checkers, and so on.

Additional Materials: Be sure that you have plenty of high quality colored chalk, large sheets of unlined white paper, and that your students have colored pencils, pens, or crayons. Plan well ahead to be sure you have any needed materials that are not supplied by CSMP.

Keep all teacher materials where they are easily accessible. You may find the guides easier to use if they are put in looseleaf

binders : the books are perferated so that pages can be removed easily. The demonstration Minicomputer boards and number line should always be on display in the room.

27. Q: Much of the CSMP material is new to me. How do I learn enough to teach a class?

A: Every CSMP teacher should have been trained by his or her coordinator. Kindergarten teachers receive eight hours training, first grade teachers sixteen hours, second grade teachers twenty-four hours, third grade teachers thirty-two hours, fourth grade teachers forty hours and fifth grade teachers forty-four hours. During the training sessions the CSMP content is taught to new teachers and suggestions are given for organization of their classes.

If the suggested training schedule has been followed, many lessons taken from the CSMP curriculum, or very similar lessons, have been presented. The schedule also makes use of the spiral approach so that training sessions aim not only to teach CSMP mathematics but to familiarize teachers with the curriculum and pedagogy of CSMP.

28. Q: What should be done when there is a substitute teacher?

A: Some school systems have included in their CSMP training program people who frequently substitute. When this is not the case, it would be very difficult for the untrained substitute to teach from CSMP materials. One solution is to arrange for another CSMP teacher to teach math to your class while the substitute teacher teaches another subject to that teacher's class. If that can't be arranged and you will only miss one or two days, you may find certain CSMP material, uncompleted workbooks for example, that the substitute can manage. Otherwise, use those days to cover material not included in the CSMP curriculum, such as telling time. If you absence will be an extended one, arrangements should be made with your coordinator to train a substitute, rearrange schedules

to put a CSMP teacher in your class, or otherwise provide for your students.

29. Q: The program makes use of the student's creative freedom. How much room is there for the teacher's ingenuity?

A: CSMP not only allows for your ingenuity but depends on it. While the lessons may appear to be very rigid by presenting a dialogue for teacher and student, it should be remembered that these are suggested questions, answers, and comments based on our experience with development classes. Even if you were to follow your part of the "script" your students are likely to offer responses which differ from those we have listed. It is the student's response that will determine the content and direction of your next question or comment.

In a traditional program your creativity is called upon the first time you develop a particular lesson, and to a lesser extent, as you revise lessons in succeeding years. There is little room for spur-of-the-moment creativity once a lesson begins. The CSMP curriculum, by creating a free and open situation emphasizing the student's creative freedom, continually presents you with unanticipated student response and calls on your ingenuity at all times.

TO THE PARENTS OF CSMP STUDENTS

Your child is participating in the Comprehensive School Mathematics Program's (CSMP) elementary school curriculum, developed under a contract with the National Institute of Education. While the program cannot be fully explained in so brief a format, it is hoped that this letter will give you some feeling for the substance of your child's mathematics education.

The content goals of the CSMP K-6 program include a development of the standard number systems and their operations (including an early introduction to the integers, rational numbers and decimal numbers) interwoven with a two-pronged development of geometry (a rich variety of experiences in combinatorial and metric geometry together with a carefully developed sequence of activities in parallelogram (affine) geometry). On an equal footing with the above, the CSMP program includes a strong component of probability, statistics, and combinatorics which extends throughout grades 1-6. Powerful pedagogical devices are exploited providing several nonverbal languages which render the unifying mathematical ideas underlying the content accessible to very young children. Through these languages, children are able to apply mathematical ideas effectively and creatively to explore the content directly, unemcumbered by the terminology and notation which usually pervade the early stages of the curriculum and stand as a barrier between the children and the content. These languages include:

1) <u>The Language of Strings</u> This "language" mimics the basic methods of collecting and classifying data relevant to the problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of set.

2) <u>The Language of Arrows</u> This graphical language models the fundamental process involved in comparing and analyzing sets and operations on them. That is, it deals with the mathematical notions of relations and functions including, but not restricted to, ordering relations and the numerical processes of adding to, subtracting from, multiplying by, and sharing equally among.

3) <u>The Language of the Papy Minicomputer</u> The Papy Minicomputer, an extremely effective paper abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to a multitude of numerical algorithms, including the standard algorithms for the basic numerical operations.

Among other things, these languages allow for the development of the basic operations on numbers in a natural way based on previous experiences with the languages. They also provide a context in which situations arise that call for the repeated use of arithmetic operations without resorting to drill exercises, which most children find boring.

Another tool, not unique to but used by CSMP, is the hand-calculator. The hand-calculator allows the child to study problems that call for calculation beyond the child's ability or in which the calculation is so extensive as to cause the child to lose interest. The calculator is also used in a way similar to the use of languages mentioned above. Properly used, it can help lead to the development of important mathematical ideas and present situations for the interesting use of mental or paper and pencil arithmetic.

The content is learned in an atmosphere of constant applications utilizing a "pedagogy of situations".* These vary from simple story situations in the early primary grades to challenging applications and nontrivial simulations of real world problems in the upper grades.

One of the tenets of CSMP's development philosophy is that mathematics is a unified whole and should be learned as such. As a result, along with CSMP's concern with the variation in ability levels in the typical heterogeneous class, the curriculum is structured in a spiral format throughout the K-6 program. The content is completely sequenced in spiral form so that each student is brought into contact with each area of content continuously throughout the program. This consciously precludes atomizing the content and mastering each bit before

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* Inspired by the work of the mathematician and psychologist, Caleb Gattegno.

continuing to the next. Rather, each student spirals through repeated exposures to the content, building interlocking experiences of increasing sophistication.

Another tenet of CSMP's development philosophy, based on years of experimentation in the classroom, is that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation, and individual independent experiences. Individualization throughout the K-6 program is achieved through graded series of workbooks integrated into the basic sequence of lessons. Due to the spiral nature of the curriculum, each student can engage in a workbook series without losing touch with the progress of the class while at the same time working his or her way up through the series to the limits of his or her ability.

A feature unique to the CSMP program is a collection of storybooks written by Frederique Papy (STORIES BY FREDERIQUE) and a collection of story-workbooks (THE CSMP LIBRARY). These probe the affective domain of learning and bring children into contact with mathematics at levels until now unexplored.

CSMP does not have its own system of student assessment and progress reporting. While standardized testing indicates that CSMP students develop the "basic skills" as early or earlier than non-CSMP students, we hope you will not be obsessed with your child's development of these "skills". Each child is an individual and should not be judged against norms but on what he or she can and does accomplish. Many extremely important areas --- development of self-confidence, ability to think and reason, etc. --- are very difficult to measure and grade. If you have any questions regarding the progress of your child, his or her teacher is in the best position to answer those questions.

THE CSMP STAFF

WORKBOOKS

Comprehensive School Mathematics Program

Mathematics for the Intermediate Grades Part III

Workbooks

CEMREL, Inc.

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WORKBOOK

INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the four content strands. In the workbook strand, however, it is this individualized work which becomes the chief end of the lessons. It has been our goal in this strand to provide the students with opportunities to

- review many of the ideas they have met in the content strands
- apply their acquired knowledge to new situations requiring various kinds of strategic thinking
- learn how to read and use mathematics workbooks and story-workbooks.

The following workbooks are provided.

A Variety of Problems 1
A Variety of Problems 2
A Variety of Problems 3
A Variety of Problems 4
A Variety of Problems 5
A Variety of Problems 6

These six workbooks are written with problems of varying levels of difficulty (*, **, and ***) all in one booklet. Approximately the first 10 pages of each booklet are easy (*) problems, the next 10 to 12 pages are average (**) problems, and the last 10 pages are more challenging (***) problems.

The following story-workbooks are provided.

Number 1,000's Dream
Seven Secret Numbers

Three lessons are based on a new game, the Arrow Game, involving the students immediately in analyzing situations involving relations similar to the String Game analysis of situations involving sets.

We suggest that you devote one day a week to this strand. We provide a complete suggested schedule for "W" lessons immediately following this introduction. It is suggested that you follow this schedule more or less as given, even if many of your students have not finished a particular workbook after the two or three days that are alotted to it. As a general rule, students at this age do not enjoy remaining on the same topic for too long. And besides, it is a basic principle of the CSMP spiral approach that a variety of experiences is preferable to any attempt to master a given topic by continued "plugging away" at it. This principle applies as much (if not more so) to the workbook strand as it does elsewhere in the curriculum. If you do feel, however, that more time is needed for a particular workbook, you may find it convenient to continue work on that workbook during one (or more) of the adjustment days that occur in the suggested schedules for the four content strands.

For each of the story-workbooks dealt with in this part of the Teacher's Guide, there are collective lessons that accompany the story-workbook. These are followed by an answer key for that story-workbook. There are also some short collective (warm-up) lessons suggested for the second lesson on each "A Variety of Problems" workbook. Following these suggestions, there are answer keys for the workbook. All answer keys are presented in reduced format, each page actually containing reduced copies of several (usually four) pages of the original booklet, with solutions written in.

For each workbook we suggest that all students start work at the easiest level of difficulty (i.e., on page 1) and then work through as many pages as they can handle during the two lessons scheduled to that workbook. You can expect that, under normal circumstances, about two-thirds of the students will work through about 20 to 25 pages, and about one-third will reach the most challenging problems at the end of the booklet.

The workbooks provide an excellent instrument with which you can assess the progress of your students on a week-by-week basis. To use this instrument effectively certainly does not require that you check every page and every problem for each student. You should develop the sort of procedure for checking student's work with which you can feel most comfortable. This might take

the form of checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or simply looking carefully at the first few pages to be sure the students have understood the general idea of the problems in a particular book. In our experience, an effective procedure is to clip to the front cover of each checked workbook a strip of paper on which can be indicated your assessment of the students' work on each page you check. For this purpose, you might use the following symbols.

meaning "well done" or "correct"

meaning "something wrong, try again"

meaning "did you forget this problem"

or "do you need a hint?"

On the next page we show the front cover of one student's A Variety of Problems 1 workbook after it has been checked.

Immediately following the suggested schedule for "W" lessons we provide a progress chart to help you keep track of

- where each of your students is on any particular workbook day
- how many pages each student has completed under each title
- your personal degree of satisfaction with the student's work on each workbook or story-workbook.

NOTE: This is the way Robert's workbook was returned to him at the beginning of the second lesson devoted to this workbook. The blanks on lines 18 - 32 indicate that he had not yet worked on these pages. Before Robert begins on these pages during the second lesson, he should correct his mistakes on page 15 and finish his work on pages 5, 7, 9, 10 and 16.

In particular you should be on the watch to see if, for any of your students, even the easiest problems in the workbooks are too difficult. Such students would probably benefit a great deal if you were able to devote some time with them to use some of the suggestions made in the book " Math Play Therapy".

We now provide a suggested schedule for "W" lessons. The placement of the various workbooks has been determined by bearing in mind what knowledge is assumed and where the lessons that deal with that information occur in the suggested schedule for the four content strands. Hence, you are warned to be particularly careful when considering possible modification of this suggested schedule. Whatever modification you might like to make, we strongly suggest that you use one day each week for the workbooks.

Here is a list of some of the important points to bear in mind when working in this strand.

- All students start at the beginning of each workbook and progress as far as they can.
- All students begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students are expected to complete a given workbook. Only the faster students will reach the most challenging problems. The slower students may succeed only to do the easiest problems although you should not assume that this will automatically be the case--surprises are not at all uncommon in this area.
- Read the introductory material for each workbook and story-workbook and give the short introductory collective lesson(s).

SUGGESTED SCHEDULE FOR W - LESSONS

WEEK	Workbooks	Materials
1	**1W** A Variety of Problems 1 p. 1	Teacher: None Student: Pencil; colored pencils; hand-calculator (optional); "A Variety of Problems 1"
2	**2W** A Variety of Problems 1 p. 2	T: Colored chalk S: Pencil; colored pencils; hand-calculator (optional); "A Variety of Problems 1"
3	**3W** A Variety of Problems 2 p. 13	T: None S: Pencil; colored pencils; hand-calculator (optional); "A Variety of Problems 2"
4	**4W** A Variety of Problems 2 p. 14	T: Colored chalk S: Pencil; colored pencils; hand-calculator (optional); "A Variety of Problems 2"
5	**5W** Arrow Game #1 p. 25	T: Colored chalk; Arrow Game Analysis Sheets S: Pencil; Worksheet N85
6	**6W** A Variety of Problems 3 p. 61	T: None S: Pencil; colored pencils; hand-calculator (optional); "A Variety of Problems 3"
7	**7W** A Variety of Problems 3 p. 62	T: Colored chalk S: Pencil; colored pencils; hand-calculator (optional); "A Variety of Problems 3"
8	**8W** Number 1,000's Dream p. 73	T: Minicomputer set; Weighted Checkers (powers of 2) S: "Number 1000's Dream"
9	**9W** Number 1,000's Dream p. 83	T: Minicomputer set; Weighted Checkers (powers of 2); colored chalk S: "Number 1,000's Dream"; paper; pencil

SUGGESTED SCHEDULE FOR W - LESSONS

WEEK	Workbooks	Materials
10	**10W** A Variety of Problems 4 p. 91	T: None S: Pencil; 30cm ruler; hand-calculator (optional); "A Variety of Problems 4"
11	**11W** A Variety of Problems 4 p. 92	T: Colored chalk S: Pencil; 30cm ruler; hand-calculator (optional); "A Variety of Problems 4"
12	**12W** Seven Secret Numbers p. 105	T: Colored chalk S: "Seven Secret Numbers"; pencil
13	**13W** A Variety of Problems 5 p. 119	T: None S: Pencil; colored pencils; "A Variety of Problems 5"
14	**14W** A Variety of Problems 5 p. 120	T: Colored chalk S: Pencil; colored pencils; "A Variety of Problems 5"
15	**15W** Arrow Game #2 p. 41	T: Colored chalk; Arrow Game Analysis Sheets S: Pencil; Worksheet N86
16	**16W** A Variety of Problems 6 p. 131	T: None S: Pencil; colored pencils; hand-calculator (optional); "A Variety of Problems 6"
17	**17W** A Variety of Problems 6 p. 132	T: Colored chalk S: Pencils; colored pencils; hand-calculator (optional); "A Variety of Problems 6"
18	**18W** Arrow Game #3 p. 53	T: Colored chalk; Arrow Game Analysis Sheets S: Pencil; Worksheets N87, N88, N89, N90, N91

Workbook Progress Chart

Name	A Variety of Problems 1	A Variety of Problems 2	A Variety of Problems 3	Number 1,000's Dream

Workbook Progress Chart

Name	A Variety of Problems 4	Seven Secret Numbers	A Variety of Problems 5	A Variety of Problems 6

A VARIETY OF PROBLEMS 1

A VARIETY OF PROBLEMS 1

This workbook has problems of varying levels of difficulty (*, **, and ***) all in one booklet — approximately the first 10 pages of which are easy (*) problems, the next 10 to 12 pages are average (**) problems and the last 10 pages are more challenging (***) problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for this workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about 20 to 25 pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Lesson 1

Give each student a copy of the workbook "A Variety of Problems 1" and let them work on their own.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

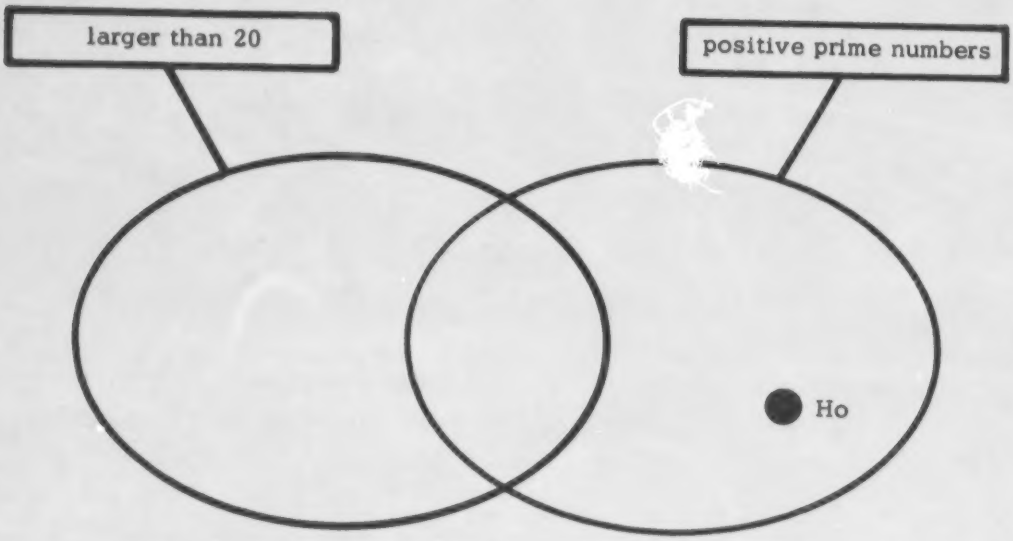
2W

Lesson 2

T: Today we have a detective story about a secret number named "Ho".

First clue

Draw this string picture on the board.



T: What does the string picture tell us about Ho?

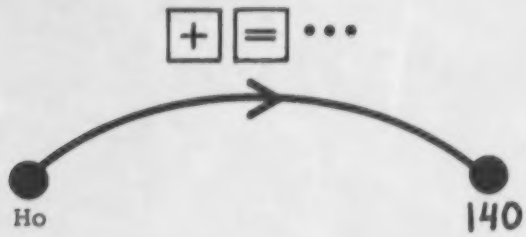
S: Ho is a positive prime number and Ho is not larger than 20.

T: Which numbers could Ho be?

List the numbers that Ho could be on the board as the students name them. It might be necessary to remind the students that a positive prime number has exactly two positive divisors. The class should conclude that Ho could be 2, 3, 5, 7, 11, 13, 17, or 19.

Second clue

Draw this arrow picture on the board.



T: If I put Ho on the display of a hand-calculator and press + = = and so on as many times as I like, 140 will appear on the display. Which numbers could Ho be?

Give the students a minute to think about the clue.

S: Ho could be 5, because I can count by fives and get 140.

T: Yes, Ho could be 5. What are some other numbers Ho could be?

S: Ho could be 2, because I can count by twos and get 140.

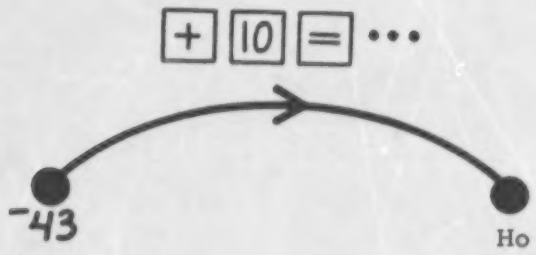
S: Ho could be 7, because 7 x 20 = 140.

The class should conclude that Ho could be 2, 5, or 7.

2W

Third clue

Draw this arrow picture on the board.



T: Ho is in this arrow picture. Who is Ho?

Again, allow a minute for the students to think about the new clue.

S: I started from ⁻43 and counted by tens: ⁻43, ⁻33, ⁻23, ⁻13, ⁻3, 7. Ho must be 7.

The class should conclude that Ho is 7.

Return the students' copies of "A Variety of Problems 1". Ask them to correct their mistakes (if any), and then to continue working in the workbooks.

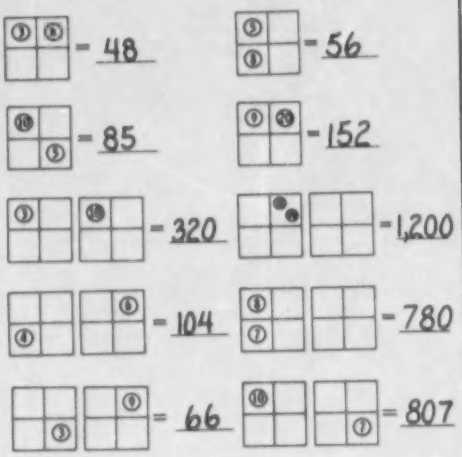
If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the conclusion of this lesson, collect the workbooks for your progress check. You may want to make this workbook available later for students to work in at their leisure.

A Variety of Problems 1

Put any number you wish on the Minicomputer using exactly two of these checkers:

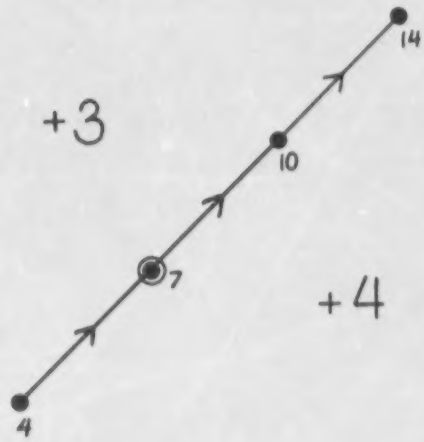
② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩ ⑳



2

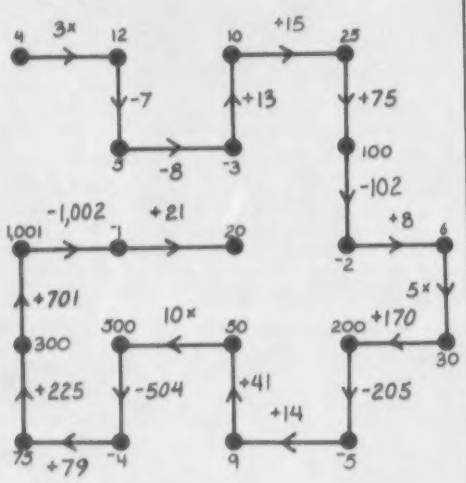
(Many solutions are possible.)

7 is the only odd number in this picture. Where is 7? Label all the dots.



3

Label each arrow.

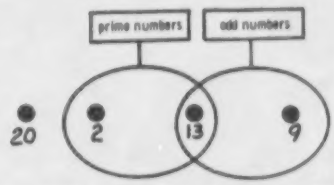


4

(Other labels are possible for some of the arrows.)

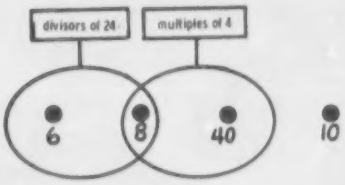
Put each of these numbers in this string picture.

9; 2; 20; 13



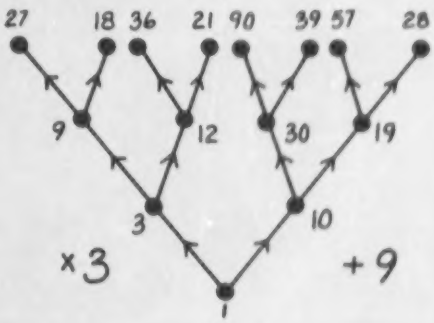
Put each of these numbers in this string picture.

10; 6; 8; 40



5

Label the dots.



In this arrow picture, Pif and Paf are the only multiples of 7. Pif is smaller than Paf.

Pif is 21 ; Paf is 28 .

6

How many triangles can you find in each figure ?
Write the number in the blank.

5

There are more than three here.

5

There are more than four here.

10

9

7

First clue

Klik is one of these numbers on the Minicomputer.

- 28

- 56

- 18

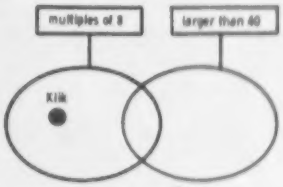
- 24

- 48

- 36

Second clue

Klik is in this string picture.



Klik is 24 .

8

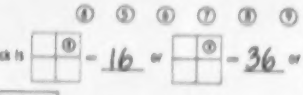
First clue

Tick is a square number larger than 10 and smaller than 100.

Tick is 16 or 25 or 36 or 49 or 64 or 81 .

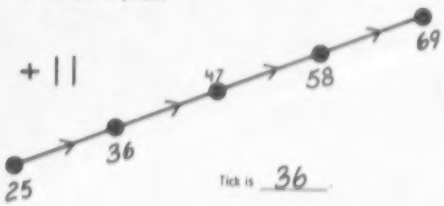
Second clue

Tick can be put on the Minicomputer using exactly one of these checkers :



Third clue

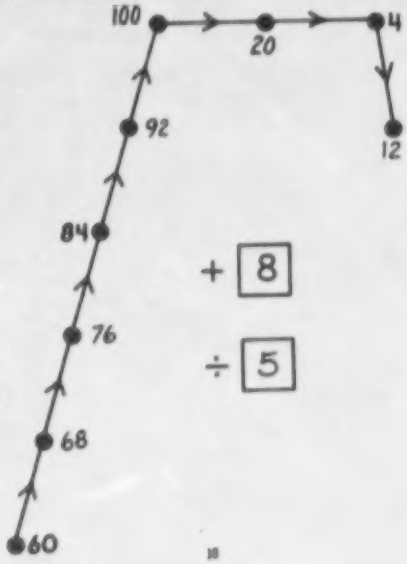
Tick is in this arrow picture.



Tick is 36 .

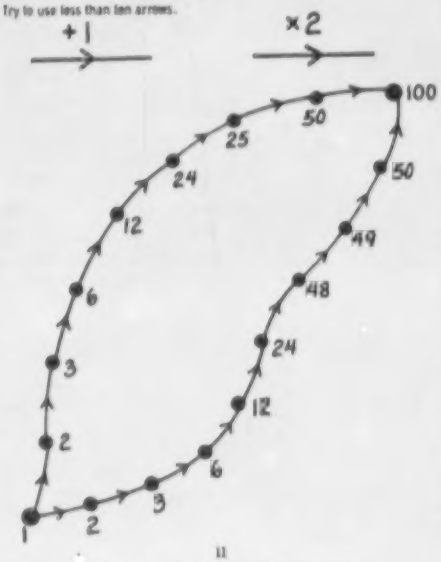
9

Fill in the boxes for the arrows and label the dots.



10

Build a red-blue road from 1 to 100.
Try to use less than ten arrows.

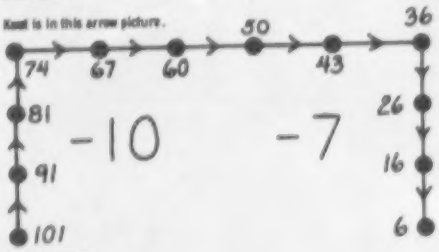


11

Many solutions are possible.
A solution using eight arrows
and a solution using nine
arrows are shown.

First clue

Kaat is in this arrow picture.



Second clue

Kaat is a square number.
Kaat is 81 or 36 or 16.

Third clue

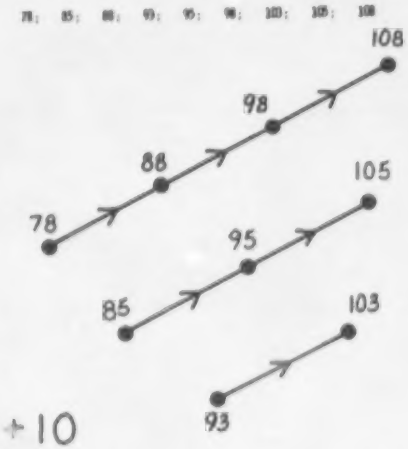
Kaat can be put on the ones' board of the Minicomputer using exactly one (9)-checker and no other checkers.

Kaat is = 36.

12

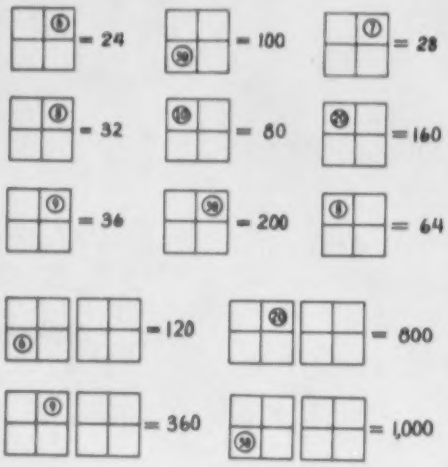
Put each of these numbers in this arrow picture.

78; 85; 88; 93; 95; 98; 103; 105; 108



13

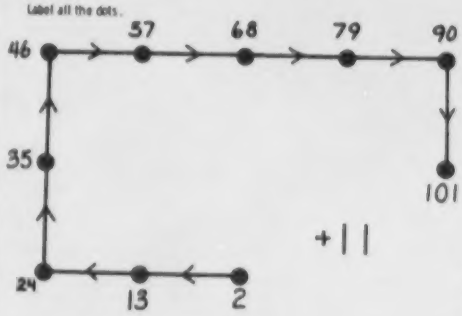
Put each number on the Minicomputer using exactly one of these checkers:



14

(Other solutions are possible.)

Label all the dots.

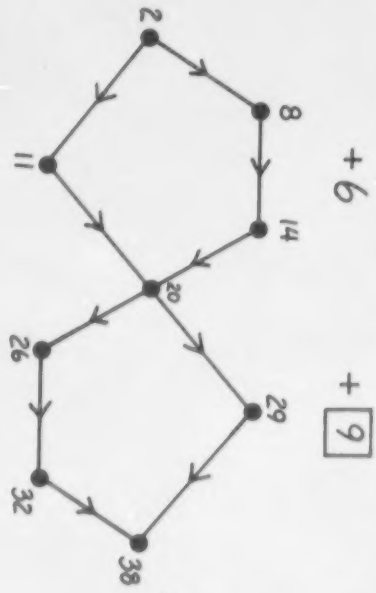


In this arrow picture, Ry is the only multiple of 10 and Ru is the only multiple of 7.

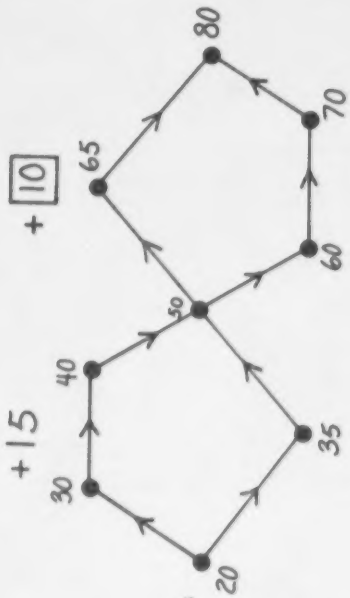
Ry is 90 ; Ru is 35 .

15

Label the dots and fill in the box for the blue arrows.



16



17

Label the dots and fill in the box for the red arrows.

First clue

One of the symbols +, −, × is hidden in each blank of the hand-calculator sentence. A symbol may be used more than once.

8		5		2	−	Zip

Zip is 15 or 11 or 26 or 1 or 5 or 6 or 80 or 42 or 38.

Second clue

Zip is a prime number.

Zip is 11 or 5.

Third clue

+ = . . .

Zip ——→ 1,000

Zip is 5.

18

These are quilt patterns. How many triangles can you find in each pattern? Write your answers in the blanks.

Maltese cross

16

Double X

20

Evening star

12

19

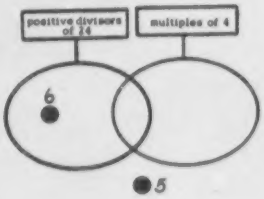
The red tag is one of these:

~~positive prime numbers~~
~~positive divisors of 20~~
~~multiples of 4~~
~~larger than 23~~
~~smaller than 23~~
positive divisors of 24

The blue tag is one of these:

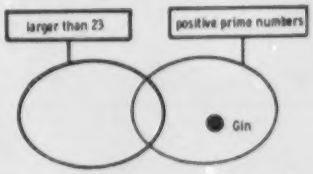
~~positive prime numbers~~
~~positive divisors of 20~~
multiples of 4
~~larger than 23~~
~~smaller than 23~~
~~positive divisors of 24~~

Complete the tags.



20

First clue



Gin is 2 or 3 or 5 or 7 or 11 or 13 or 17 or 19 or 23.

Second clue

+ = . . .

Gin ——→ 154

Gin is 2 or 7 or 11.

Third clue

+ 10 = . . .

−83 ——→ Gin

Gin is 7.

21

First clue

Clo's name can be written using all these symbols, each of them exactly once.

+ × 2 3 8) (

Clo is 22 or 14 or 30 or 19 or 40 or 26.

In this chart write at least one name using all the symbols for each number that Clo could be. One is done for you.

2×(3+8) 22	(2×3)+8 14	(8+2)×3 30
(2×8)+3 19	8×(3+2) 40	(8×3)+2 26

Second clue

Clo is a prime number.

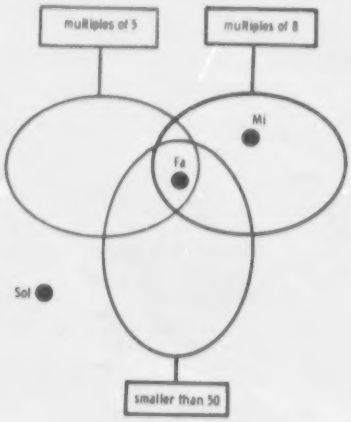
Clo is 19.

22

Mi, Fa, Sol are three of these numbers:

24; 25; 26; 40; 55; 56; 57; 80.

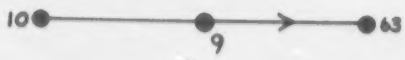
Mi is 56; Fa is 40; Sol is 57.



23

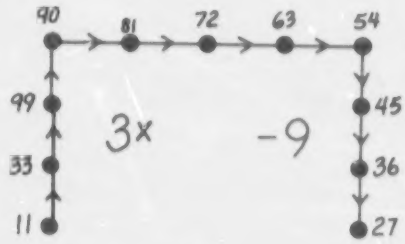
Build a red-blue road between these pairs of numbers. Use at most three arrows or cords in each road.

is a divisor of → +1 or −1



24

Label all the dots.



In this picture, Nick and Nack are the only numbers that can be put on the ones' board of the Minicomputer using exactly one of these checkers:

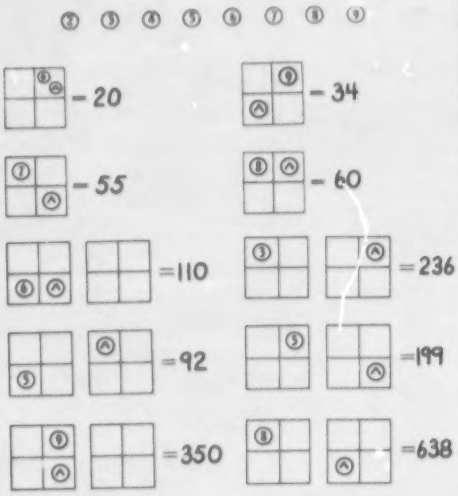
② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Nick is smaller than Nack.

Nick is [⑨] = 36. Nack is [⑨] = 72.

25

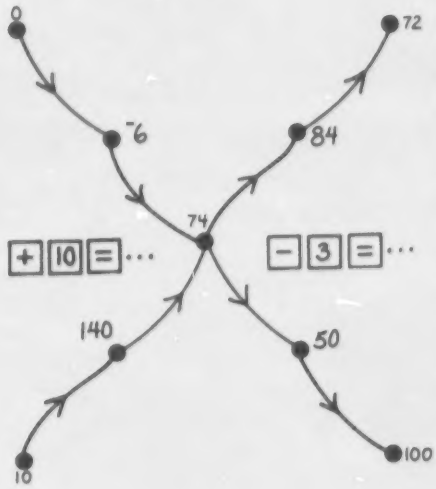
Put each number on the Minicomputer using exactly one negative checker and exactly one of these checkers:



26

(Other solutions are possible.)

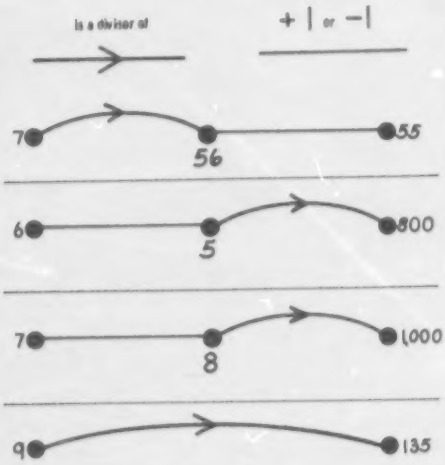
Label the dots. Many answers are possible.



27

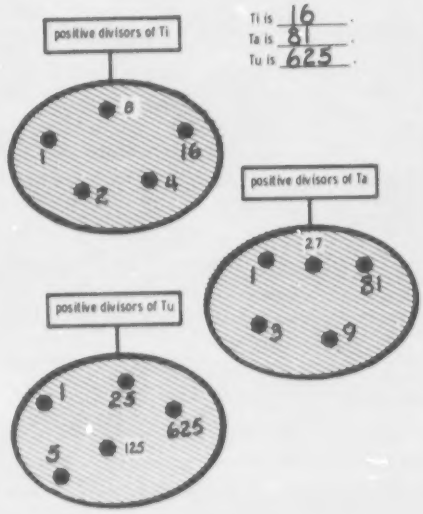
(Many solutions are possible.)

Build a red-blue road between these pairs of numbers. Use at most one arrow and one cord in each road.



28

Label the dots.

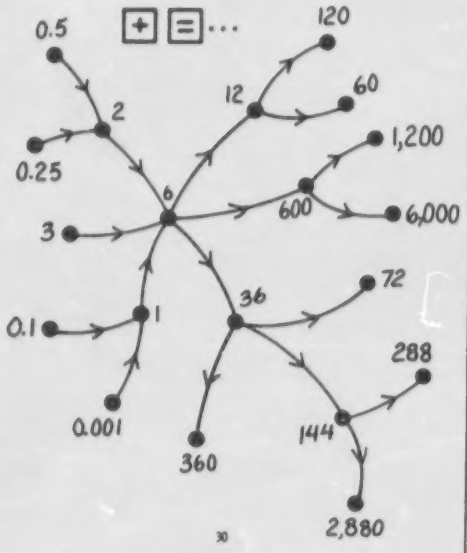


Ti is 16.
Ta is 81.
Tu is 625.

29

(The order in which the dots are labeled may vary.)

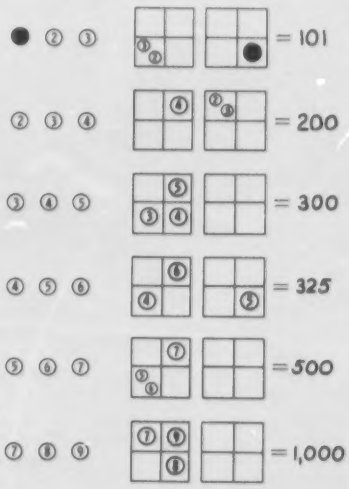
Label the dots. Many answers are possible.



30

(Many solutions are possible.)

Put each number on the Minicomputer using each of the three checkers to the left of the boards.

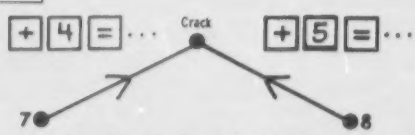


31

(Other solutions are possible.)

Crack is a secret number.

First clue



Crack could be 23 ; 43 ; 63 ; 83 ; 103 ; 123 ; 143 ; 163 ; 183 ; 203 ; 223 ; 243 ; 263 ; 283 ; and so on.

Second clue

Crack is a multiple of 3.
Crack could be 63 ; 123 ; 183 ; 243 ; 303 ; 363 ; 423 ; and so on.

Third clue

Crack is between 900 and 1,100.
Crack is 903 or 963 or 1,023 or 1,083 .

Fourth clue

If you add 1 to Crack, you will find a square number.

Crack is 1,023 .

32

A VARIETY OF PROBLEMS 2

This workbook has problems of varying levels of difficulty (*, **, and ***) all in one booklet — approximately the first 10 pages of which are easy (*) problems, the next 10 to 12 pages are average (**) problems and the last 10 pages are more challenging (***) problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for this workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about 20 to 25 pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Lesson 1

Give each student a copy of the workbook "A Variety of Problems 2" and let them work on their own.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

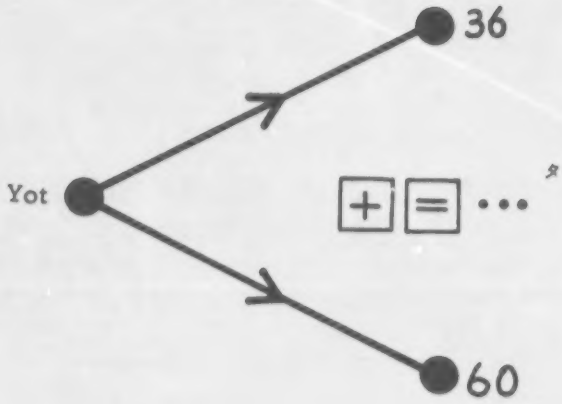
4W

Lesson 2

T: Yot is a secret number.

First clue

Draw this arrow picture on the board.



T: This is a detective story about a secret WHOLE number named "Yot". Yot is in this arrow picture. Which numbers could Yot be? Why?

Let the students have a minute to consider the situation.

S: Yot could be 3. If you start with 3 on the display of a hand-calculator and press [+] [=] [=] [=] and so on, 36 will appear on the display. If you continue pressing [=], 60 will appear.

T: How many times 3 to get to 36?

S: 12 times, because 12 x 3 = 36.

---

* See the General Introduction Section 2(f) on the Role and Use of Hand-Calculators for a discussion of the [+] [=] ... relation.

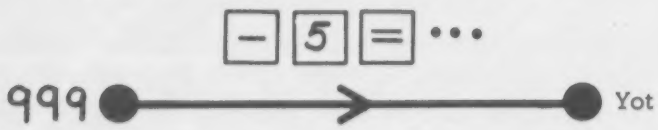
T: How many times 3 to get 60?

S: 20 times, because 20 x 3 = 60.

Continue in this manner until the students conclude that Yot could be 1, 2, 3, 4, 6, or 12.

Second clue

Draw this arrow picture on the board.



T: The second clue is that if you put 999 on the display of a hand-calculator and press − 5 = ... as many times as you like, Yot will appear on the display. Who is Yot?

Give the class a few minutes to consider this situation. The class should conclude that Yot is 4. Ask several students to explain how they found Yot. Two possible explanations are given below.

S: When 5 is subtracted again and again, the last digit of the number on the display is either 9 or 4. So Yot must be 4.

S: I started from 999 and subtracted 900, so I know 99 will appear on the display. Then I subtracted 90, so 9 appears and 9 - 5 = 4. Yot is 4.

Accept any reasonable explantion along these lines.

4W

Return the students' copies of "A Variety of Problems 2". Ask them to correct their mistakes (if any), and then to continue working in the workbooks.

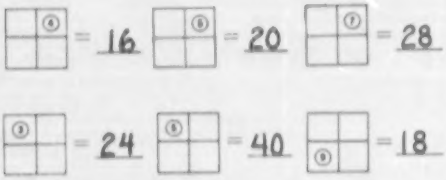
If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the conclusion of this lesson, collect the workbooks for your progress check. You may wish to make this workbook available later for students to work in at their leisure.

A Variety of Problems 2

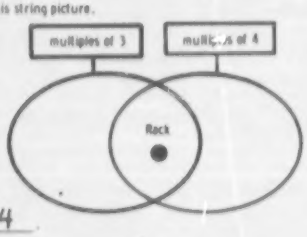
First clue

Rack is one of these numbers on the Minicomputer.



Second clue

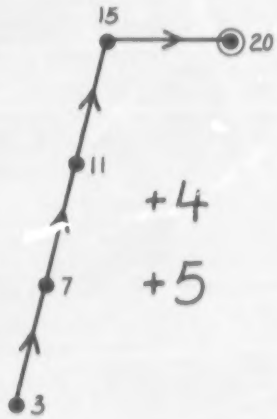
Rack is in this string picture.



Rack is 24.

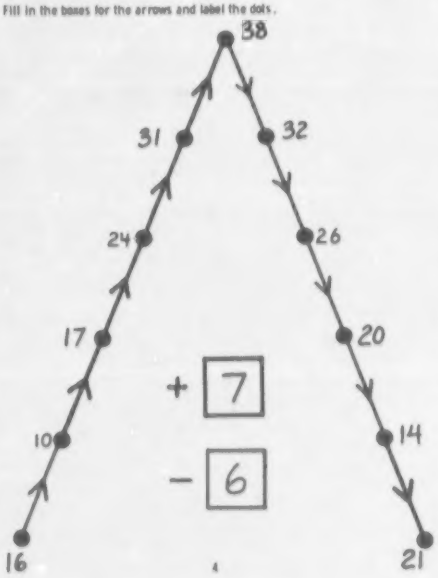
2

20 is the only multiple of 4 in this picture.
Where is 20? Label all the dots.



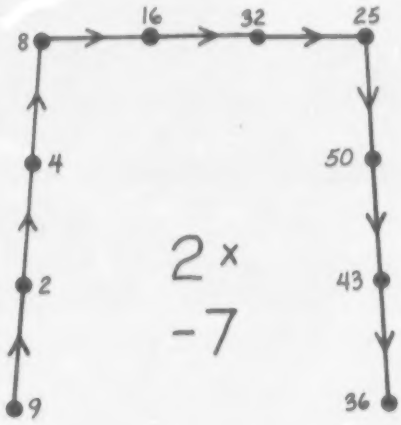
3

Fill in the boxes for the arrows and label the dots.



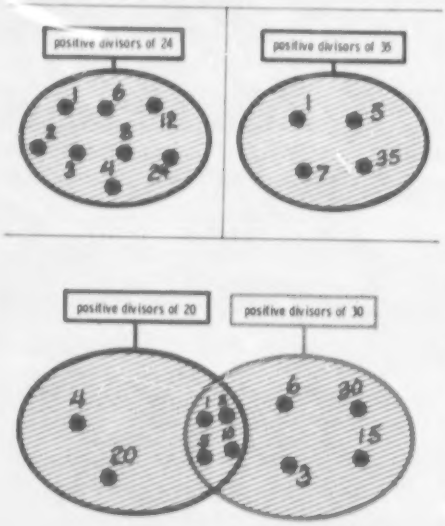
4

Label the dots.
The only positive prime numbers in this picture are 2 and 43.



5

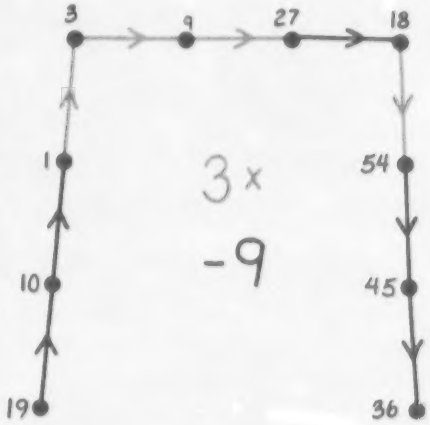
Label the dots.



6

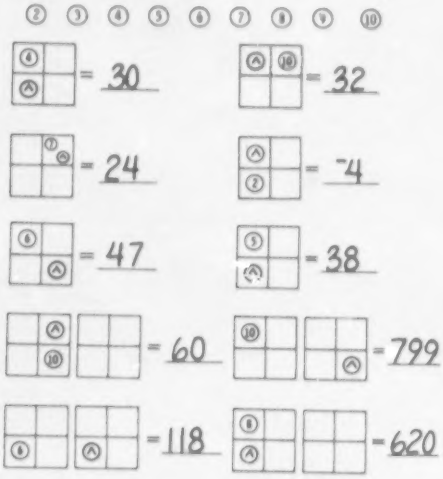
Label the dots.
In this picture, exactly three numbers are not multiples of 3.

They are 1, 10, and 19.



7

Put any number you wish on the Minicomputer using exactly one negative checker and exactly one of these checkers:

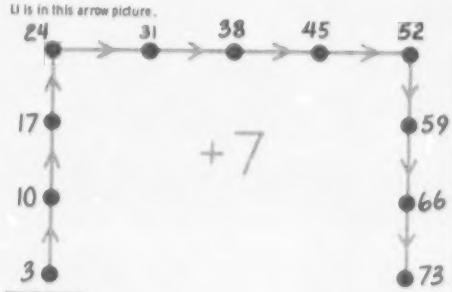


8

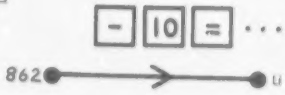
(Other solutions are possible.)

First clue

Li is in this arrow picture.



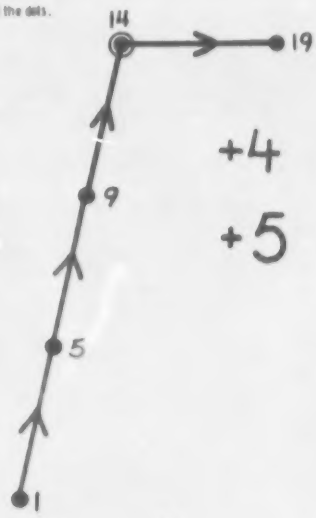
Second clue



Li is 52.

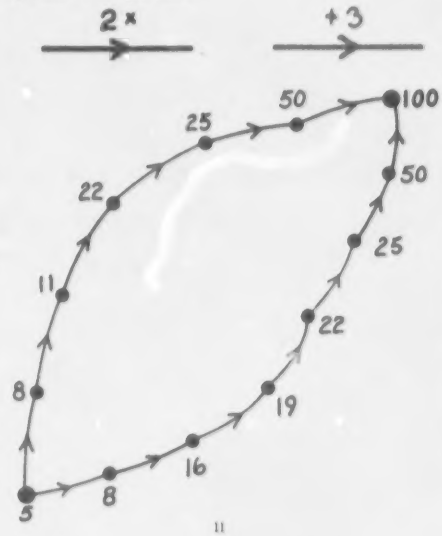
9

14 is the only even number in this picture.
Where is 14 ?
Label all the dots.



10

Build a red-blue road from 5 to 100.
Try to use less than eight arrows.

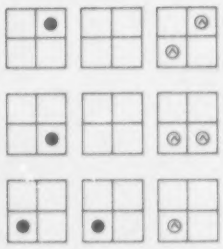


11

Many solutions are possible. A solution using six arrows and a solution using seven arrows are shown.

Write "EVEN" or "ODD" in the blank for each number.

101 + 102 + 103	even
52 + 53 + 54	odd
2 x 175	even
3 x 123	odd
4 x 51	even
10 x 99	even

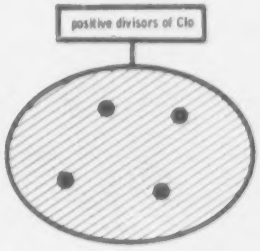


even

odd

even

12



Write at least six numbers that Clo could be.

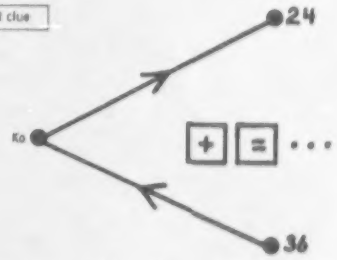
6 ; 8 ; 10 ;
14 ; 15 ; 21 ; and so on.

13

Clo can be any number that is the product of two distinct prime numbers or is the third power of a prime number; e.g., 6 = 3 x 2; 8 = 2^3 = 2 x 2 x 2; 10 = 5 x 2; and so on.

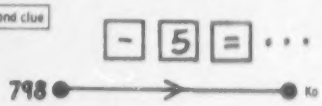
Ko is a secret WHOLE number.

First clue



Ko is 1 or 2 or 3 or 4 or 6 or 12.

Second clue

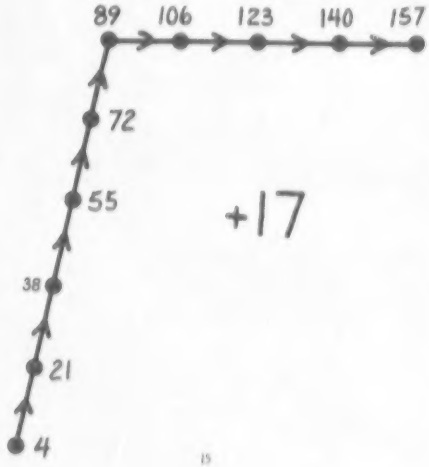


Ko is 3.

14

Label all the dots.
In this arrow picture, Clin is the only multiple of 10 and Clan is the only multiple of 3.

Clin is 140. Clan is 72.



15

First clue

Lou can be put on the ones' board of the Minicomputer using exactly one of these checkers:

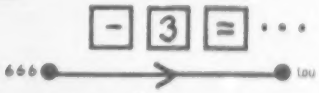
Lou is 2 or 4 or 8 or 16 or 5 or 10 or 20 or 40 or 6 or 12 or 24 or 48.

Second clue

Lou is a divisor of 30.

Lou is 2 or 5 or 6 or 10.

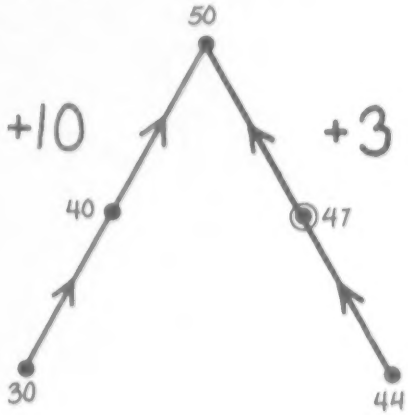
Third clue



Lou is 6.

16

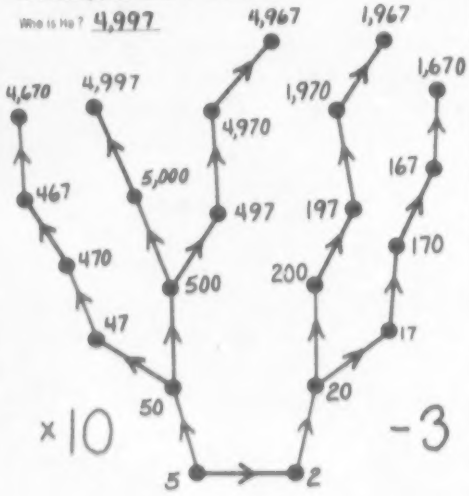
47 is the only odd number in this picture.
Where is 47?
Label all the dots.



17

Label all the dots.
Ho is the largest odd number in this arrow picture.

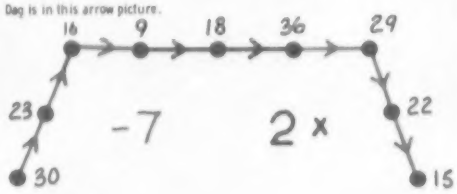
Who is Ho? 4,997



18

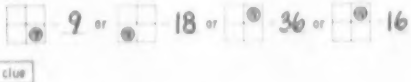
First clue

Dag is in this arrow picture.



Second clue

Dag can be put on the ones' board of the Minicomputer using exactly one of these checkers:

Dag is  9 or 18 or 36 or 16

Third clue

Dag is not a square number.

Dag is 18

19

How many tulips, each like , can you find? Be careful! There are more than eight.

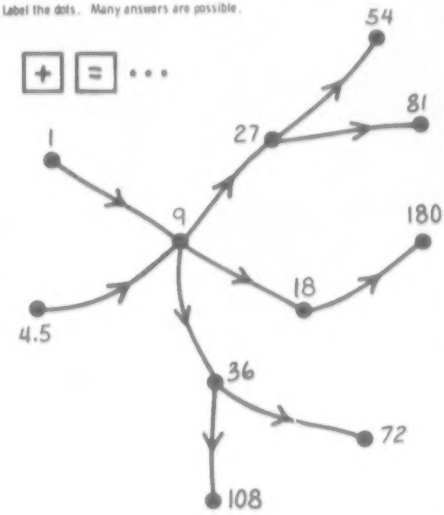
I find 15 tulips.

How many triangles are in this design? 24

BASKET

20

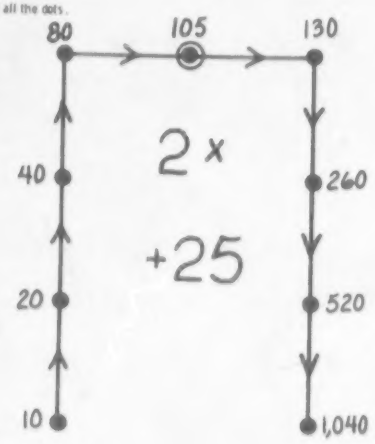
Label the dots. Many answers are possible.



21

(Many solutions are possible.)

In this picture all the dots are for whole numbers.
105 is the only odd number in this picture.
Where is 105 ?
Label all the dots.



22

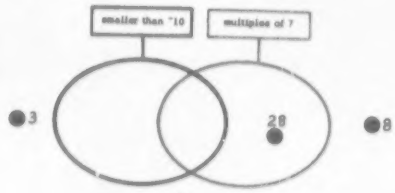
The red tag is one of these:

[redacted]
[redacted]
[redacted]
[redacted]
[redacted]
smaller than 10
[redacted]
[redacted]
[redacted]

The blue tag is one of these:

[redacted]
[redacted]
[redacted]
[redacted]
[redacted]
[redacted]
[redacted]
multiples of 7
[redacted]

Complete the tags.

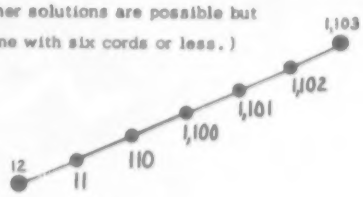


23

Build a blue-red road between these pairs of numbers.
Use as few cords as possible. Use only whole numbers.

+1 or −1 ×10 or ÷10

(Other solutions are possible but none with six cords or less.)

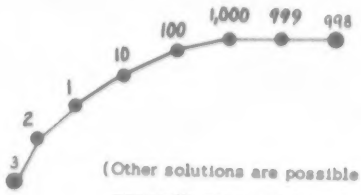


24

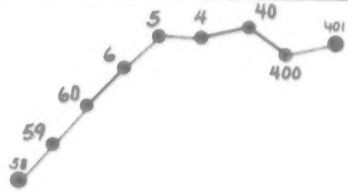
(Other solutions are possible but none with five cords or less.)

Build a blue-red road between these pairs of numbers.
Use as few cords as possible. Use only whole numbers.

+1 or −1 ×10 or ÷10



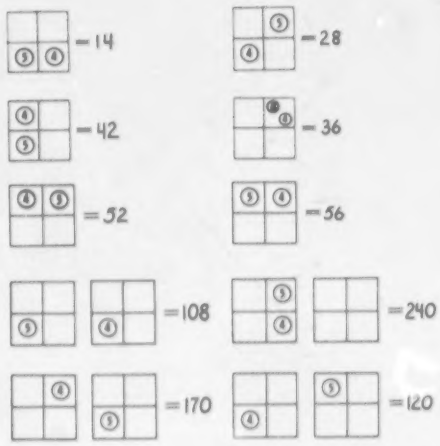
(Other solutions are possible but none with seven cords or less.)



25

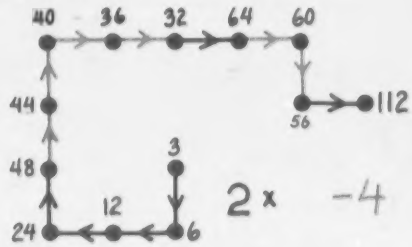
(Other solutions are possible but none with eight cords or less.)

Put each number on the Minicomputer using just these two checkers, each exactly once. ④ ⑤



26

Label all the dots.



2 x −4

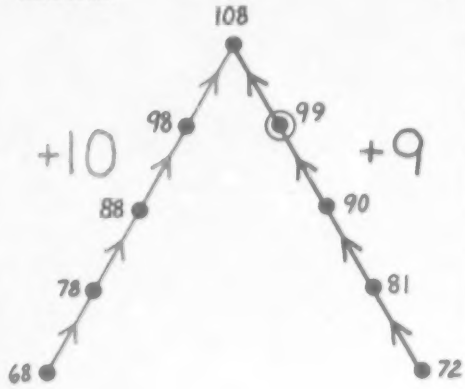
In this arrow picture, there are exactly three numbers that CANNOT be put on the ones' board of the Minicomputer with just one of these checkers:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩

They are 44, 60, and 112.

27

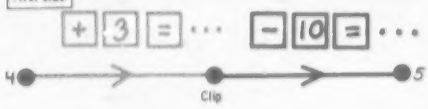
There are exactly two odd numbers in this picture and 99 is the larger of the two.
Where is 99?
Label all the dots.



28

Clip is a secret number.

First clue



Clip could be 25, 55, 85, 115, 145, 175, 205, 235, 265, 295, 325, 355, 385, 415, 445, and so on.

Second clue

Clip is between 600 and 700.

Clip is 625 or 655 or 685.

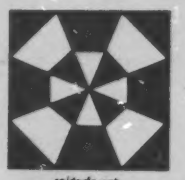
Third clue

Clip is a square number.

Clip is 625.

29

How many triangles can you find in each quilt pattern ?
Write your answers in the blanks.

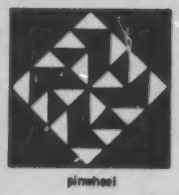


spider's web

20

pine tree

29

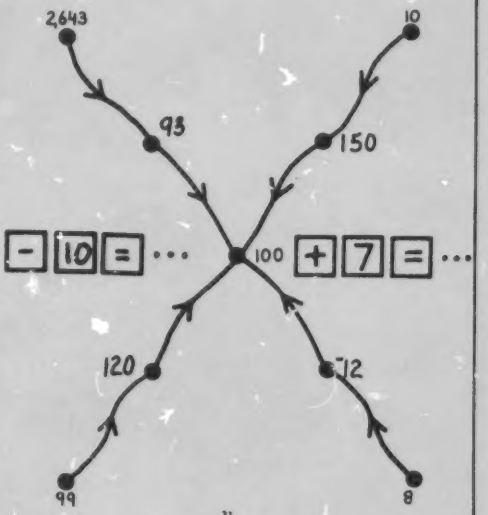


pinwheel

28

30

Label the dots. Many answers are possible.



31

(Many answers are possible.)

First clue

Cho is a secret whole number smaller than 150.

Cho is 10 or 15 or 35 or 55 or 65 or 85 or 95 or 115 or 125 or 145.

Second clue

If you subtract 1 from Cho, you will get a square number.

Cho is 10 or 65 or 145.

Third clue

The square of Cho is larger than 5,000. Cho is 145.

32

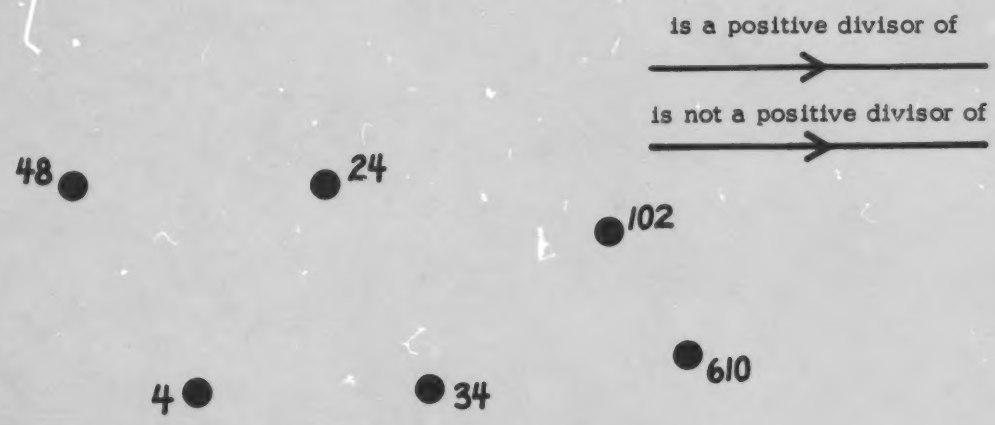
ARROW GAME

ARROW GAME #1

NOTE: Throughout this lesson the arrows that are shown in black should be drawn in white (or in the color of your regular chalk).

Exercise 1

Draw this picture on the board. Spread out the dots so that when arrows are drawn the picture does not become too crowded.



T: Do you see where we could draw a red arrow or a white arrow?

Invite students to point out where arrows could be drawn and to draw them. Each time an arrow is drawn ask the student to explain why the arrow is red or white. For example,

S: A red arrow can be drawn from 4 to 24 because 4 is a positive divisor of 24 (or because $6 \times 4 = 24$).

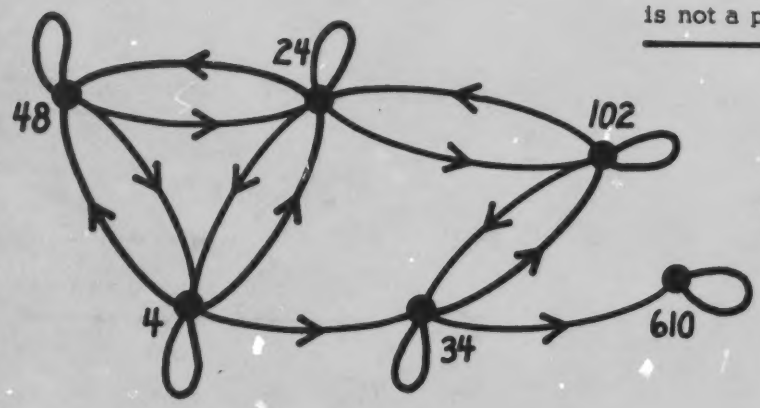
S: A white arrow can be drawn from 4 to 34 because 4 is not a positive divisor of 34.

5W

Continue this activity as long as there is student interest. The next illustration shows a partially complete arrow picture in which all the red arrows have been drawn, but not all of the white arrows.

is a positive divisor of

is not a positive divisor of



Exercise 2

Tape a demonstration Arrow Game analysis sheet to your chalkboard and distribute Worksheet N85.

Draw a red unlabeled key arrow on the board.

IS LARGER THAN	IS SMALLER THAN	IS	
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	10×	+10	2×
+2	$\frac{2}{3}$×	$\frac{3}{2}$×	[+][=]···
[+][2][=]···	[−][2][=]···	[+][3][=]···	[−][3][=]···
[+][4][=]···	[−][4][=]···	[+][5][=]···	[−][5][=]···

T: The red arrow is for a secret relation; one of these 22 relations (point to the chart). You have the same list of relations on your worksheet. I will give you some clues and you must determine the secret relation.

NOTE: For this exercise, the secret relation is [-] [3] [=] . . . . Do not tell this to the students.

T: Here is the first clue

Draw this arrow on the board.

The students should analyze this game in much the same way as the String Game -- crossing off relations that the red arrow cannot be for.

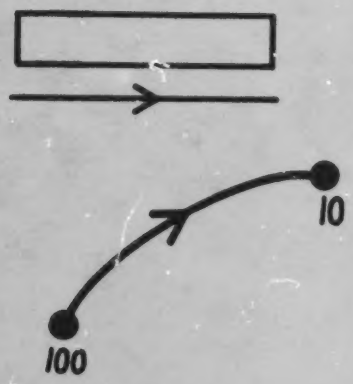
T: What information does this clue give us about the red arrow?

S: The red arrow could be for "is larger than", because 100 is larger than 10.

S: The red arrow cannot be for "is smaller than", because 100 is not smaller than 10.

Use a red marker to cross off the relations on the list of Arrow Game relations that the red arrow cannot be for as they are mentioned. Encourage students to cross off the relations on their lists, too.

5W



IS LARGER THAN	~~IS SMALLER THAN~~	IS	
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	$10\times$	$\div 10$	$2\times$
$\div 2$	$\frac{2}{3}\times$	$\frac{3}{2}\times$	[+][=]···
[+][2][=]···	[−][2][=]···	[+][3][=]···	[−][3][=]···
[+][4][=]···	[−][4][=]···	[+][5][=]···	[−][5][=]···

T: Are there any other relations the red arrow cannot be for?

S: The red arrow cannot be for $\frac{2}{3}\times$, because $\frac{2}{3}\times 100$ is about 66.

S: The red arrow cannot be for $\frac{3}{2}\times$, because $\frac{3}{2}\times 100 = 150$.

T: Could the red arrow be for [−][4][=] ... ?

Let the students consider your question for a minute.

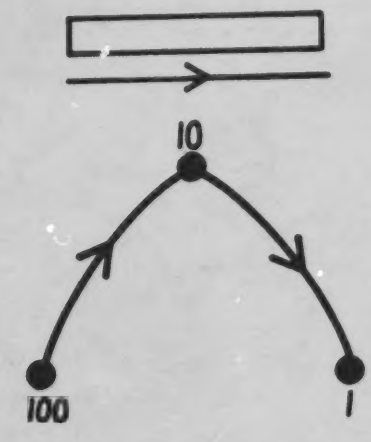
S: I know I can get 60 if I press [=] 10 times, then 20 after another ten, then 16, 12, and 8, but not 10. So the red arrow cannot be for [−][4][=] ... .

S: The red arrow cannot be for [−][4][=] ... , because 100 - 10 = 90 and 90 is not a multiple of 4.

Your students should find that this clue allows them to eliminate (cross off their lists) 14 of the 22 relations. This is indicated by the following illustration of what your Arrow Game list should look like after this clue is fully analyzed.

IS LARGER THAN	IS SMALLER THAN		
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	$10\times$	$\div 10$	$2\times$
$\div 2$	$\frac{2}{3}\times$	$\frac{3}{2}\times$	$+\boxed{1}\boxed{=}\cdots$
$+\boxed{2}\boxed{=}\cdots$	$-\boxed{2}\boxed{=}\cdots$	$+\boxed{3}\boxed{=}\cdots$	$-\boxed{3}\boxed{=}\cdots$
$+\boxed{4}\boxed{=}\cdots$	$-\boxed{4}\boxed{=}\cdots$	$+\boxed{5}\boxed{=}\cdots$	$-\boxed{5}\boxed{=}\cdots$

Add to your picture a red arrow from 10 to 1.



T: The second clue is given by this red arrow starting at 10 and ending at 1. What new information does this clue give us? On your lists, cross off the relations that the red arrow cannot be for.

Let the students have a few minutes to consider the second clue; then continue the analysis as above. This clue eliminates four more relations from your list.

S: 10 is only 9 larger than 1, so "is at least 20 larger than" may be crossed off.

5W

S: " [-] [5] [=] ..." may be crossed off. If you start from 10, you will get 5, then 0 -- but not 1.

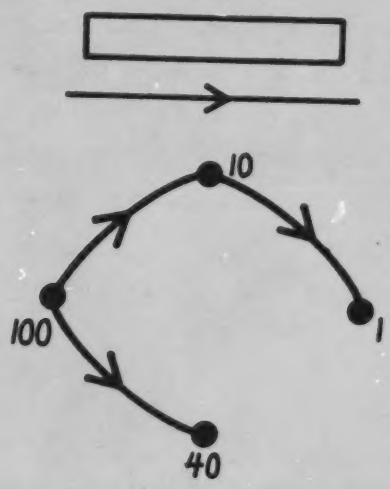
S: 1 x 1 = 1, so 10 is not the square of 1. We can cross off "is the square of".

S: If you start from 10 and press [-] [2] [=] ... , you will get 2, then 0 -- but not 1. Cross off " [-] [2] [=] ...".

After the second clue has been fully analyzed, your Arrow Game analysis sheet should look like this.

IS LARGER THAN	IS SMALLER THAN		
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	10 ×	÷ 10	2 ×
÷ 2	$\frac{2}{3}$ ×	$\frac{3}{2}$ ×	[+] [ ] [=] ...
[+] [2] [=] ...	[-] [2] [=] ...	[+] [3] [=] ...	[-] [3] [=] ...
[+] [4] [=] ...	[-] [4] [=] ...	[+] [5] [=] ...	[-] [5] [=] ...

Add to your picture a red arrow from 100 to 40.



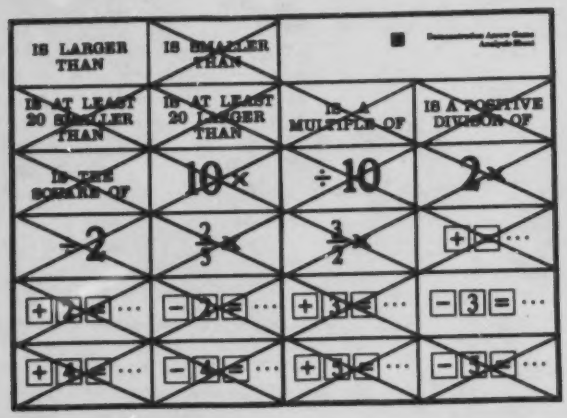
T: What new information does this arrow give us?

Allow a couple of minutes for students to independently consider the new clue.

S: 100 ÷ 10 = 10, not 40. Cross off "÷10".

S: 100 is not a multiple of 40, so we can cross off "is a multiple of".

The third clue only eliminates the two (remaining) relations mentioned above. Your analysis sheet should now have only two relations not crossed off.

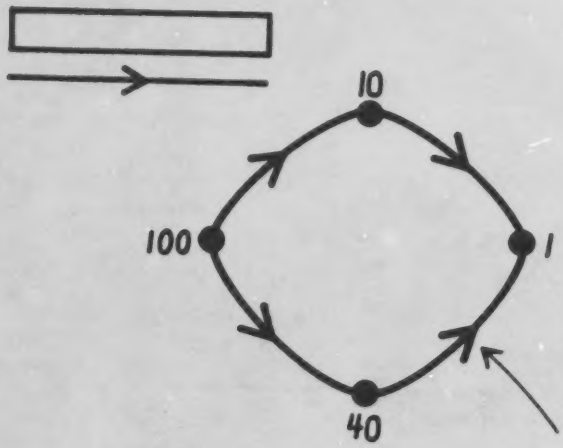


T: The red arrow is for one of these two relations, "is larger than" or [−][3][=] . . . . Now you will have to add some arrows to our picture. You can use these numbers or some other numbers of your choice, but remember, you are trying to find the secret relation.

Let students trace any arrow they wish on the board (it may be necessary for them to draw and label some dots), then you draw the arrow in red or white (or the color of your regular chalk). Draw a white arrow when there is not a red arrow, that is, for the relation "is <u>not</u> in the relation [−][3][=] . . .". When you draw a white arrow explain that the white arrow means that the arrow cannot be red. We cannot predict what arrows your students will trace. However, we will conclude the description of this exercise by reporting the end of the game that was played in one of our development classes.

5W

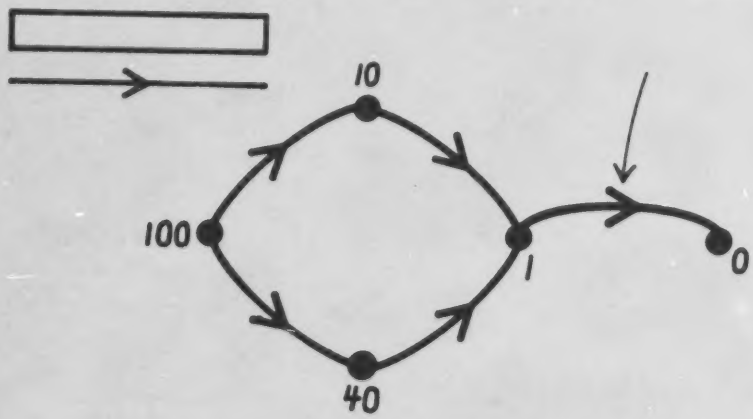
A student traced an arrow from 40 to 1. The teacher drew a red arrow starting from 40 and ending at 1, because 40 - 1 = 39 and 39 is a multiple of 3.



The students were given a few minutes to analyze this new piece of information, and found that this arrow did not help to eliminate either of the two remaining possibilities.

It is not necessary to comment on the quality of such a play. This kind of play may be common for a while until the students develop a better feeling for how the game is played.

Another student labeled a dot "0" and traced an arrow starting at 1 and ending at 0. Because $1 - 3 = {}^{-}2$ (i.e., $1 - 3 \neq 0$), the teacher drew the arrow in white (black in the picture below).

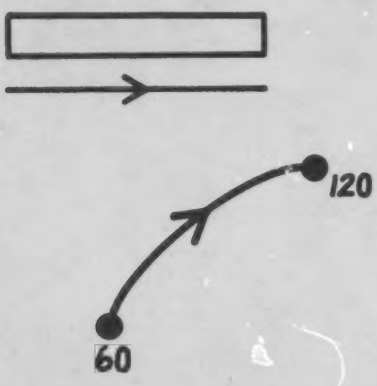


T: What new information does this clue provide?

S: The red arrow cannot be for "is larger than", because 1 is larger than 0 and the arrow is not red, so the red arrow must be for [-][3][=] . . . .

Exercise 3

Erase the board, put up a new Arrow Game analysis sheet, and draw this picture on the board. Ask the students to turn over their worksheets to find another list of the 22 Arrow Game relations.



IS LARGER THAN	IS SMALLER THAN	[redacted]	
IS AT LEAST 30 SMALLER THAN	IS AT LEAST 30 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	$10\times$	$\div 10$	$2\times$
$\div 2$	$\frac{2}{3}\times$	$\frac{3}{2}\times$	[+][=] ···
[+][2][=] ···	[−][2][=] ···	[+][3][=] ···	[−][3][=] ···
[+][4][=] ···	[−][4][=] ···	[+][5][=] ···	[−][5][=] ···

NOTE: For this game the secret relation is $2x$. Don't tell this to the students.

T: The red arrow is for one of these 22 relations. The first clue is on the board. What information does this clue give us about the red arrow? On your lists, cross off the relations that the red arrow cannot be for.

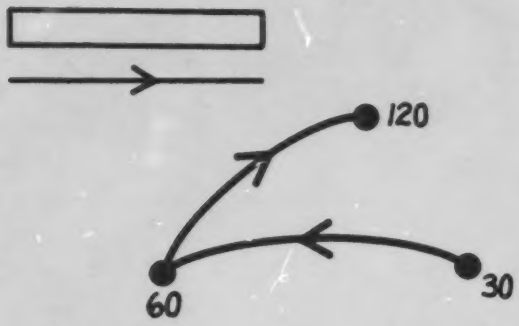
Let the students have several minutes to consider this clue, as there are 13 relations which can be crossed off. Then proceed with a collective discussion of this clue. Encourage students to explain why a relation can or cannot be crossed off.

5W

The following illustration shows what your Arrow Game analysis Sheet should look like after all 13 relations that the red arrow cannot be for have been crossed off.

~~IS LARGER THAN~~	IS SMALLER THAN		
IS AT LEAST 20 SMALLER THAN	~~IS AT LEAST 20 LARGER THAN~~	~~IS A MULTIPLE OF~~	IS A POSITIVE DIVISOR OF
~~IS THE SQUARE OF~~	~~10×~~	~~÷10~~	2×
~~÷2~~	~~$\frac{2}{3}$×~~	~~$\frac{3}{2}$×~~	[+][=]···
[+][2][=]···	~~[−][2][=]···~~	[+][3][=]···	~~[−][3][=]···~~
[+][4][=]···	~~[−][4][=]···~~	[+][5][=]···	~~[−][5][=]···~~

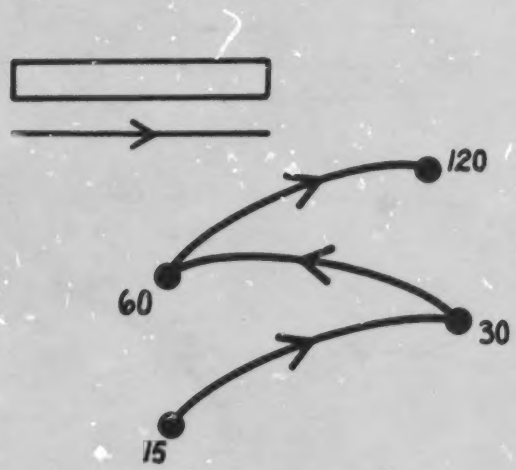
Add to your picture a red arrow from 30 to 60.



T: What new information does this clue provide?

The second clue only provides one additional piece of information. Because 60 - 30 = 30 and 30 is not a multiple of four, [+][4][=] ... may be eliminated.

Add to your picture a red arrow from 15 to 30.



T: Who can tell me some relations that can be crossed off?

S: " [+] [2] [=] ..." can be crossed off. If you start with 15 and add 2 each time, you will get only odd numbers.

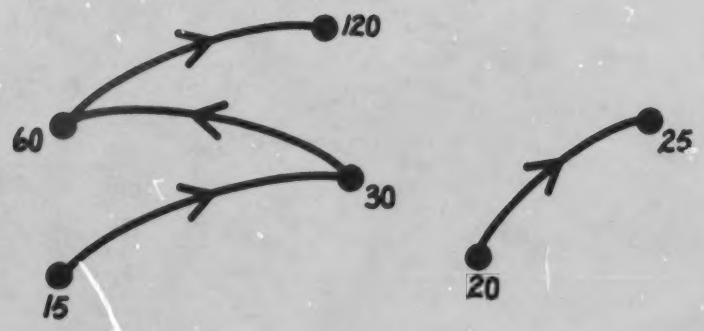
S: 15 is only 15 smaller than 30, so "is at least 20 smaller than" can be crossed off.

Be sure you cross off these two relations as they are mentioned. Your analysis sheet should look like this.

~~IS LARGER THAN~~	~~IS SMALLER THAN~~	■	
~~IS AT LEAST 20 SMALLER THAN~~	~~IS AT LEAST 20 LARGER THAN~~	~~IS A MULTIPLE OF~~	IS A POSITIVE DIVISOR OF
~~IS THE SQUARE OF~~	~~10×~~	~~÷10~~	2×
~~÷2~~	~~$\frac{2}{3}$×~~	~~$\frac{3}{2}$×~~	[+][=]···
~~[+][2][=]···~~	~~[−][2][=]···~~	[+][3][=]···	~~[−][3][=]···~~
~~[+][4][=]···~~	~~[−][4][=]···~~	[+][5][=]···	~~[−][5][=]···~~

5W

For the fourth clue, draw this white arrow (shown in black below).



T: The fourth clue is given by a <u>white</u> arrow. Which relations can be crossed off?

Allow a minute or two for the students to work independently.

S: 20 is smaller than 25 and the arrow is not red, so we can cross off "is smaller than".

S: Cross off " [+] [5] [=] ...", because 20 + 5 = 25 and the arrow is <u>not</u> red.

T: Could the red arrow be for 2 x ?

S: Yes, because 2 x 20 is <u>not</u> 25 and the arrow from 20 to 25 is white.

The relations, "is a positive divisor of", [+] [=] ... , and [+] [3] [=] ... may be discussed in a similar manner.

Your analysis sheet should look like this after you have crossed off "is smaller than" and " [+] [5] [=] ...".

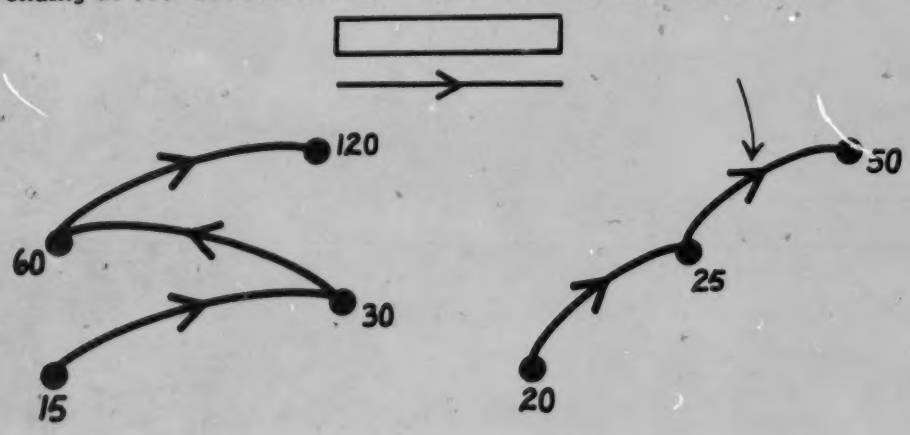
~~IS LARGER THAN~~	~~IS SMALLER THAN~~	■ Demonstration Answer Sheet Analysis Sheet	
~~IS AT LEAST 20 SMALLER THAN~~	~~IS AT LEAST 20 LARGER THAN~~	~~IS A MULTIPLE OF~~	IS A POSITIVE DIVISOR OF
~~IS THE SQUARE OF~~	~~10×~~	~~÷10~~	2×
~~÷2~~	~~$\frac{2}{3}$×~~	~~$\frac{3}{2}$×~~	[+][=]···
~~[+][2][=]···~~	~~[−][2][=]···~~	[+][3][=]···	~~[−][3][=]···~~
~~[+][4][=]···~~	~~[−][4][=]···~~	~~[+][5][=]···~~	~~[−][5][=]···~~

T: There are still four possibilities. Now it is your turn to provide the clues. Who would like to trace an arrow?

Invite students to trace any arrow they wish on the board; then you draw the arrow in red or white (or the color of your regular chalk).

In one of our development classes, the game was concluded as described below.

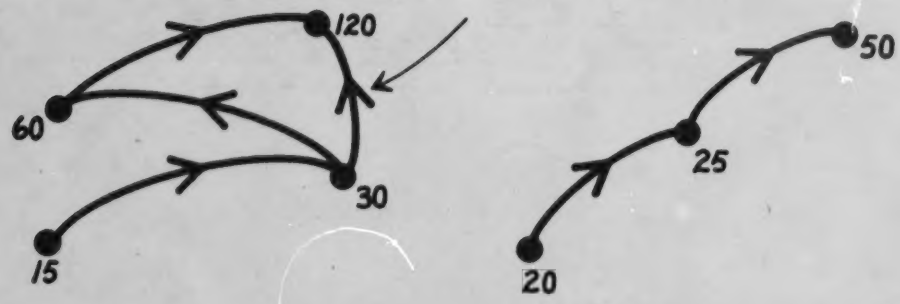
A student drew a dot and labeled it "50"; then traced an arrow starting at 25 and ending at 50. Because 2 x 25 = 50, the teacher drew the arrow in red.



5W

Since 50 - 25 = 25 and 25 is not a multiple of three, $\boxed{+}\boxed{3}\boxed{=}$ ... was eliminated. This is the only relation that the clue eliminated.

Another student traced an arrow starting from 30 and ending at 120. The teacher drew the arrow in white (black in the picture below) because $2 \times 30 \neq 120$.



This clue solved the mystery. Since 120 is a multiple of 30 and the arrow is white (i.e., not red), $\boxed{+}\boxed{=}$ ... was eliminated. Likewise, "is a positive divisor of" was crossed off, because 30 is a positive divisor of 120 and the arrow is white.

The students concluded that the secret relation is 2 x .

112

T: The third clue is given by this <u>white</u> arrow (trace it).
Which relations can be crossed off ?

S: The red arrow cannot be for "is larger than".

T: Why?

S: Because 19 is larger than 4 and the arrow from 19 to 4 is not red.

Of the remaining relations, the third clue eliminates only "is larger than".
Be sure you cross it off.

Four relations should be remaining.

IS LARGER THAN	IS SMALLER THAN		
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	$10\times$	$\div 10$	$2\times$
$\div 2$	$\frac{2}{3}\times$	$\frac{3}{2}\times$	$\boxed{+}\boxed{\times}\cdots$
$\boxed{+}\boxed{2}\boxed{=}\cdots$	$\boxed{-}\boxed{2}\boxed{=}\cdots$	$\boxed{+}\boxed{3}\boxed{=}\cdots$	$\boxed{-}\boxed{3}\boxed{=}\cdots$
$\boxed{+}\boxed{4}\boxed{=}\cdots$	$\boxed{-}\boxed{4}\boxed{=}\cdots$	$\boxed{+}\boxed{5}\boxed{=}\cdots$	$\boxed{-}\boxed{5}\boxed{=}\cdots$

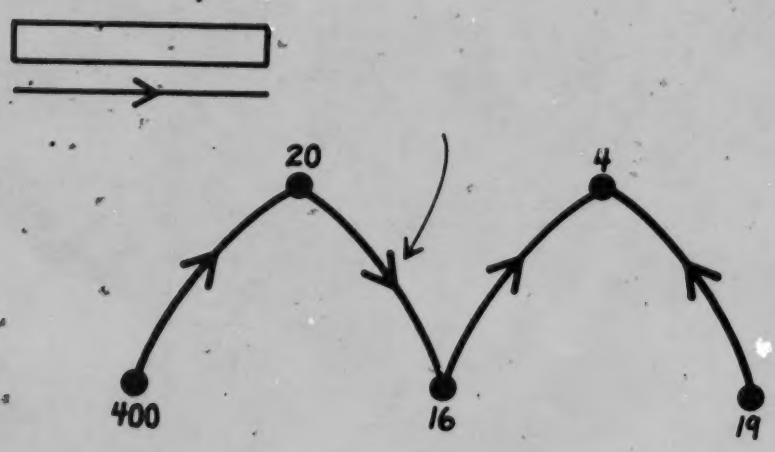
5W

Invite students to trace any arrow they wish on the board. Encourage them to think about using numbers that are not in the arrow picture. Then, tell them whether the arrow is red or white (be sure to draw the arrow).

Remember, for this game the secret relation is $\boxed{-}\ \boxed{2}\ \boxed{=}$ . . . . Do not tell this to the class. Let them try to discover the secret relation from the colors of the arrows they trace. Encourage the class to try to find the secret relation with as few additional clues as they can.

We cannot predict which arrows your students will draw. However, we will conclude the description of this lesson by completing and analyzing the game as it happened in one of our development classes.

A student traced an arrow starting at 20 and ending at 16. Because 20 - 16 = 4 and 4 is a multiple of 2, the teacher drew the arrow in red.



ARROW GAME #2

Exercise 1

Draw this arrow picture on the board.

T: What could this red arrow be for?

There are many relations that the red arrow could be for. Encourage the students to explore this very open situation. It will be up to you to keep the discussion moving along; ask a leading question when it seems appropriate.

In our development classes, these relations were suggested: $3x$; $+ 80$; "is less than"; $\boxed{+}\ \boxed{10}\ \boxed{=}$ ...; "is a positive divisor of"; "is 80 smaller than"; and $\boxed{+}\ \boxed{=}$ .... .

T: Could the red arrow be for $\frac{1}{2}x$ followed by $+ 100$?

S: Yes, because $\frac{1}{2} \times 40 = 20$ and $20 + 100 = 120$.

T: What other relations could the red arrow be for?

S: $\frac{1}{4}x$ followed by $+ 110$.

S: $\frac{1}{8}x$ followed by $24x$. NOTE: This is the same as $3x$, but there is no need to insist on this fact.

15W

Be sure to check each suggestion made by a student.

T: Could the red arrow be for "is <u>not</u> the double of"?

S: Yes, because 2 x 40 = 80 and not 120.

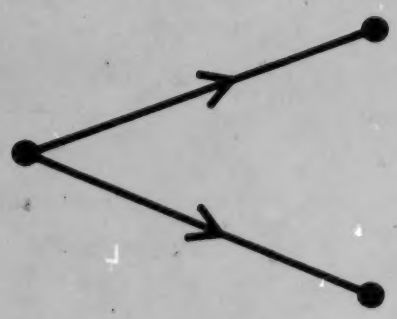
T: Can you think of any other relations the red arrow could be for?

S: "Is not more than".

S: "Is at least 20 smaller than".

<u>Exercise 2</u>

Draw this on the board.



T: What could the red arrows be for? Don't forget the red arrows are for the same relation.

S: "Is less than".

S: "Is greater than".

S: Any relation that is not a function.

There are, of course, an unlimited number of possibilities, but some other relations that the red arrow could be for are: "is a multiple of", "is a

divisor of", [+] [5] [=] ... , [-] [3] [=] ... , [+] [=] ... , and 4 x followed by + 1 <u>or</u> ⁻1.

When there is a lull in the discussion, ask:

T: Can you think of any non-numerical relations that the red arrow could be for?

Let the students have a few minutes to consider this situation. If no suggestions are forthcoming, ask:

T: Could the red arrow be for the relation "I love you"?

S: Yes, one person can love two or more other people.

T: What other relations could the red arrow be for?

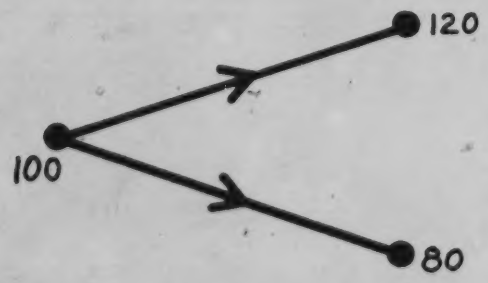
The above suggestion should spark many non-numerical relations, such as: "you are my friend"; "you are my relative"; "you are my teacher"; etc.

T: Can you think of any relations that the red arrow could <u>not</u> be for?

The red arrow cannot be for any relation that is a function; for example, "you are my mother (father)"; "you are my maternal grandmother (grandfather)"; + 5; 7 x; etc.

<u>Exercise 3</u>

Draw this arrow picture on the board.

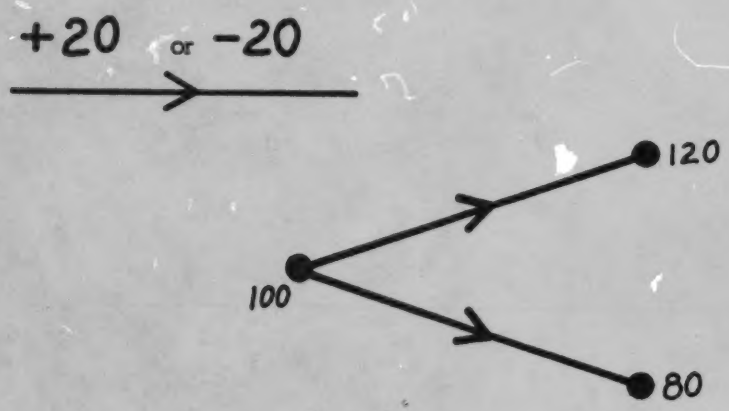


15W

T: What could the red arrows be for?

S: + 20 <u>or</u> - 20.

Add this red key-arrow to your picture.



T (tracing the appropriate arrows): 100 + 20 = 120. 100 - 20 = 80.
Good. What other relations could the red arrows be for?

S: [+] [4] [=] ... <u>or</u> [-] [4] [=] ... .

T: Could the red arrows be for "is not a positive divisor of"?

S: Yes. 100 is not a positive divisor of 120 or of 80.

T: Can you think of any other relations the red arrows could be for?

S: "Is not a multiple of".

S: "Is at least 20 smaller than <u>or</u> is at least 20 larger than".

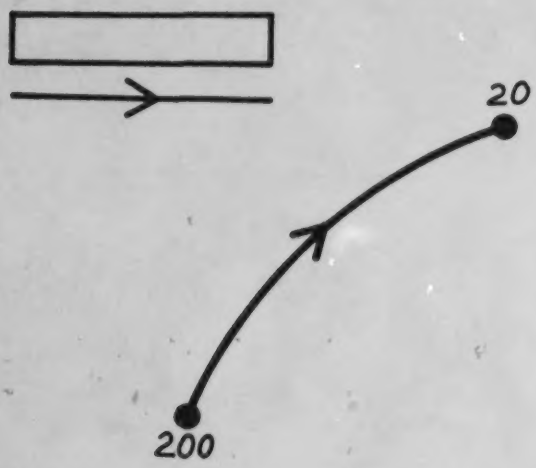
<u>Exercise 4</u>

Distribute Worksheet N86, tape an Arrow Game analysis sheet to your chalkboard, and draw a red unlabeled key arrow on the board.

NOTE: For this exercise the secret relation is "is a multiple of". Do not tell this to the students.

T: The red arrow is for one of these 22 relations (point to the chart). I will give you some clues and you will have to determine the secret relation. Here is the first clue.

Draw this red arrow.



IS LARGER THAN	IS SMALLER THAN	[illegible]	
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	$10\times$	$\div 10$	$2\times$
$\div 2$	$\frac{2}{3}\times$	$\frac{3}{2}\times$	$\boxed{+}\boxed{\blacksquare}\cdots$
$\boxed{+}\boxed{2}\boxed{\blacksquare}\cdots$	$\boxed{-}\boxed{2}\boxed{\blacksquare}\cdots$	$\boxed{+}\boxed{3}\boxed{\blacksquare}\cdots$	$\boxed{-}\boxed{3}\boxed{\blacksquare}\cdots$
$\boxed{+}\boxed{4}\boxed{\blacksquare}\cdots$	$\boxed{-}\boxed{4}\boxed{\blacksquare}\cdots$	$\boxed{+}\boxed{5}\boxed{\blacksquare}\cdots$	$\boxed{-}\boxed{5}\boxed{\blacksquare}\cdots$

Let the students have a few minutes to cross off the relations that the red arrow cannot be for. If some students have difficulty getting started, ask:

T: Could the red arrow be for "is larger than"?

S: Yes, 200 is larger than 20.

T: Could the red arrow be for "is smaller than"?

S: No, cross it off.

T: Could the red arrow be for "is at least 20 smaller than"?

S: No, 200 is larger than 20.

15W

T: Tell me some other relations the red arrow cannot be for.

Let the students make as many suggestions as they can. Encourage them to explain why the red arrow cannot be for a particular relation. It may be necessary for you to help the explanation along. For example, in our development class, a student suggested that [-] [3] [=] ... should be eliminated.

T: Let's check to see if " [-] [3] [=] ..." should be crossed off. Imagine that 200 is on the display of a hand-calculator and that we press [-] [3] , then press [=] ten times. What number would be on the display?

S: 170, because 10 x 3 = 30 and 200 - 30 = 170.

T: Good. What number would be on the display if we were to press [=] ten more times?

S: 140, because 170 - 30 = 140.

T: If we press [=] ten more times?

S: 110.

T: If we press [=] ten more times?

S: 80.

T: If we press [=] twenty more times?

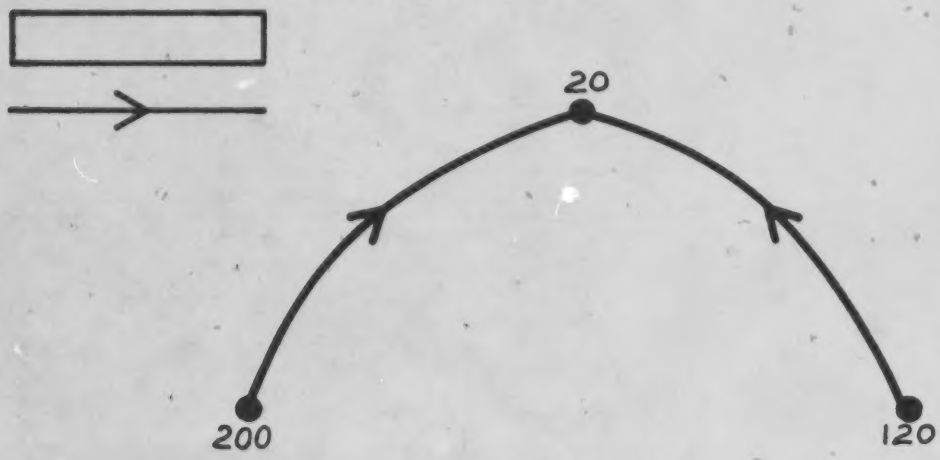
S: 20, because 80 - 60 = 20, so we cannot cross " [-] [3] [=] ..." off.

Some students may notice that 200 - 20 = 180 which is a multiple of 3 and so " [-] [3] [=] ..." cannot be crossed off. It is not necessary for you to raise this point if no one mentions it.

The first clue eliminates 14 of the 22 relations. Continue the analysis until all 14 have been crossed off on the analysis sheet as in the next illustration.

IS LARGER THAN	IS SMALLER THAN		
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	10 ×	÷ 10	2 ×
÷ 2	$\frac{2}{3}$ ×	$\frac{3}{2}$ ×	[+] [=] ···
[+] [2] [=] ···	[−] [2] [=] ···	[+] [3] [=] ···	[−] [3] [=] ···
[+] [4] [=] ···	[−] [4] [=] ···	[+] [5] [=] ···	[−] [5] [=] ···

Add to your picture a red arrow from 120 to 20.



15W

T: The second clue is this red arrow (tracing the arrow from 120 to 20). What new information does this clue give us?

Allow a few minutes for the students to work independently.

S: The red arrow cannot be for ÷10.

T: Why?

S: 120 ÷ 10 = 12, not 20.

T: Anything else?

S: The red arrow cannot be for [−] [3] [=] . . . .

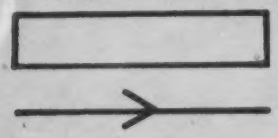
T: Why?

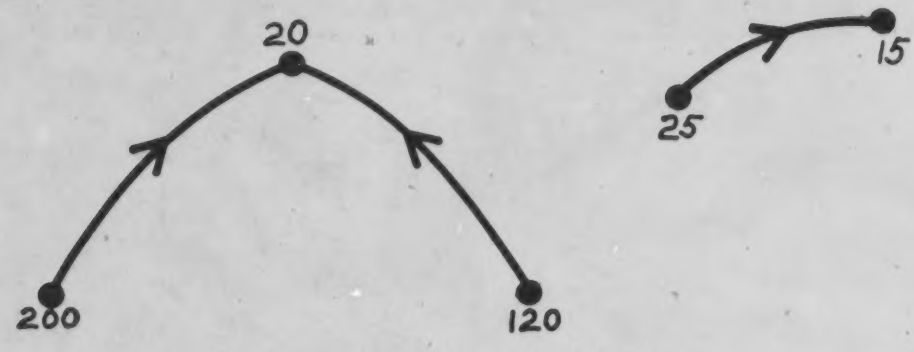
S: 120 - 20 = 100, but 100 is not a multiple of 3.

The second clue only eliminates the two relations discussed above. You should now have 16 relations crossed off on your analysis sheet.

IS LARGER THAN	IS SMALLER THAN	■ Demonstrating Arrow Clues Analysis Sheet	
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	10 ×	÷ 10	2 ×
÷ 2	$\frac{2}{3}$ ×	$\frac{3}{2}$ ×	[+] [ ] [=] ···
[+] [2] [=] ···	[−] [2] [=] ···	[+] [3] [=] ···	[−] [3] [=] ···
[+] [4] [=] ···	[−] [4] [=] ···	[+] [5] [=] ···	[−] [5] [=] ···

Draw this white arrow (shown in black) on the board.





T: The third clue is given by this white arrow (trace it). The white arrow means we <u>cannot</u> draw an arrow from 25 to 15 in red.

T: Tell me some relations that the red arrow cannot be for.

S: The red arrow cannot be for "is larger than".

T: Why?

S: Because 25 is larger than 15, and the arrow is <u>not</u> red.

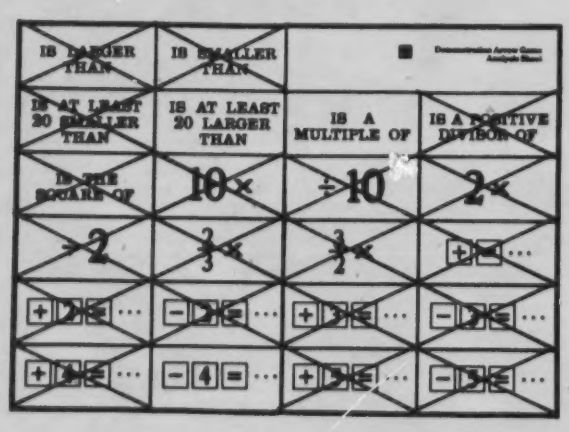
T: Are there other relations that can be eliminated?

S: If we put 25 on the display and press [-] [2] [=] [=] [=] [=] [=] , we get 15. So if the red arrows were for the relation [-] [2] [=] . . . , then the arrow should be in red. Since it's in white, the relation cannot be [-] [2] [=] . . . .

S: [-] [5] [=] . . . because if I start from 25 then 20 will appear, then 15, so the arrow would be red if the relation were [-] [5] [=] . . . . Since it's in white, the relation cannot be [-] [5] [=] . . . .

15W

These three relations, "is larger than", [-] [2] [=] ..., and [-] [5] [=] ..., are the only relations that the third clue eliminates. Your analysis sheet should now have all but three relations crossed off.

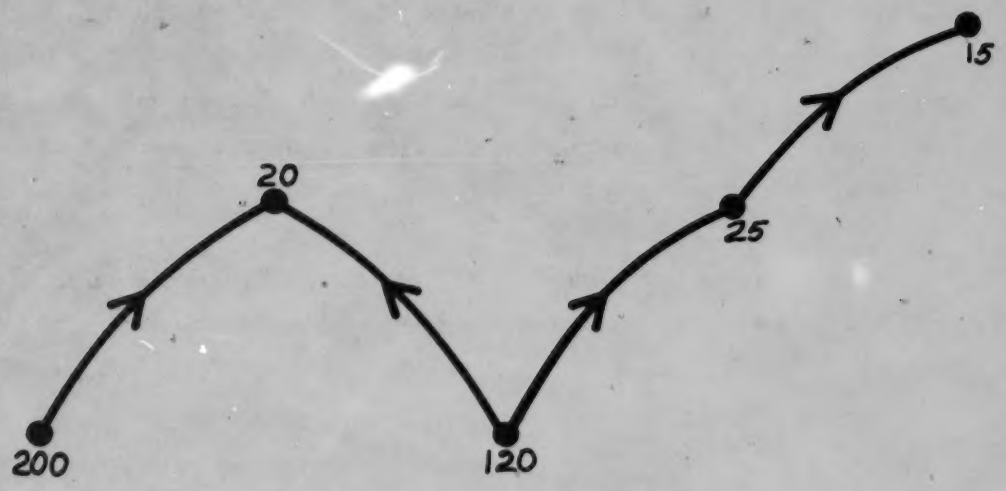


Invite students to trace any arrow they wish on the board. Encourage the students to think about using numbers that are not already in the arrow picture. Then tell them if the arrow is red or white (be sure that this arrow is drawn).

Recall that for this game, the secret relation is "is a multiple of". Therefore, an arrow should be red if and only if the starting number is a multiple of the ending number. An arrow should be white if the starting number is <u>not</u> a multiple of the ending number.

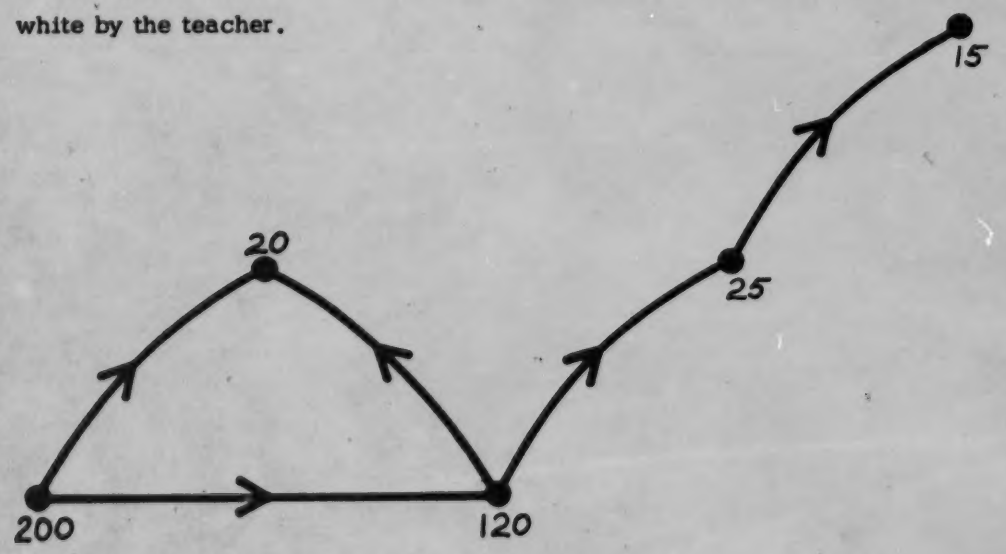
We cannot predict what arrows your students will draw. However, we will conclude the description of this lesson by completing and analyzing this game the way it was completed in one of our development classes.

A student traced an arrow from 120 to 25.



Since 120 is not a multiple of 25, the arrow was drawn in white. With this new information, the class was able to eliminate the relation "is at least 20 larger than", because 120 is at least 20 larger than 25 and the arrow from 120 to 25 is white.

Another student traced an arrow from 200 to 120 which was then drawn in white by the teacher.



This arrow was drawn in white because 200 is <u>not</u> a multiple of 120. With this clue, the class eliminated [-] [4] [=] ... (200 - 120 = 80 and 80 is a

multiple of 4) and concluded that the secret relation is "is a multiple of".

It is interesting to note that if the arrow from 200 to 120 had been drawn first, the secret relation would have been determined without any additional arrows being needed.

If there is time remaining in the lesson, the Workbook "A Variety of Problems 3" could be returned and the remaining time spent working on problems in the workbooks.

ARROW GAME #3

Exercise 1

Draw this arrow picture on the board.

T: What could this red arrow be for?

There are many relations that the red arrow could be for. Encourage your students to be creative. It will be up to you to keep the discussion moving along; ask a leading question when it seems appropriate.

In our development classes these relations were suggested: 5 x ; + 20; "is a positive divisor of"; + 19.999... (or + 19.$\dot{9}$); "you are my square"; and [+] [4] [=] . . . .

T: The red arrow could be for 3 x followed by what?

S: + 10, because 3 x 5 = 15 and 15 + 10 = 25.

T: What other relations could the red arrow be for?

S: 4 x followed by + 5, because 4 x 5 = 20 and 20 + 5 = 25.

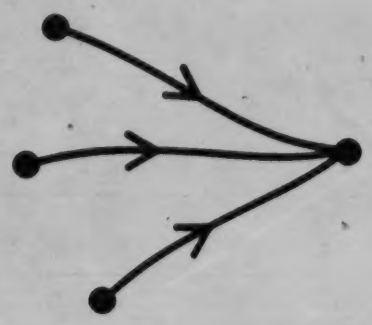
S: [+] [5] [=] . . . .

S: "Is at least 10 smaller than".

Encourage the class to participate in checking each suggestion that is made.

Exercise 2

Draw this arrow picture on the board.



T: What could the red arrows be for?

Again let the students suggest several numerical and non-numerical relations. For example,

S: "Is larger than".

S: $\boxed{+}\boxed{1}\boxed{=}$ . . . .

S: "Is a multiple of".

S: "You are my mother".

T: If the red arrows were for "you are my mother", where would there be a mother in this picture?

S: The ending dots of all the arrows would be for mothers.

T: Would the three other dots be for boys or girls?

S: Can't tell.

T: Can you think of any other relations the red arrows could be for?

S: "You are my teacher".

S: "I love you".

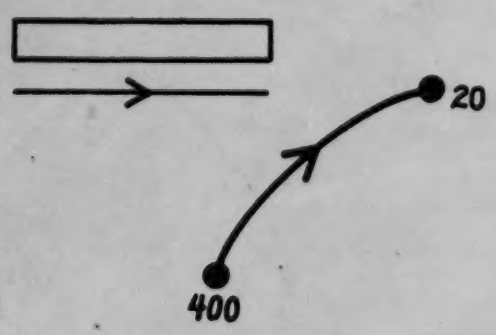
Exercise 3

Distribute Worksheet N87. Tape a demonstration Arrow Game analysis sheet to your chalkboard, and draw a red key arrow on the board.

NOTE: For this exercise the secret relation is [-] [2] [=] . . . . Do not tell this to the students.

T: The red arrow is for one of these 22 relations. I will give you some clues and you will have to determine the secret relation. Here is the first clue.

Draw this arrow on the board.



18 W

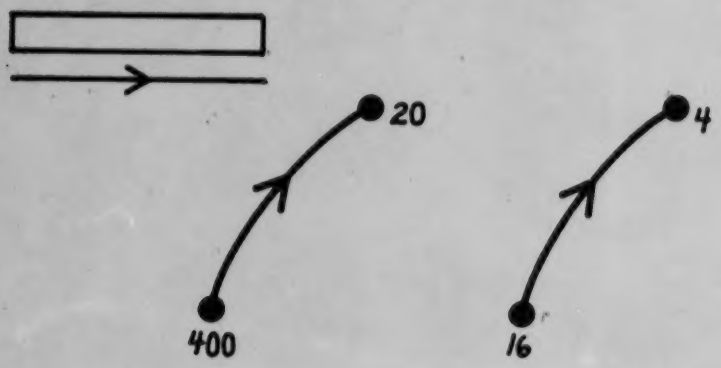
Before collectively discussing this clue, let the students have a few minutes to cross off on their worksheets the relations that the red arrow cannot be for.

Ask students to make many suggestions of relations that can be crossed off on the demonstration analysis sheet. Encourage them to explain why the red arrow cannot be for a particular relation. At times it may be necessary for you to help the explanation along.

The first clue eliminates 15 of the 22 relations as shown below. Continue the analysis until all 15 have been eliminated.

IS LARGER THAN	IS SMALLER THAN		
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	$\times 10$	$\div 10$	$\times 2$
$\div 2$	$\times \frac{2}{3}$	$\times \frac{3}{2}$	$\boxed{+}\boxed{=}\cdots$
$\boxed{+}\boxed{2}\boxed{=}\cdots$	$\boxed{-}\boxed{2}\boxed{=}\cdots$	$\boxed{+}\boxed{3}\boxed{=}\cdots$	$\boxed{-}\boxed{3}\boxed{=}\cdots$
$\boxed{+}\boxed{4}\boxed{=}\cdots$	$\boxed{-}\boxed{4}\boxed{=}\cdots$	$\boxed{+}\boxed{5}\boxed{=}\cdots$	$\boxed{-}\boxed{5}\boxed{=}\cdots$

Add to your picture a red arrow from 16 to 4.



T: The second clue is given by this red arrow (trace the arrow starting at 16 and ending at 4).
What new information does this clue tell us?

Give the class a few minutes to work independently; then ask for suggestions.

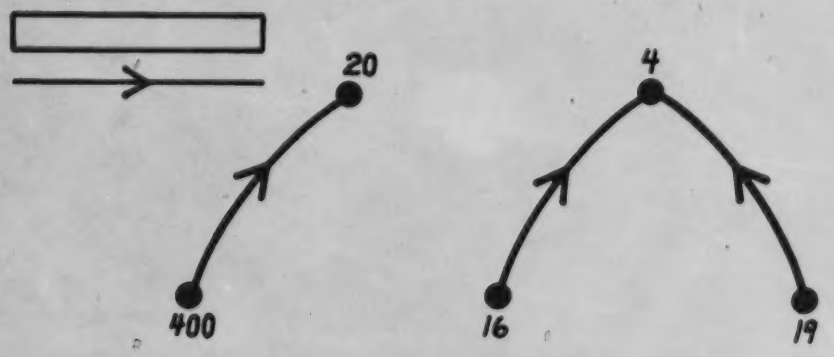
S: The red arrow cannot be for "is at least 20 larger than", because 16 is only 12 larger than 4 and the arrow is red.

S: Cross off " [-] [5] [=] ... ". If you start at 16 and subtract fives, you will get 11, 6, and 1, but not 4.

The second clue eliminates only the two relations discussed above. You should now have 17 relations crossed off the demonstration analysis sheet.

IS LARGER THAN	IS SMALLER THAN		
IS AT LEAST 20 SMALLER THAN	IS AT LEAST 20 LARGER THAN	IS A MULTIPLE OF	IS A POSITIVE DIVISOR OF
IS THE SQUARE OF	10 ×	÷ 10	2 ×
÷ 2	$\frac{2}{3}$ ×	$\frac{3}{2}$ ×	[+] [=] ···
[+] [2] [=] ···	[-] [2] [=] ···	[+] [3] [=] ···	[-] [3] [=] ···
[+] [4] [=] ···	[-] [4] [=] ···	[+] [5] [=] ···	[-] [5] [=] ···

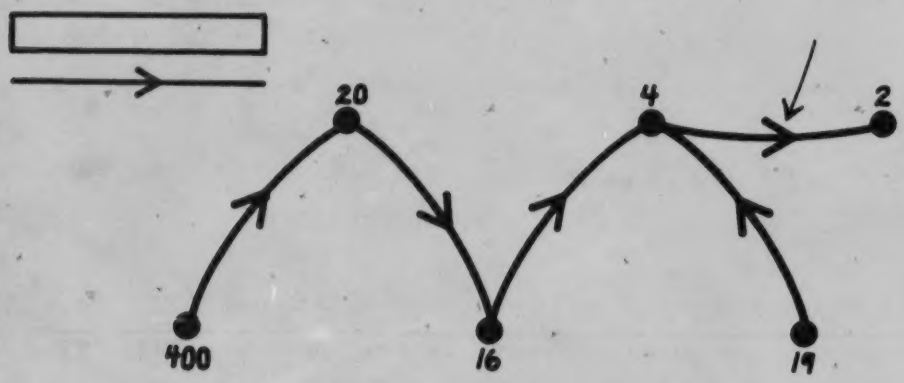
Draw this white arrow (shown in black below) on the board.



18 W

This clue eliminates two relations: "is a multiple of" and "is the square of".

Another student drew a dot and labeled it "2", then traced an arrow starting at 4 and ending at 2. The teacher drew the arrow in red, because 4 - 2 = 2 and 2 is a multiple of 2.



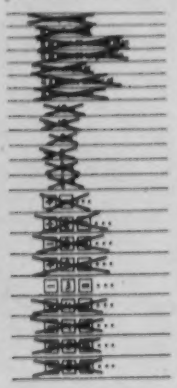
This clue eliminated [-] [4] [=] ... and the students concluded that the secret relation is [-] [2] [=] ... .

Worksheets N88, N89, N90, and N91 are available.

Name _____ **N 88** *

Complete the arrow label.

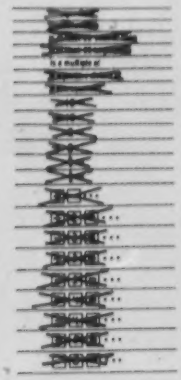
The red arrow could be for:



Name _____ **N 89** **

Complete the arrow label.

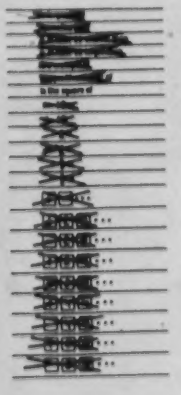
The red arrow could be for:



Name _____ **N 90** ***

Complete the arrow label.

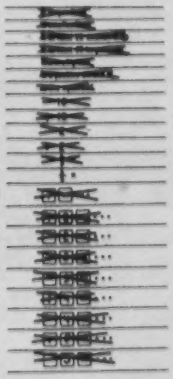
The red arrow could be for:



Name _____ **N 91** ****

Complete the arrow label.

The red arrow could be for:



A VARIETY OF PROBLEMS 3

This workbook has problems of varying levels of difficulty (*, **, and ***) all in one booklet — approximately the first 10 pages of which are easy (*) problems, the next 10 to 12 pages are average (**) problems and the last 10 pages are more challenging (***) problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for this workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about 20 to 25 pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Lesson 1

Give each student a copy of the workbook "A Variety of Problems 3" and let them work on their own.

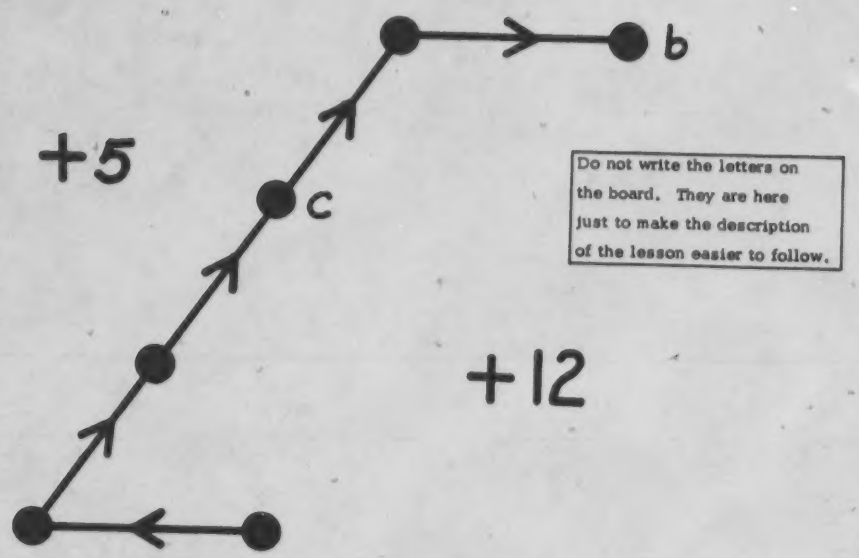
If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

7 W

Lesson 2

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: 36 is the only multiple of 6 in this arrow picture. Where is 36? Label all the dots.

Give the students a minute to think about this problem.

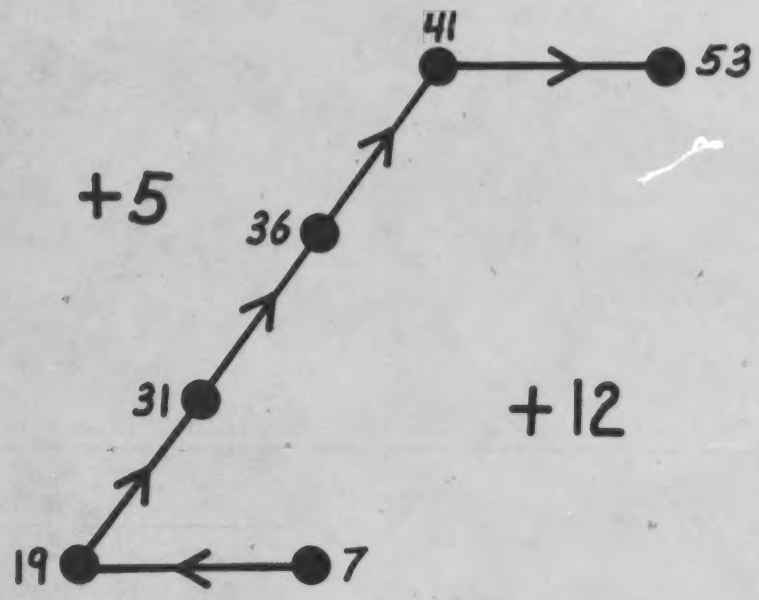
T (pointing to b): Could 36 be here?

S: No. If 36 were there, then 24 would be in the picture because 36 - 12 = 24. But 24 is another multiple of 6, so 36 cannot be there.

Encourage the students to continue discussing this problem until they decide that 36 is at c.

S (pointing to c): 36 is here. If 5 is added or subtracted from 36, you get 41 or 31. Adding or subtracting 12 from 41 or 31 will not give a multiple of 6.

Complete the activity by labeling the dots in the arrow picture. A completed picture is shown below.



Return the students' copies of "A Variety of Problems 3". Ask them to correct their mistakes (if any), and then to continue working in the workbooks.

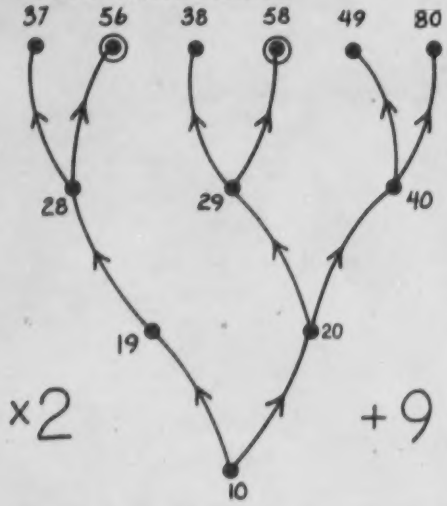
If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the conclusion of this lesson, collect the workbooks for your progress check. You may wish to make this workbook available later for students to work in at their leisure.

A Variety of Problems 3

Label the dots.

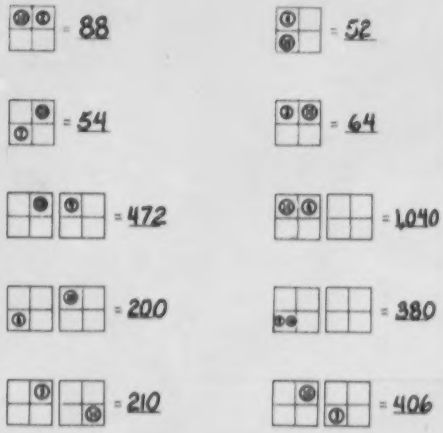
56 and 58 are in this picture. Circle their dots.



2

Put any number you wish on the Minicomputer using one (10)- checker and exactly one of these checkers:

(2) (3) (4) (5) (6) (7) (8) (9)



3

Each student will most likely complete this page in a way different from anyone else. The answers given above are just a sample way to complete this page.

☐ has area 1 cm^2.

Use one color to color all shapes of area 1 cm^2.

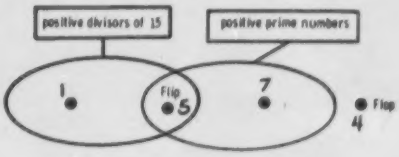
Use another color to color all shapes of area 2 cm^2.

4

(Any two colors will suffice. We used red and blue.)

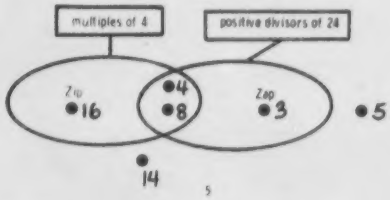
Flip and Flop are two of these numbers: 1; 4; 5; 7.

Flip is 5; Flop is 4.



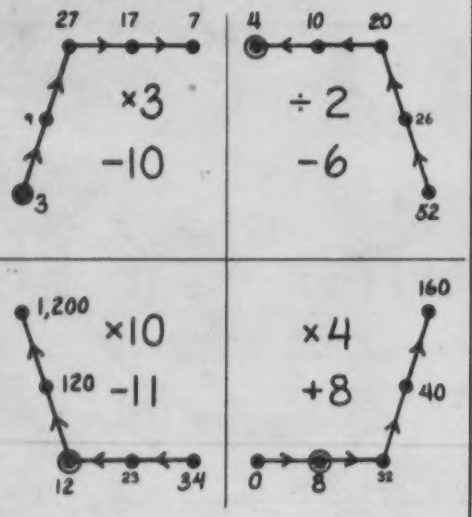
Zip and Zap are two of these numbers: 3; 4; 5; 8; 14; 16.

Zip is 16; Zap is 3.



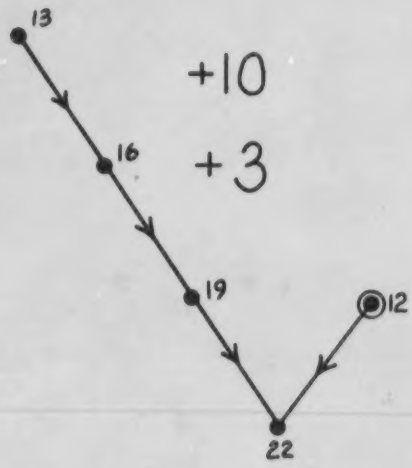
5

A positive divisor of 24 is in each of these pictures. In each picture, circle its dot. Label all the dots.



6

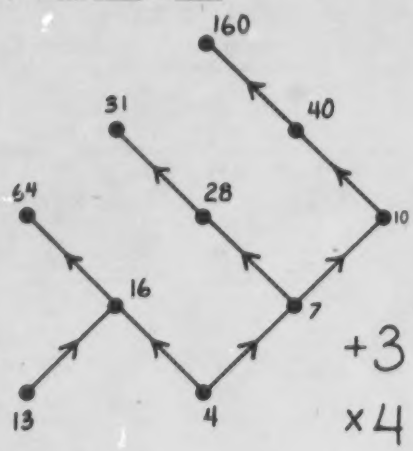
12 is the only multiple of 3 in this picture. Where is 12?
Label all the dots.



7

Label all the dots.
There are exactly three prime numbers in this arrow picture.

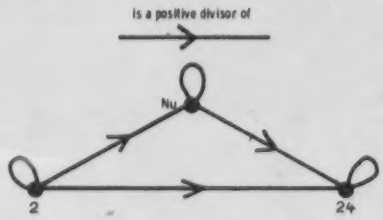
They are 7 ; 13 ; and 31 .



8

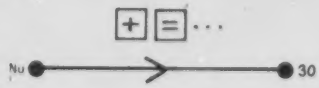
Nu is a secret number.

First clue



Nu is 4 or 6 or 8 or 12

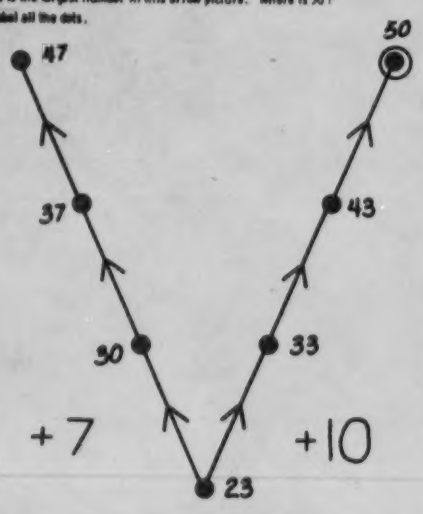
Second clue



Nu is 6

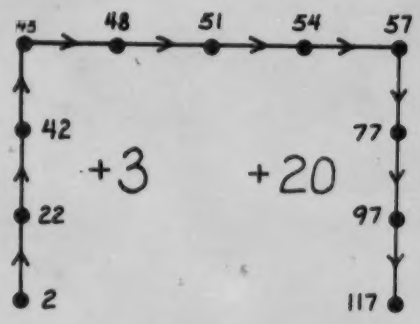
9

50 is the largest number in this arrow picture. Where is 50?
Label all the dots.



10

Label the dots.



Paz is the only number in the arrow picture above that can be put on the ones' board of the Minicomputer using exactly one of these checkers:

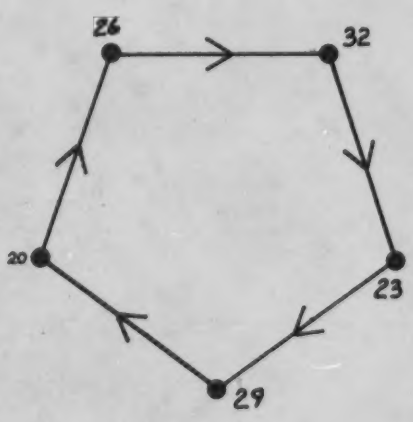
③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Paz = ⑧ = 48.

11

Label the dots and fill in the box for the red arrow.

+6 − $\boxed{9}$



12

28;	✗	18;	36;	✗	24;	✗
32;	✗	48;	64;	✗	56;	40

From this list, cross out the five numbers that cannot be put on the ones' board of the Minicomputer using exactly one of these checkers:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Put the nine other numbers on the ones' board of the Minicomputer using exactly one of these checkers.

⑦ = 28 ⑨ = 18 ⑥ = 36

⑥ = 24 ⑧ = 32 ⑧ = 48

⑧ = 64 ⑦ = 56 ⑦ = 40

13

(Other solutions are possible.)

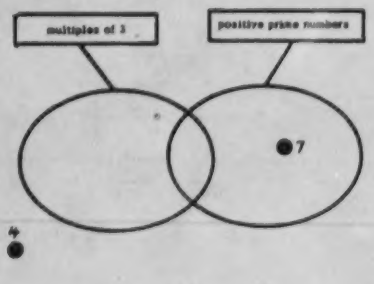
The red tag is one of these:

[redacted]
[redacted]
multiples of 3
[redacted]
[redacted]
[redacted]

The blue tag is one of these:

positive prime numbers
[redacted]
[redacted]
[redacted]
[redacted]
[redacted]

Complete the tags.



14

you are my mother →

you are my teacher →

DECODE

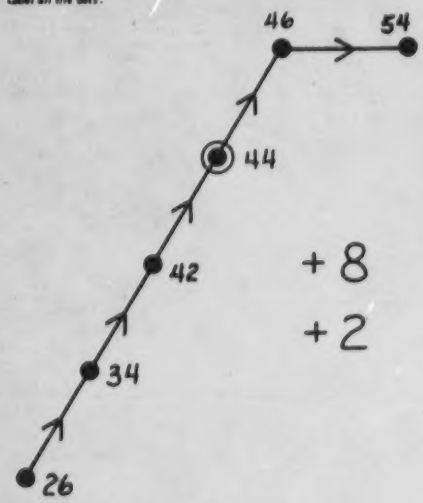
My teacher's mother is my mother's teacher.

My mother teaches my teacher's mother.

15

(There are other ways to express these messages.)

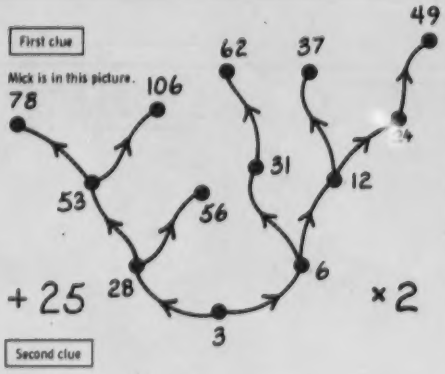
All the numbers in this arrow picture are WHOLE numbers.
44 is the only multiple of 4 in this picture. Where is 44 ?
Label all the dots.



16

First clue

Mick is in this picture.



Second clue

[−] [20] [=] . . .

572 ● ——→ ● Mick

Mick is 12

17

☐ has area 1 cm².

Use one color to color all shapes of area 1½ cm². gray

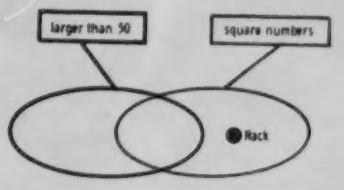
Use another color to color all shapes of area 2 cm². red

Use a third color to color all shapes of area 3 cm². blue

18

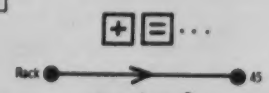
(Any three colors will suffice.
We used red, blue and gray.)

First clue



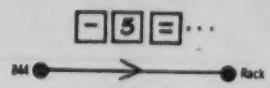
Rack is 1 or 4 or 9 or 16 or 25 or 36 or 49.

Second clue



Rack is 1 or 9.

Third clue



Rack is 9.

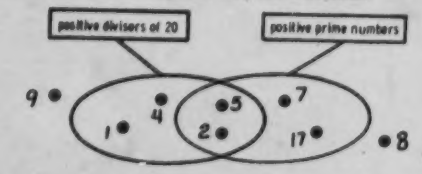
19

Put each of these numbers in this string picture.

4; 5; 7; 8

(The numbers 4, 5, 7, 8 must go in the regions where they are in this solution, but can be re-arranged within that region.)

Label the other dots to fit the picture. Many answers are possible.

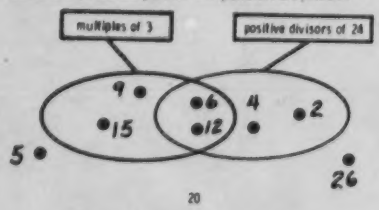


Put each of these numbers in this string picture.

4; 5; 6; 9

(The numbers 4, 5, 6, 9 must go in the regions where they are in this solution, but can be re-arranged within that region.)

Label the other dots to fit the picture. Many answers are possible.

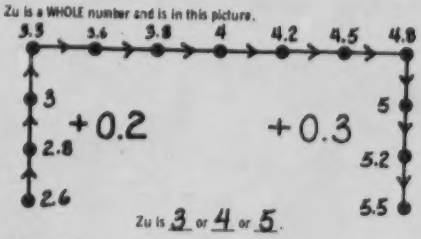


20

(Many other solutions are possible
for both problems.)

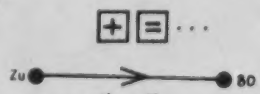
First clue

Zu is a WHOLE number and is in this picture.



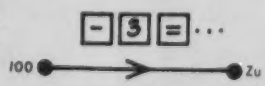
Zu is 3 or 4 or 5.

Second clue



Zu is 4 or 5.

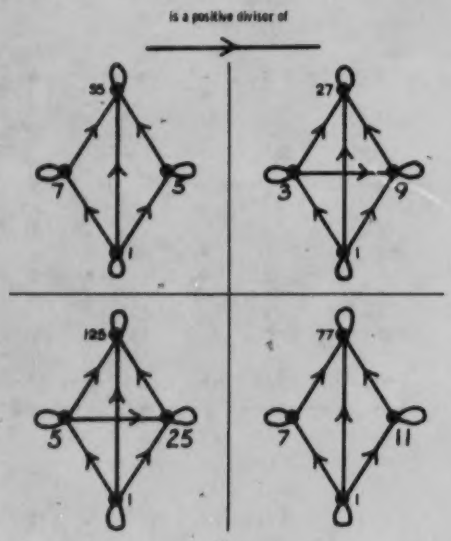
Third clue



Zu is 4.

21

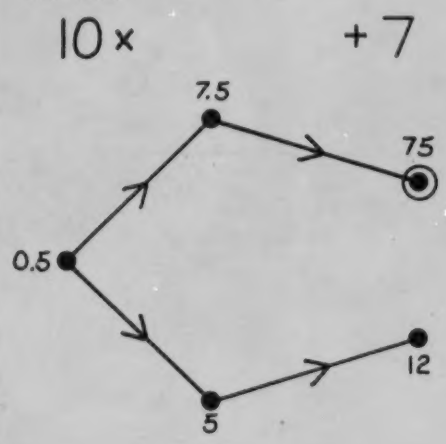
Label the dots.



22

In the first problem, the "5" and "7" can be interchanged. In the last problem, the "7" and "11" can be interchanged. In the other two problems, the numbers must be placed as they are here.

In this picture, all the numbers are positive and 75 is the largest.
Where is 75 ?
Label all the dots.

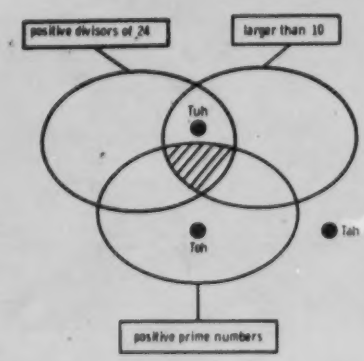


23

Tah, Teh and Tuh are three of these numbers:

1; 2; 3; 6; 7; 10; 12; 14; 20; 48

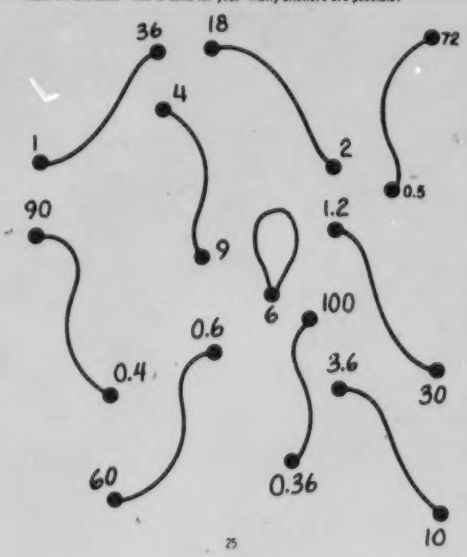
Tah is 10; Teh is 7; Tuh is 12.



One region of this diagram should be hatched. Hatch it.

24

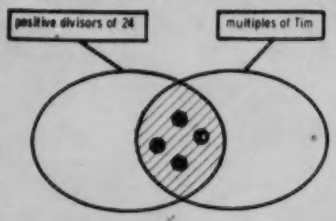
Two numbers are joined by a red cord if and only if their product is 36.
Label all the dots. One is done for you. Many answers are possible.



25

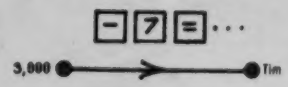
(Many other solutions are possible.)

First clue



Tim is 3 or 4.

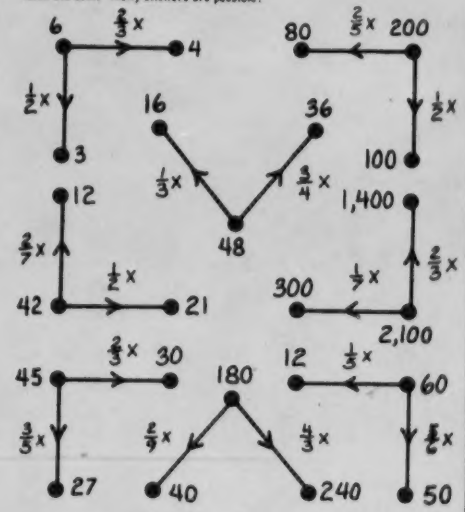
Second clue



Tim is 4.

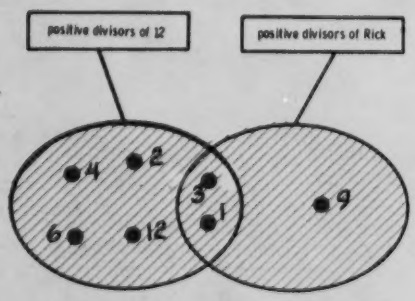
24

All the dots are for WHOLE numbers.
Label the dots. Many answers are possible.



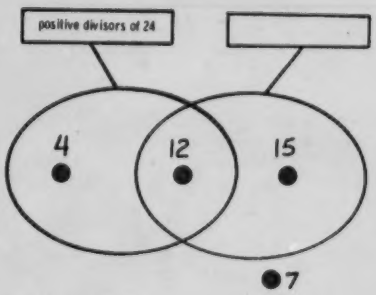
27

(Many other solutions are possible.)



Rick is 9.

28



The blue tag is one of these:

multiples of 3 | larger than 10

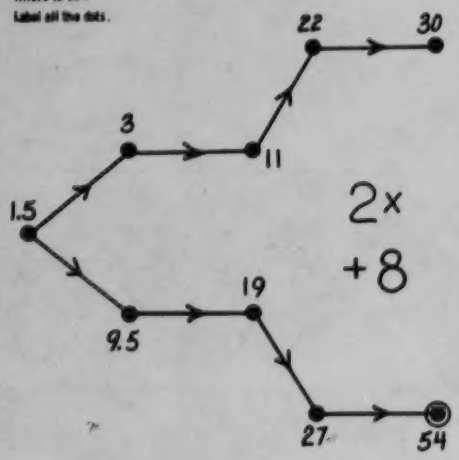
With the given information, four of these numbers can be placed in the string picture.

12; 15; 7; 6; 4

Put the four numbers in the picture.

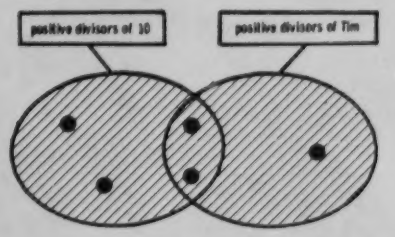
29

In this picture, all the numbers are positive and 54 is the largest.
Where is 54?
Label all the dots.



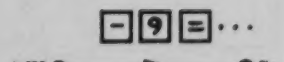
30

First clue



Tim is 4 or 25.

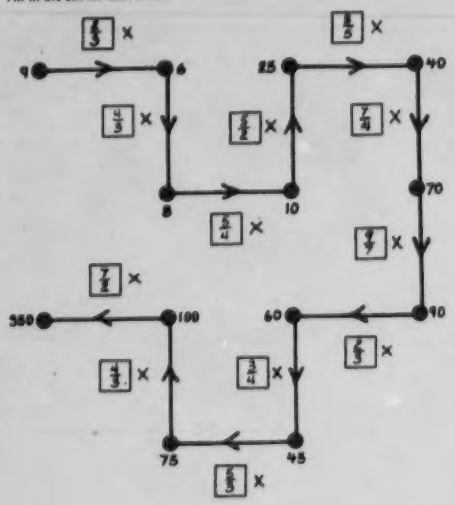
Second clue



Tim is 4.

31

Fill in the box for each arrow.



32

The answers above are those that may be given by students.
It is possible that other correct names could be given instead;
e.g., $\frac{2}{3} = \frac{6}{9} = \frac{8}{12} = \frac{40}{60} = \ldots$.

1000'S DREAM

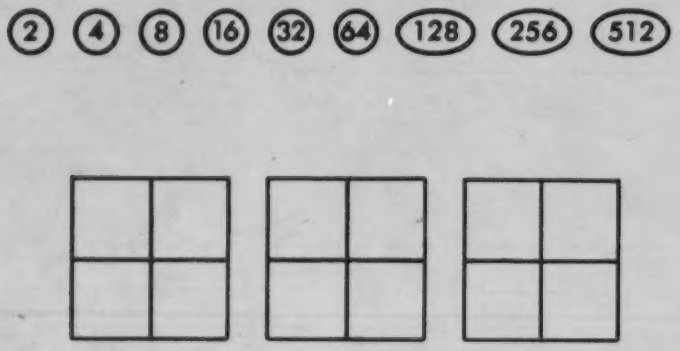
NUMBER 1,000'S DREAM

Lesson 1

Do not distribute copies of the story-workbook "Number 1,000's Dream" during this first lesson.

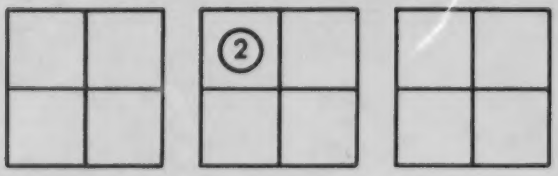
Exercise 1

Display three Minicomputer boards and your set of these checkers:



T: Today we will use these new checkers. I will put a number on the Minicomputer and you must tell me which number it is.

Display this configuration and ask which number is on the Minicomputer.

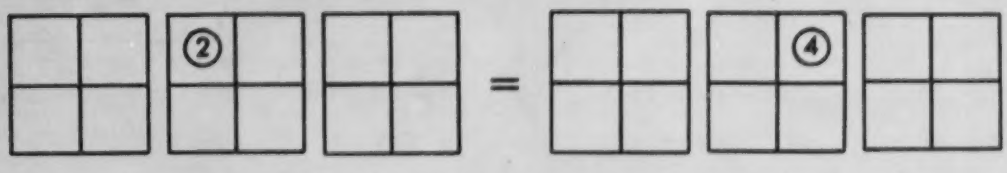


S: 160, because 2 x 80 = 160.

8W

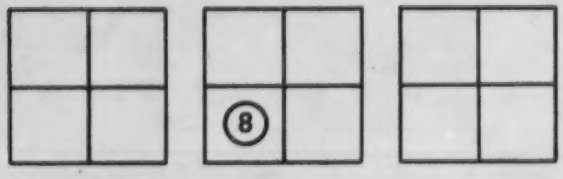
T: Good. Can you put the same number on the Minicomputer using a (4)-checker instead of the (2)-checker?

As a student makes the trade emphasize that 2 x 80 = 4 x 40 = 160.

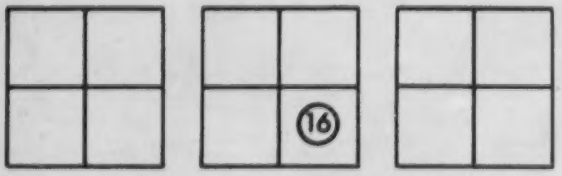


T: Can you show us the same number by trading the (4)-checker for an (8)-checker?

Invite a student to make the trade.



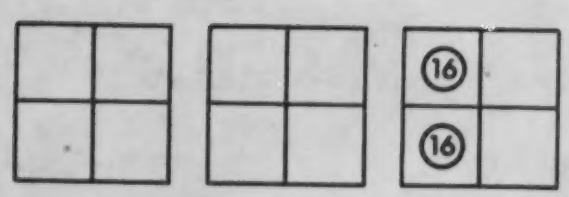
T: Show the same number by trading the (8)-checker for a (16)-checker.



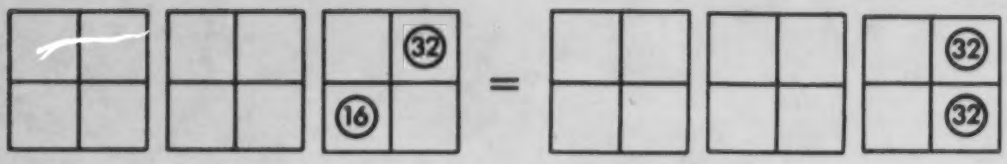
T: Show the same number using two (16)-checkers instead of one (16)-checker.

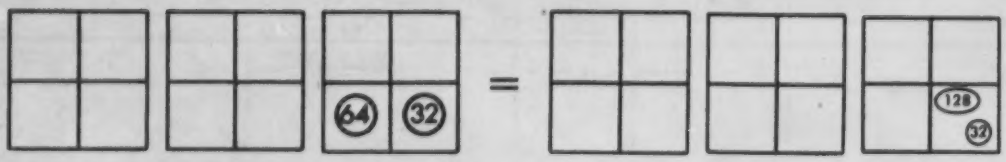
Give the students a minute to consider this problem as it involves a trade different from the previous ones. Then let a student make the trade (i.e., make the move).

147



Continue the trading -- each time asking a student to come to the demonstration Minicomputer and to make a trade. A sequence of trades is given below.





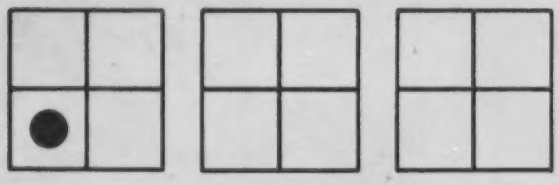
Although the intermediate trades might not be the same as those above, you should finish with a (128)-checker and a (32)-checker on the 1-square.

T: We started with two checkers on the 80-square and finished with 160 checkers on the 1-square.

8W

Exercise 2

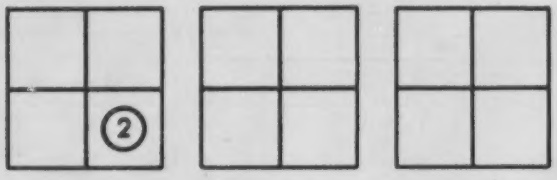
Put 200 on the Minicomputer with one regular checker.



T: Imagine that we are visiting the World of Numbers. After the numbers awake each day, they perform exercises. Watch 200 as it exercises on the Minicomputer.

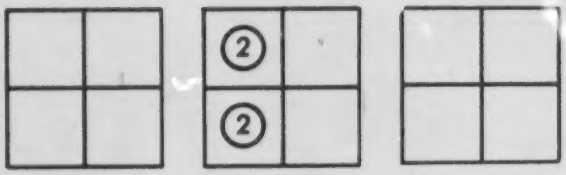
It jumps.

Show this jump (trading the regular checker for a (2)-checker).



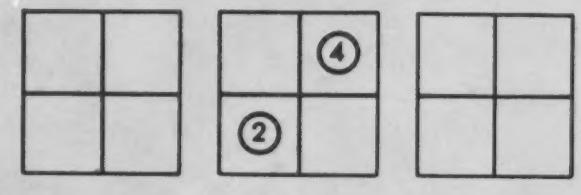
T: Then it jumps again.

Show another trade.

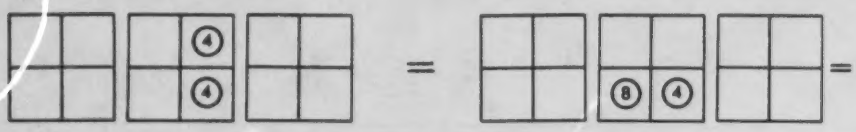


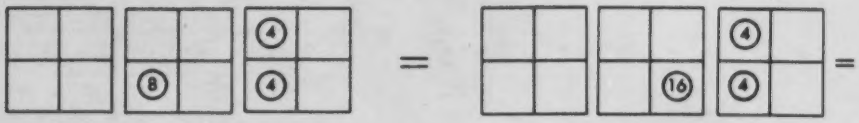
T: Here is 200 presented as (2 x 80) + (2 x 20).
Can you show me another jump by trading a (2)-checker for a (4)-checker?

Call on a student to make one of the possible trades; for example,



Continue making trades as you did in Exercise 1. Below we show how the trades might be made.





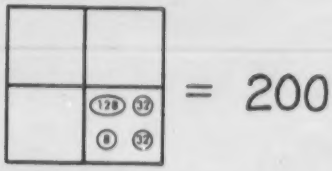
Up until this point, you have controlled the trades that were made by asking questions such as, "Who can show the same number by trading an (8)-checker for a (16)-checker?" Now present a more open situation. For example,

T: Make any trade you wish.

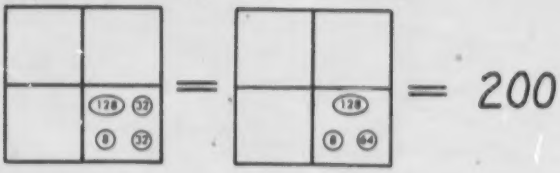
8W

Some students will continue to make the same kind of trades as have been made. However, some students may make "multiple trades"; for example, a (4)-checker on the 8-square for a (32)-checker on the 1-square.

Continue letting students make trades until all checkers are on the 1-square. In one of our demonstration classes, the following configuration was reached.



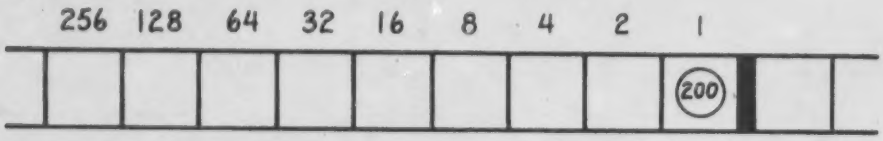
There is nothing wrong with this configuration; however, 200 can be presented with just three of these new checkers by trading the two (32)-checkers for a (64)-checker. Thus,



Leave this last configuration of new checkers on the ones' board for use later in the lesson.

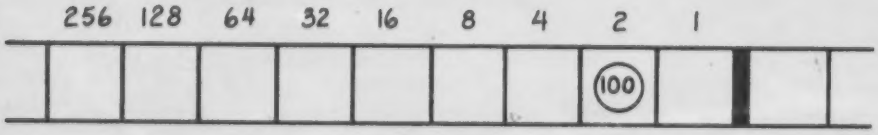
Exercise 3

Draw a binary abacus on the chalkboard* and put a (200)-checker on the 1-square. (Draw the checker on the board.)



T: Now number 200 exercises on the binary abacus. Show a jump that 200 could make.

Let the students have a few minutes to consider this situation; then ask a student to make a jump.



T: 200 on the 1-square equals 100 on the 2-square. Show another jump that 200 could make.

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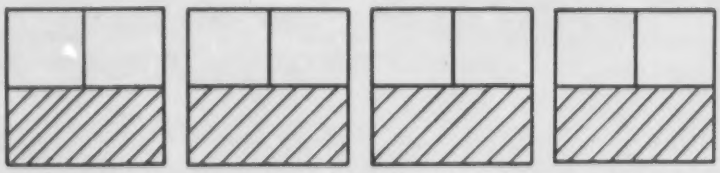
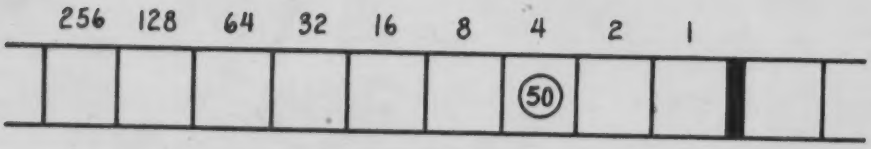
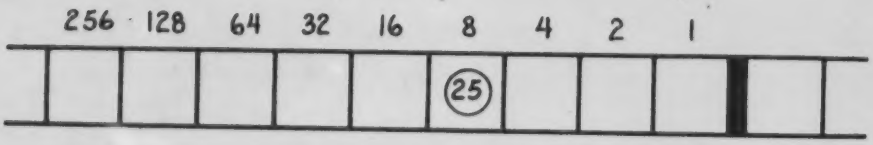
* You may prefer to construct a magnetic binary abacus using your demonstration Minicomputer boards. To do so, cover the red and white squares with paper and use only the brown and purple squares on each board.





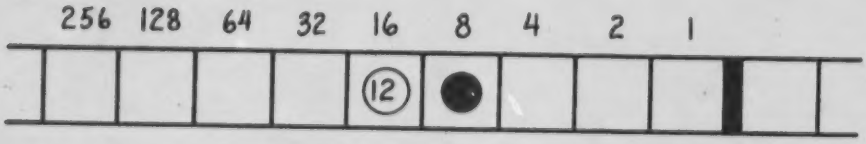
T: Another jump.



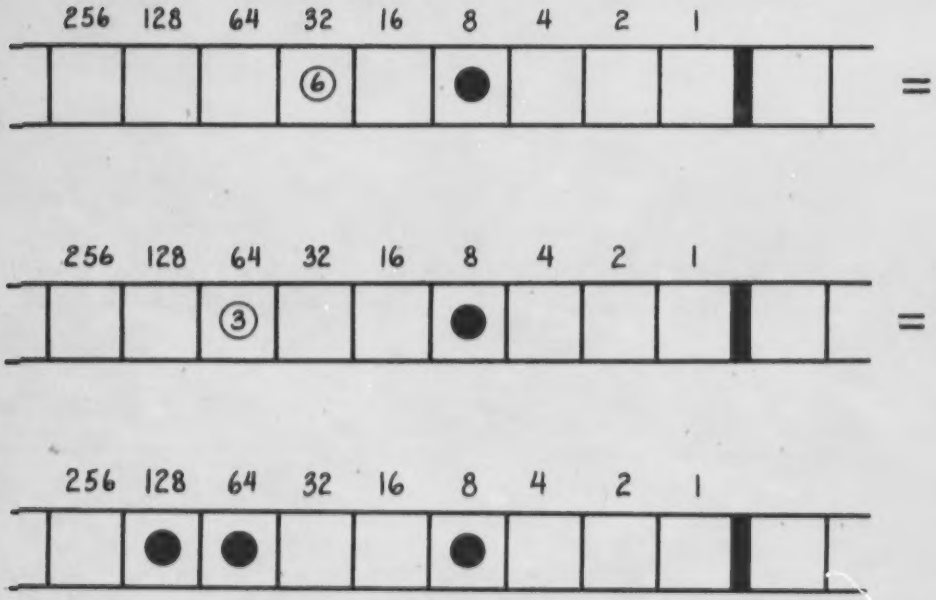
T: This is 25 checkers on the 8 - square. How can 200 jump again?

Let the students have a few minutes to consider the situation.

S: Leave one checker on the 8 - square and put 12 checkers on the 16 - square.

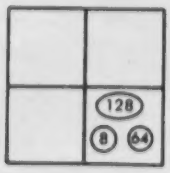


If no student suggests this jump, suggest it yourself. Continue asking students to make the jumps as in the next illustration.



Ask the students if 200 is still on the abacus. They should verify that it is by adding: 128 + 64 + 8 = 200.

At this point, display the Minicomputer board with the configuration at the end of Exercise 2.



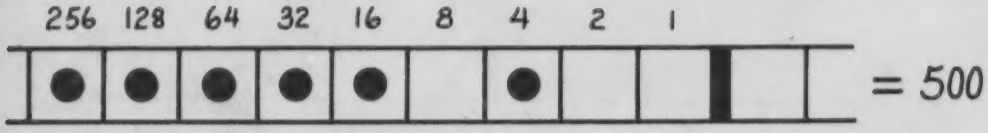
T: What do you think of these checkers on the Minicomputer and the checkers on the binary abacus?

This question should provoke some discussion. Let the students express their opinions. Some students will probably notice that the weight of the checkers on the Minicomputer corresponds to the place value of the squares of the binary abacus that have checkers on them.

8 W

Repeat this exercise with 500 and suggest that the class verify the work by adding:

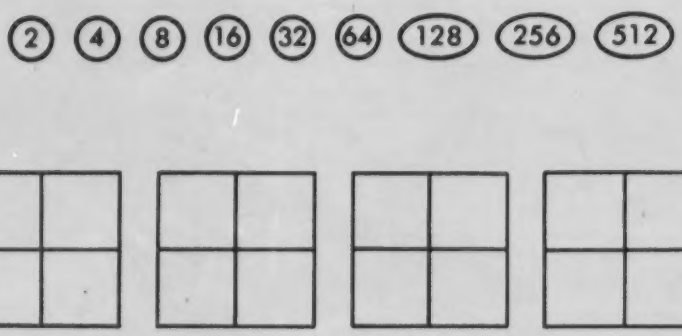
$$256 + 128 + 64 + 32 + 16 + 4 = 500.$$



Lesson 2

Distribute copies of the story-workbook "Number 1,000's Dream".

Display four Minicomputer boards and your set of these checkers:



T: Do you notice anything interesting about these checkers?

Allow the students to make some observations.

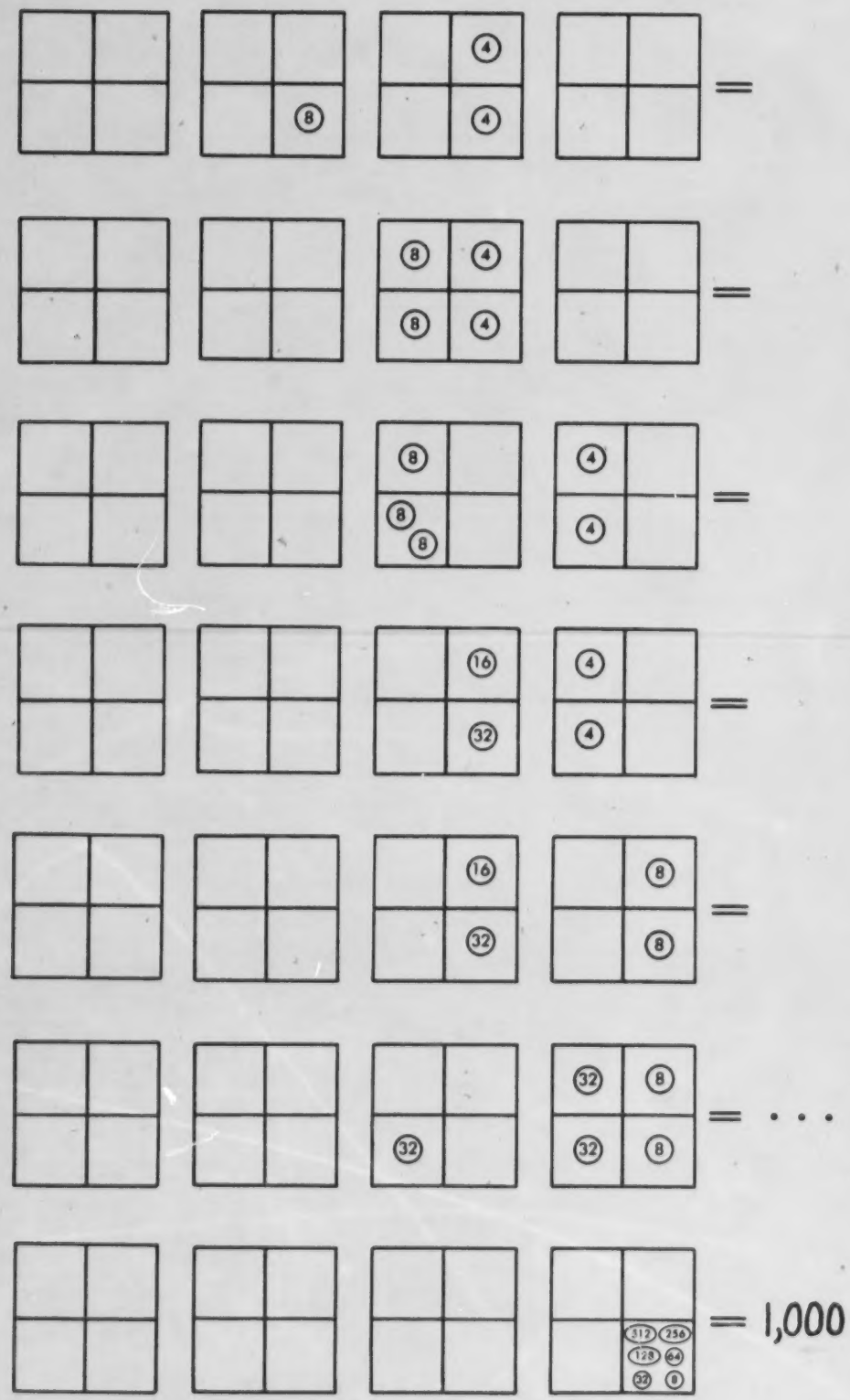
S: All the checkers are multiples of two.

S: The values double: 2, 4, 8, 16, and so on.

Ask the students to read pages 3 - 5, either aloud or silently.

Put 1,000 on the Minicomputer using one positive regular checker and ask students to make the jumps pictured on pages 3 - 5. Let the students continue making jumps. A partial sequence of jumps that could be made is shown in the next illustration. Here we start with the configuration at the bottom of page 5 in the story-workbook.

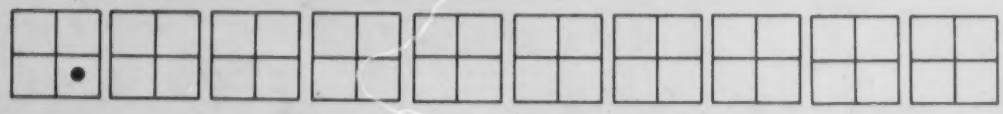
9W



When all the checkers on the Minicomputer are on the 1 - square, ask students to add the values of the checkers. They should verify that 1,000 is still on the Minicomputer.

Ask some students to read pages 6 - 11 aloud for the class.

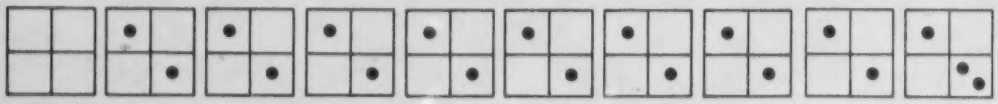
Using the four demonstration Minicomputer boards and drawing six additional Minicomputer boards on the chalkboard (or borrowing six additional Minicomputer boards), display ten Minicomputer boards.



T: What number is this?

S: One billion.

Ask the students to read page 12; then make jumps on the Minicomputer until this configuration is reached.



Continue reading pages 13 and 14.

T: What calculation does the Minicomputer suggest?

S: 999,999,999 + 1 = 1,000,000,000.

9W

Ask a student to read page 15 aloud. Then write the problems below on the board. The students should copy them and fill in the boxes.

600 = [ ] + 1

7,000 = [ ] + 1

9,500 = [ ] + 1

19,000 = [ ] + 1

99,000 = [ ] + 1

478,000 = [ ] + 1

Be sure to check these problems collectively. Solutions are given below.

600 = [599] + 1

7,000 = [6,999] + 1

9,500 = [9,499] + 1

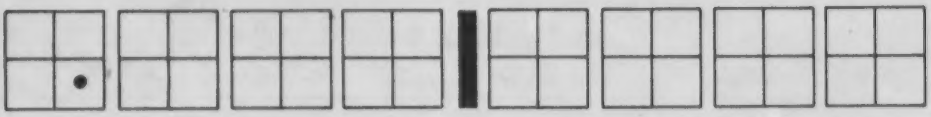
19,000 = [18,999] + 1

99,000 = [98,999] + 1

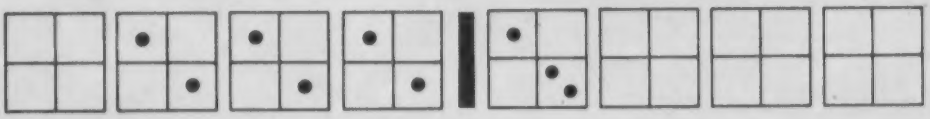
478,000 = [477,999] + 1

Continue reading pages 16 to 25.

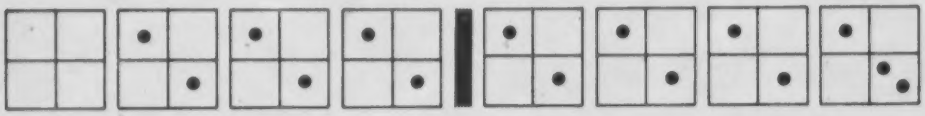
Display at least eight Minicomputer boards (additional boards will have to be borrowed or drawn on the chalkboard)



As one student reads aloud pages 26 and 27, ask others to make the jumps on the demonstration Minicomputer. Then invite a student to make the jump over the green bar.*



As a student reads aloud pages 28 and 29, ask others to continue making jumps until this configuration is reached.



T: What would happen if we had more Minicomputer boards?

S: We could continue making jumps.

---

* In our illustrations, the green bar is shown in gray; however, you should draw it with green chalk on your chalkboard.

9W

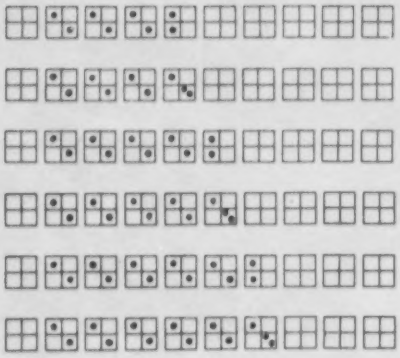
Finish the lesson with the reading of pages 30 - 32.

Write 1,000's new signature on the chalkboard for emphasis.

$$\boxed{1{,}000 = 999.9999\ldots}$$

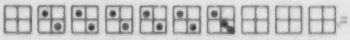
"DO YOU UNDERSTAND MY FRIEND'S EXERCISES? HE IS NOT JUMPING EXACTLY AS I DID, IS HE?"

"CAN YOU DRAW THE RESULTS OF HIS NEXT SIX JUMPS?"

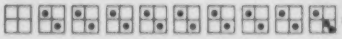


13

"I will draw the last picture for you."



"But my friend hasn't finished his game yet," adds 1,000. "He keeps on jumping until he gets two checkers on the white square of the ones' board. Then, he rests on his Minicomputer boards like this."



CAN YOU THINK OF SOME INTERESTING CALCULATIONS THAT THE MINI-COMPUTER SHOWS US? WRITE THEM BELOW.

14

See the second lesson of "Number 1,000's Dream" for solutions.

1,000 writes:

$$1{,}000{,}000{,}000 = 999{,}999{,}999 + 1$$

He adds,

"I can write a similar calculation for myself."

$$1{,}000 - 999 + 1$$

DID YOU FIND ANY OTHER INTERESTING CALCULATIONS?

15

See the second lesson of "Number 1,000's Dream" for solutions.

"I know some numbers who are even luckier than 1,000,000,000," says 1,000. "For instance, my friend 1,000,000,000,000 has thirteen boards on which to jump. Isn't that marvelous?"

1,000 thinks for a while and writes:

$$1{,}000{,}000{,}000{,}000 = 999{,}999{,}999{,}999 + 1$$

CAN YOU WRITE SIMILAR CALCULATIONS WITH EVEN LARGER NUMBERS? DO IT BELOW.

16

See the second lesson of "Number 1,000's Dream" for solutions.

A VARIETY OF PROBLEMS 4

A VARIETY OF PROBLEMS 4

This workbook has problems of varying levels of difficulty (*, **, and ***) all in one booklet — approximately the first 10 pages of which are easy (*) problems, the next 10 to 12 pages are average (**) problems and the last 10 pages are more challenging (***) problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for this workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about 20 to 25 pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Lesson 1

Give each student a copy of the workbook "A Variety of Problems 4" and let them work on their own.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

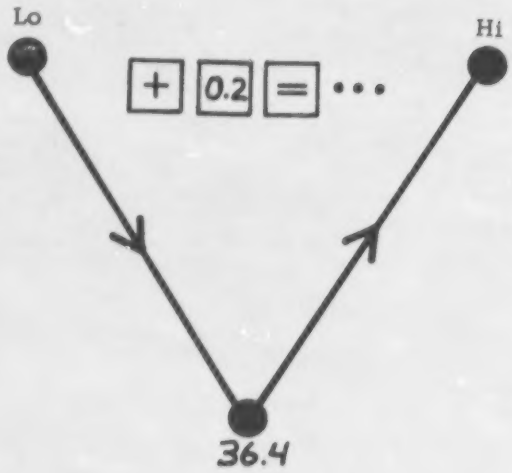
At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

11W

Lesson 2

Exercise 1

Draw this arrow picture on the board.



T: Hi and Lo are in this arrow picture. What are some numbers Hi could be?

S: 36.6.

S: 73.8.

List the numbers that students suggest on the board. These numbers might be some of the ones included in your list.

36.6; 40.2; 47.8; 73.8; 80.4

T: Do you notice anything interesting about this list?

S: The last digit is an even number.

T: Which is the largest number smaller than 100 that Hi could be?

Let the students have a minute to consider the question.

S: 99.8.

T: Which is the smallest number larger than 300 that Hi could be?

S: 300.2.

T: Hi is the largest number smaller than 500 which may be put in this arrow picture. Who is Hi?

S: 499.8.

T: Good. Which numbers could Lo be?

List the numbers that students suggest on the board. Encourage the students to name some negative numbers that Lo could be; e.g., $^{-}0.2$; $^{-}1.4$; $^{-}3.8$; etc.

T: Which is the smallest number larger than 30 that Lo could be?

S: 30.2.

T: Which is the largest number smaller than 8 that Lo could be?

S: 7.8.

T: Lo is the smallest <u>positive</u> number which may be put in this arrow picture. Who is Lo?

S: 0.2.

T: So Hi is 499.8 and Lo is 0.2.

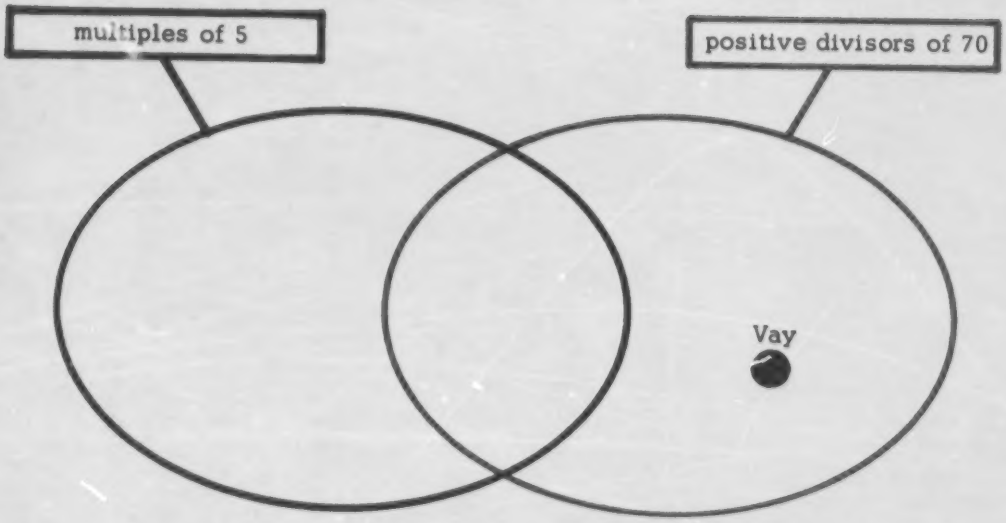
11W

Exercise 2

T: Vay is a secret number.

First clue

Draw this string picture on the board.



T: What does the string picture tell us about Vay?

S: Vay is a positive divisor of 70 and is not a multiple of 5.

T: Could Vay be 7? Why?

S: Yes, because 7 is a positive divisor of 70 and 7 is not a multiple of 5.

T: So Vay could be 7. Could Vay be 10?

S: No; 10 is in the middle, because it is both a multiple of 5 and a positive divisor of 70.

T: What are some other numbers Vay could be?

The class should conclude that Vay could be 1 or 2 or 7 or 14.

Second clue

T: The square of Vay is larger than 50. Who is Vay?

S: 14.

The class should conclude that Vay is 14.

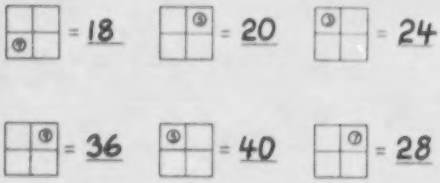
Return the students' copies of "A Variety of Problems 4". Ask them to correct their mistakes (if any), and then to continue working in the workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

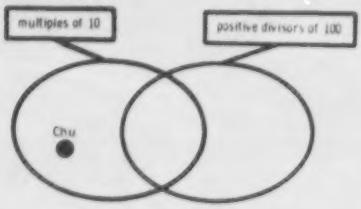
At the conclusion of this lesson, collect the workbooks for your progress check. You may wish to make this workbook available later for students to work in at their leisure.

A Variety of Problems 4

Chu is one of these numbers:



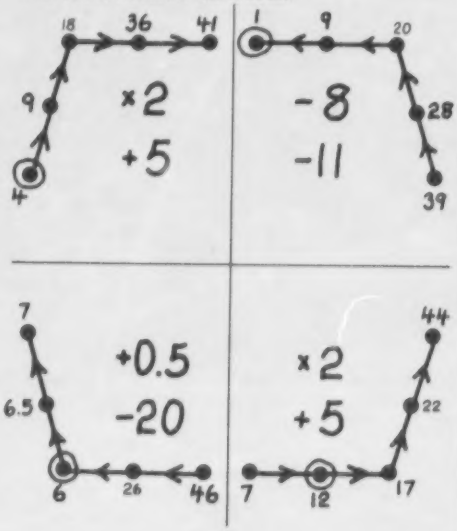
Chu is in this string picture.



Chu is 40.

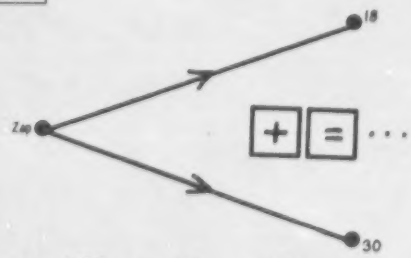
2

A positive divisor of 12 is in each of these pictures.
In each picture, circle its dot. Label all the dots.



3

First clue



Can Zap be 2? YES ; 5? NO ; 6? YES ;
9? NO ; 10? NO ; 1? YES ; 0.5? YES

Second clue

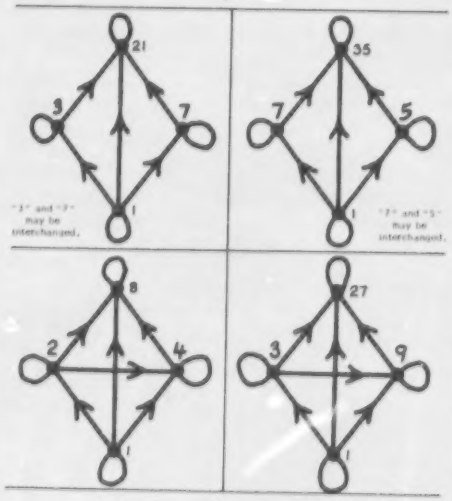
Zap is larger than 3.

Who is Zap? 6

4

Label the dots.

is a positive divisor of



5

Four small squares fill up this big square

Here they are

Use three parallelograms to fill up this boat.

Use a ruler to draw the lines.

Use one square

and four shapes like this:

to fill up the big square.

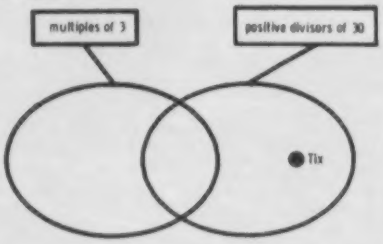
Use one square

and four shapes like this:

to fill up the big square.

6

First clue



Can Tix be 10? YES ; 6? NO ; 5? YES ;

3? NO ; 1? YES ; 4? NO ; 15? NO ;

2? YES ; 7? NO ; 8? NO .

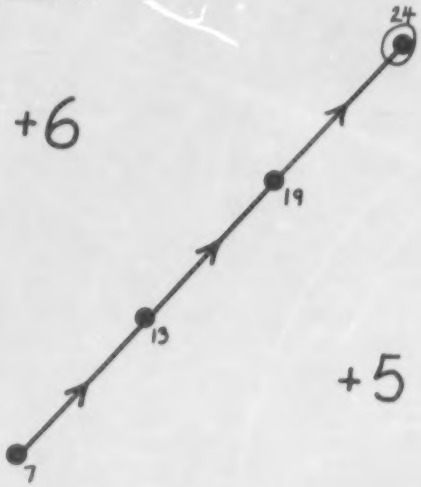
Second clue

Tix is larger than 5.

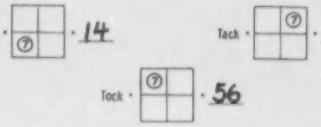
Who is Tix? 10

7

24 is the only multiple of 3 in this picture. Where is 24?
Label all the dots.



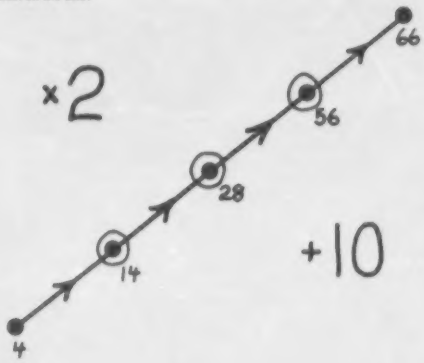
8

Tick =  = 14

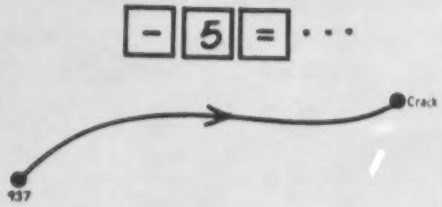
Tack = = 28

Tock = = 56

Tick, Tack, and Tock are in this picture. Circle their dots.
Label all the dots.



9



Can Crack be 927? YES ; 921? NO ; 902? YES ;

896? NO ; 857? YES 411? NO ;

412? YES ; 88? NO .

Write five numbers smaller than 200 that Crack could be.

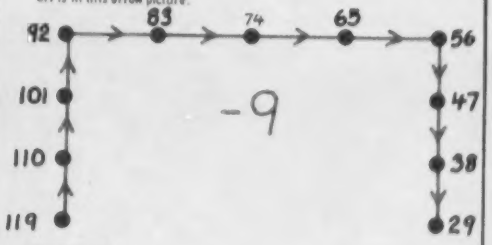
197 ; 102 ; 67 ; -3 ; -48 .

(Many other solutions are possible.)

10

First clue

Cri is in this arrow picture.



Second clue

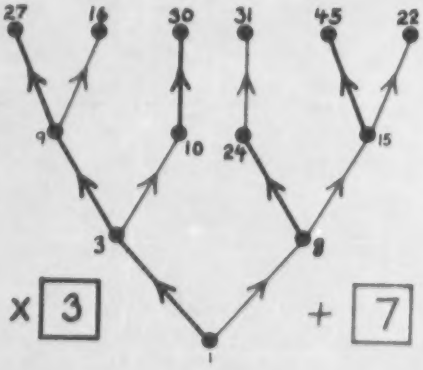
Cri can be put on the ones' board of the Minicomputer using exactly one of these checkers:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Cri = ⑦ · 56 .

11

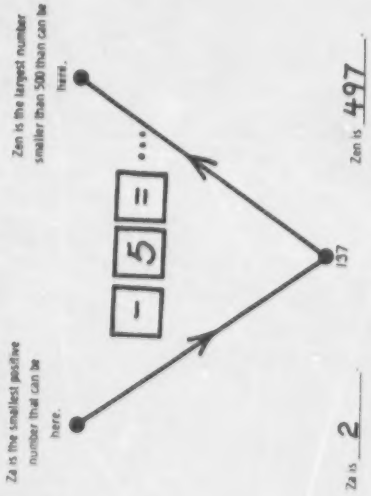
Fill in the boxes for the arrows and label the dots.



Dug is the largest prime number in this picture.

Dug is 31 .

12

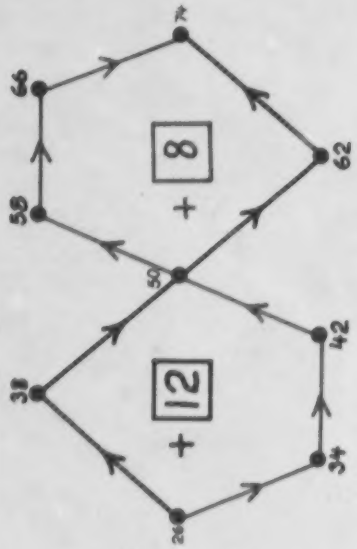


Zen is 497 .

Za is 2 .

13

Fill in the boxes for the arrows and label the dots.



14

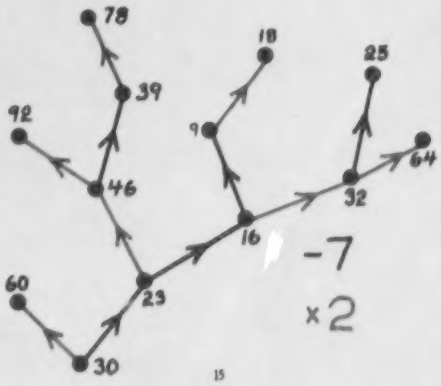
Label the dots.

Ding is the only *prime* number in this picture.

Ding is 23.

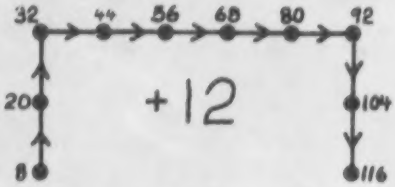
Dong is the largest square number in this picture.

Dong is 64.



15

Label the dots.



Four of the numbers in this *arrow* picture can be put on the ones' board of the Minicomputer using exactly one of these checkers:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

They are:

= 8

= 32

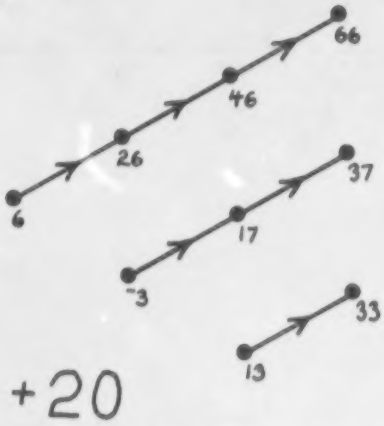
= 20

= 56

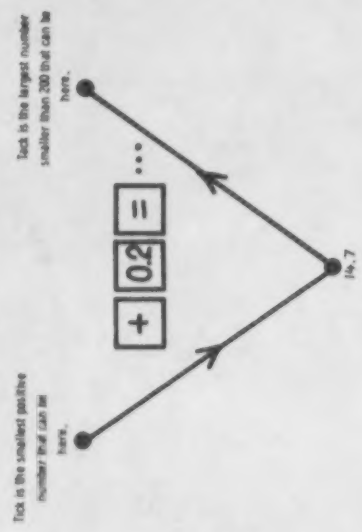
16

Put these numbers in this *arrow* picture.

-3; 6; 13; 17; 26; 33; 37; 46; 66.



17

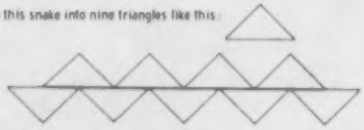


Tick is 0.1

Tick is 199.9

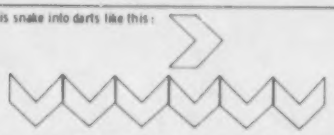
18

Divide this snake into nine triangles like this:

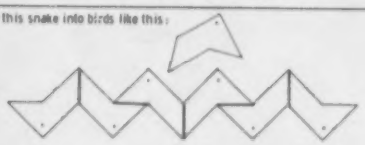


Divide this snake into seven rectangles like this:

Divide this snake into darts like this:

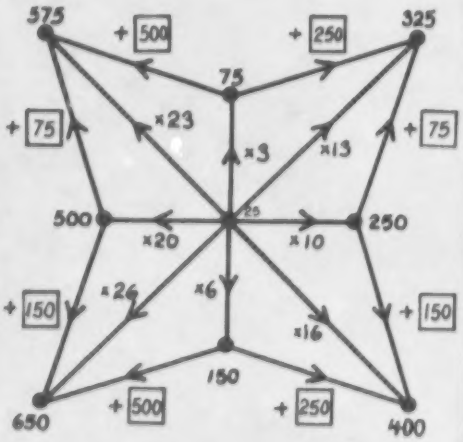


Divide this snake into birds like this:



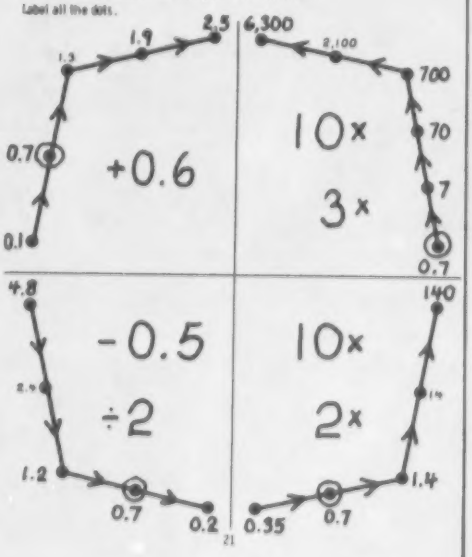
19

Label the dots and the arrows.



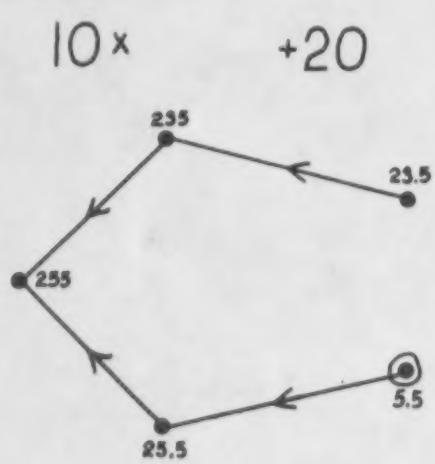
20

0.7 is in each picture. In each picture circle its dot.
Label all the dots.



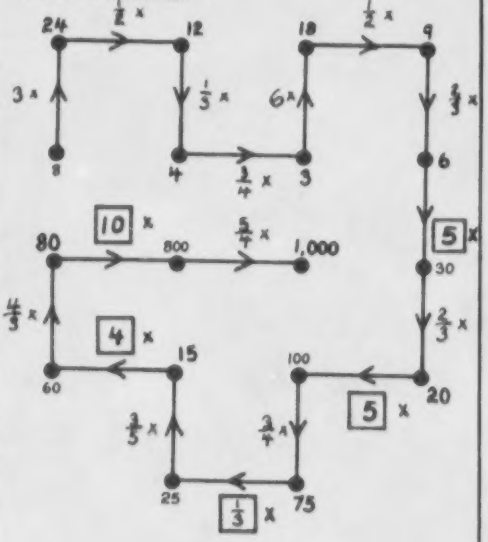
21

In this picture all the numbers are positive and 5.5 is the smallest.
Where is 5.5?
Label all the dots.



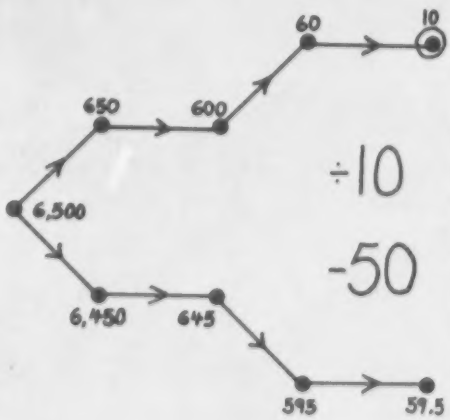
22

Label the dots and the arrows.

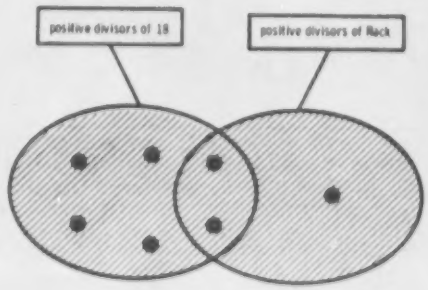


23

In this picture, all the numbers are positive and 10 is the smallest.
Where is 10?
Label all the dots.



24

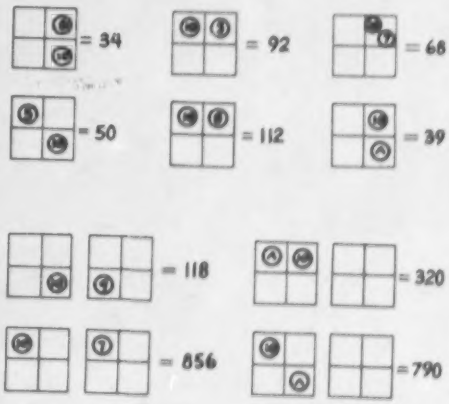


Rack is 4

25

Put each number on the Minicomputer using exactly one ⑩ - checker and one of these checkers:

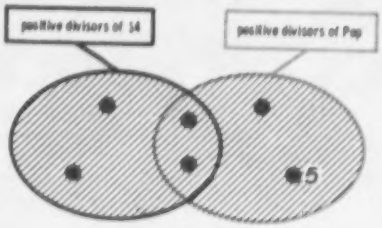
② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⓐ



(Other solutions are possible.)

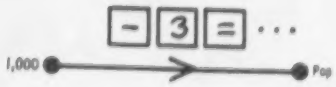
26

First clue



Pop is 10 or 35.

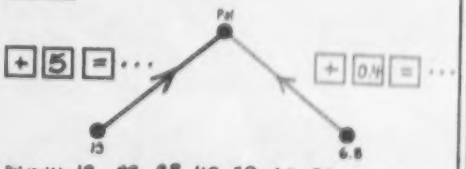
Second clue



Pop is 10.

27

First clue



Pal could be 18, 28, 38, 48, 58, 68, 78, and so on.
(Pal could be any number whose ones' digit is 8, beginning with 18.)

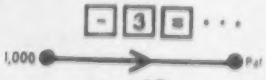
Second clue

Pal can be put on the ones' board of the Minicomputer using exactly one of these checkers:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

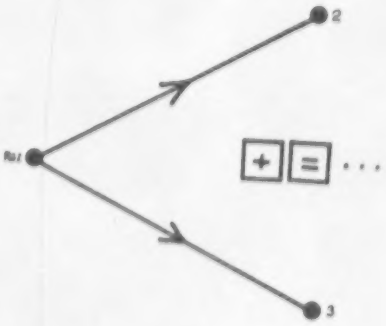
Pal is = 18 or = 28 or = 48.

Third clue



Pal is 28.

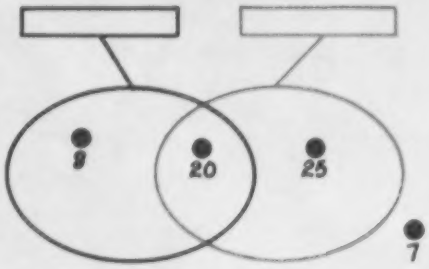
28



From this list, cross out the numbers that Raz cannot be:

0.1, 0.2, ~~0.3~~, ~~0.4~~, 0.5, 0.25,
0.01, 0.02, ~~0.03~~, 0.04, 0.05, 0.125.

29



The red tag is one of these:

- positive divisors of 80
- multiples of 4

The blue tag is one of these.

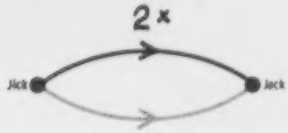
- multiples of 5
- larger than 15

With the given information, four of these numbers can be placed in the string picture.

5; (7); (8); 10; 15; 16; (20); (25)

Put the four numbers in the picture.

30



+ 3 =

Write ten numbers that Jick could be.

3 ; 6 ; 9 ; 12 ; 15 ; 18 ; 21 ; 24 ; 27 ; 30

Write ten numbers that Jick cannot be.

0 ; 1 ; 2 ; 10 ; 16 ; 25 ; 100 ; 358 ; 975 ; 1,000

Write ten numbers that Jack could be.

6 ; 12 ; 18 ; 24 ; 30 ; 36 ; 42 ; 48 ; 54 ; 60

What do you notice about Jick?

Jick can be any positive multiple of 3.

What do you notice about Jack?

Jack can be any positive multiple of 6.

31

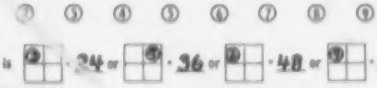
First clue

Pil could be 24 ; 36 ; 48 ; 60 ; 72 ; 84 ; 96 ; 108 and so on.

(Pil could be any multiple of 12 greater than 12.)

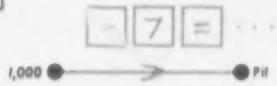
Second clue

Pil can be put on the ones' board of the Minicomputer using exactly one of these checkers:



Pil is 24 or 36 or 48 or 72.

Third clue



Pil is 48.

32

SEVEN SECRET NUMBERS

This lesson description indicates some questions and student comments that were made during the presentation of the story-workbook in our development classes. Feel free to modify the presentation in any way that you see fit.

Distribute copies of the story-workbook, SEVEN SECRET NUMBERS. Ask the students to read pages 3-5 individually and begin your collective discussion with page 5.

Page 5

T: Look at the arrow picture on page 4. Which is the largest number and which is the smallest number in the picture?

S: 157 is the largest number.

S: $^{-}3$ is the smallest number.

Collectively check answers to the questions on page 5. The class should conclude that the seven secret numbers and the four spies are all integers larger than $^{-}4$ and smaller than 158.

Pages 6 and 7

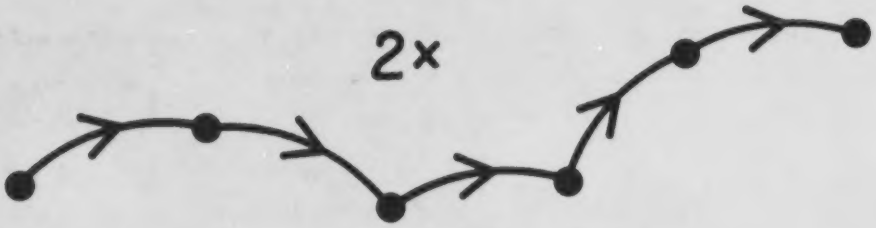
T: One of the secret numbers is easy to find in this picture. Can you find it?

S: 0 with the loop, because $2 \times 0 = 0$.

You will probably want to make a list on the board of the known secret numbers and the known spies. Be sure to continually update your list.

12W

Draw this part of the arrow picture from page 6 on the board.



T: What are your ideas for this part of the arrow picture?

S: 4 could be a starting number.

T: If 4 were the starting number, what other numbers would be in the picture?

S: 8, 16, 32, 64, and 128.

It will be helpful to write this list of numbers on the board. Students should also write the list of numbers on page 7.

4; 8; 16; 32; 64; 128

T: Any other possibilities?

Let students make suggestions. The other possibilities are:

1; 2; 4; 8; 16; 32

2; 4; 8; 16; 32; 64

3; 6; 12; 24; 48; 96

T: Could there be any other possibilities?

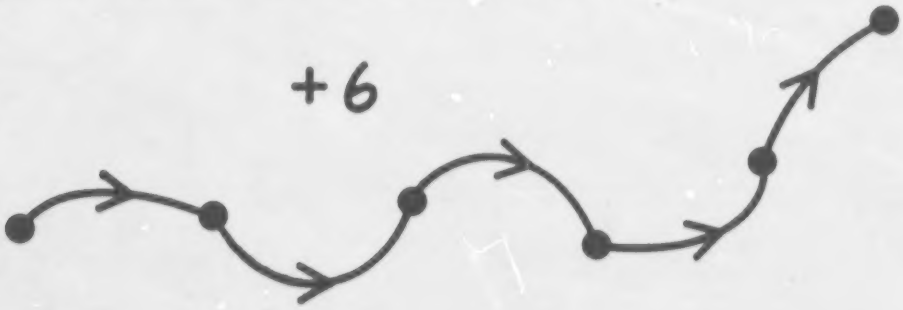
S: If we start with 5, then the other numbers would be 10, 20, 40, 80, and 160. But all the secret numbers are smaller than 158, so the starting number cannot be 5; it cannot be larger than 4.

T: Could there be any negative numbers in the picture?

S: No. If $^{-}1$ were the starting number, then the other numbers in the picture would be $^{-}2$, $^{-}4$, $^{-}9$, $^{-}16$, and $^{-}32$. But the secret numbers must be larger than $^{-}4$.

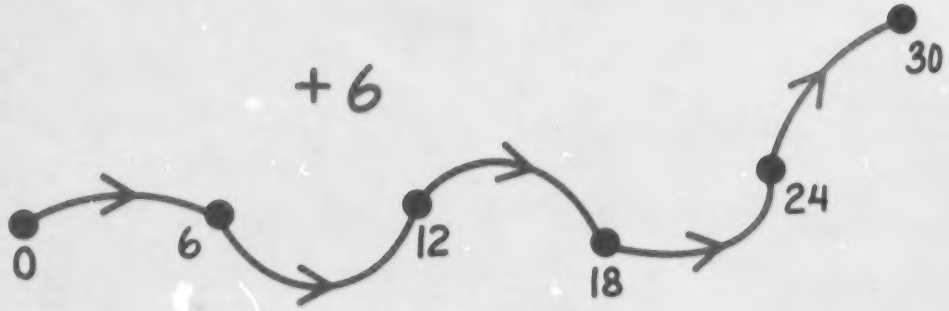
Pages 8 and 9

Draw the arrow picture from page 8 on the board.



T: With this clue we can find the seven secret numbers. Who has an idea?

Listen carefully to student suggestions. Someone should notice from the third list of numbers on page 7 (3; 6; 12; 24; 48; 96) that 6, 12, and 24 can be put in the arrow picture. Invite a student to label the dots in the arrow picture. A completed picture is shown in the next illustration.



T: Can you point to the four secret numbers?

Let a student point to "0", "6", "12", and "24".

T: Can you point to the two spies?

A student should point to "18" and "30".

T: Who are the three other secret numbers?

S: 3, 48, and 96.

The class should conclude that the seven secret numbers are 0, 3, 6, 12, 24, 48, and 96 and that two of the spies are 18 and 30.

Pages 10 and 11

T: Look carefully at page 10, and then answer the questions on page 11.

After a few minutes, ask:

T: Where are the spies?

S: In the helicopter.

180

T: Where are the secret numbers?

S: In the truck.

T: Who is the driver?

S: 0.

T: Do you remember from the previous pages who are two of the spies?

S: 18 and 30.

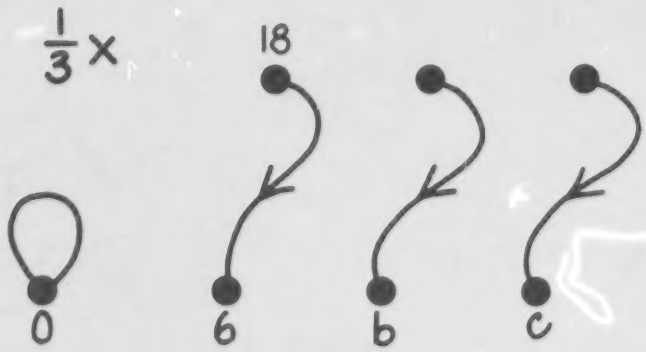
T: One of those spy numbers cannot be in the helicopter. Which one?

S: 30; $\frac{1}{3} \times 30 = 10$, but 10 is not a secret number.

T: Can you tell me one spy who must be in the helicopter? Why?

S: 18 must be in the helicopter, because there are only four spies and 30 is not in the helicopter. So all the other spies are in the helicopter.

Draw this picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

12W

T: We know that 0 and 6 are two of the secret numbers in the truck. (Point to b and c.) Which numbers could be here?

S: 3, 12, 24, 48, or 96.

T: Yes, those are the other secret numbers, but one of them <u>cannot</u> be in the truck. Which one?

S: 96 cannot be in the truck, because $3 \times 96 = 288$ and 288 is larger than 157.

T: Very good. Could 48 be in the truck? If so, then which spy would be in the helicopter watching 48?

S: 48 could be in the truck. $3 \times 48 = 144$, so 144 could be in the helicopter watching 48.

T: If 24 were in the truck, which spy would be in the helicopter watching 24?

S: 72 would be in the helicopter, because $3 \times 24 = 72$ (or $\frac{1}{3} \times 72 = 24$).

T: If 12 were in the truck, then which spy would be in the helicopter watching 12?

S: 36, because $\frac{1}{3} \times 36 = 12$.

T: And if 3 were in the truck?

S: 9 would be in the helicopter.

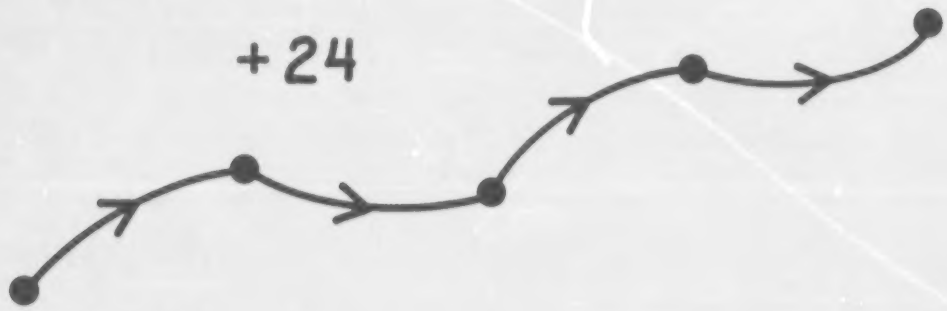
T: So 144, 72, 36, or 9 could be in the helicopter.

The class should conclude that two of the spies are 18 and 30 and that the two remaining spies are two numbers from the list:

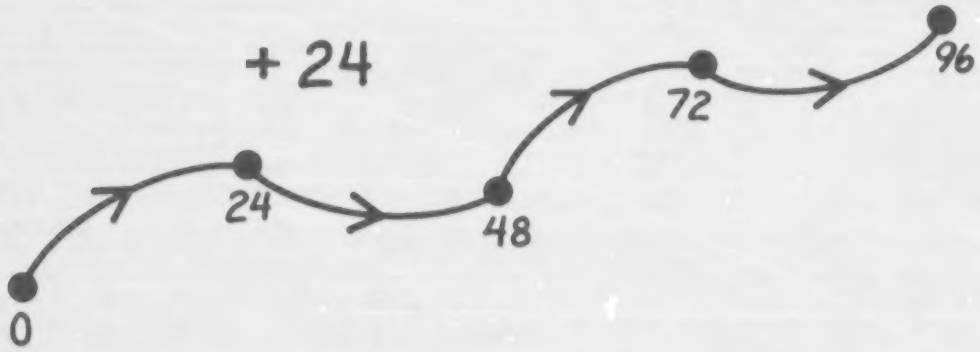
9, 36, 72, 144.

Pages 12 and 13

Draw this arrow picture from page 12 on the board.



Let the students have a few minutes to work on this clue, then ask someone to complete the picture so that it looks like the one below.



12W

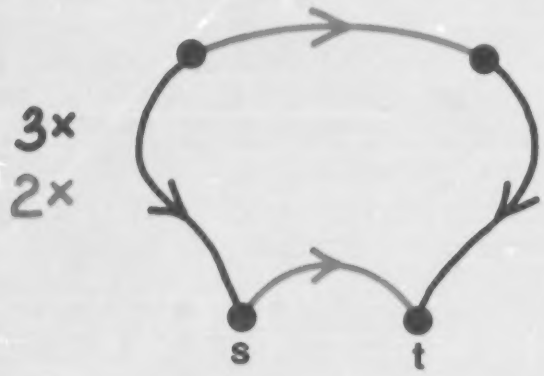
T: Who is the spy with the binoculars?

S: 72.

At this point, the class should conclude that three of the spies are 18, 30, and 72. The fourth spy is 9 or 36 or 144.

<u>Pages 14 and 15</u>

Draw this arrow picture from page 14 on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

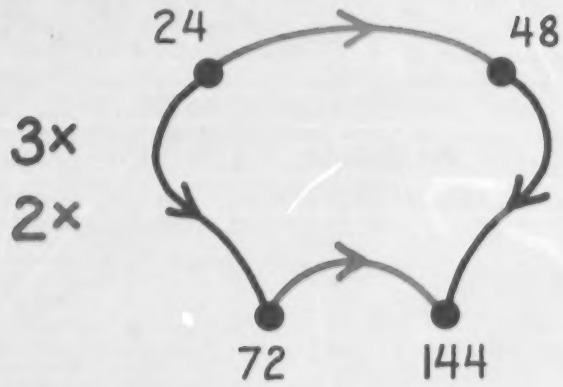
T: Who is one of the spies on the motorcycle? (72)
Who can show where 72 could be in our picture?

A student should point to s or t.

Label the dot at s "72".

T (pointing to s): If 72 is here, which other numbers are in this picture?

Ask a student to complete the picture.



T (pointing to "72"): If 72 is here, which other spy is on the motorcycle?

S: 144.

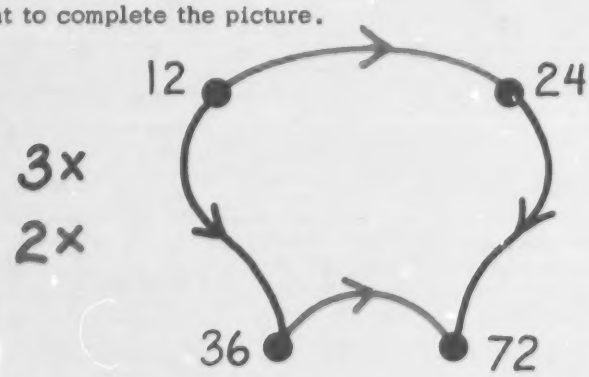
T: And which secret numbers are behind the trees?

S: 24 and 48.

Erase the numerals from your picture and label the dot at t "72".

T (pointing to t): If 72 is here, which other numbers are in this picture?

Ask a student to complete the picture.



T (pointing to "72"): If 72 is here, which other spy is on the motorcycle?

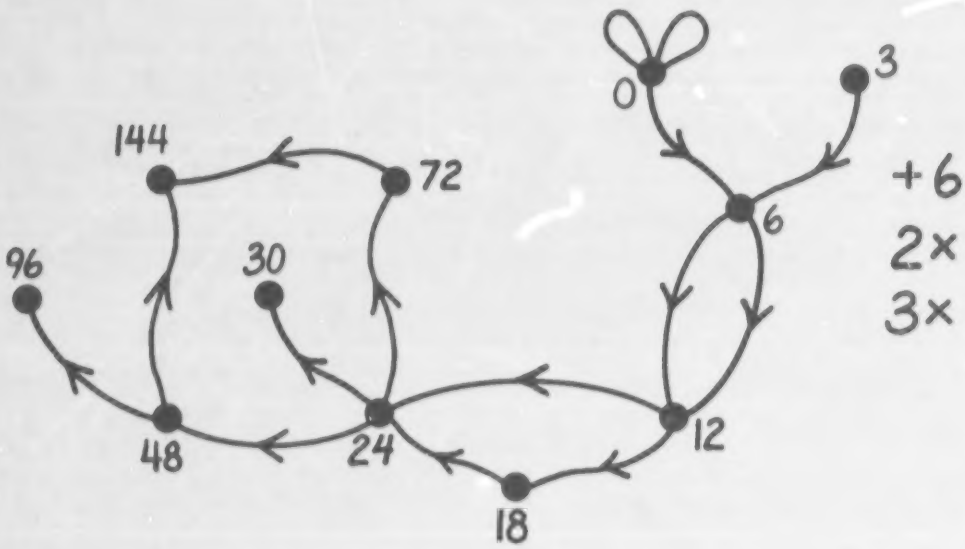
S: 36.

T: Which secret numbers are behind the trees?

S: 12 and 24.

Page 16

Copy the picture from page 16 on the board and invite students to label the dots. A completed picture is shown below.



The class should conclude that the four spies are 18, 30, 72, and 144; that the seven secret numbers are 0, 3, 6, 12, 24, 48, and 96.

If time and interest permit, suggest that the students write the names of the spies and secret numbers in each picture of their books. Also, invite them to write their own stories about these seven secret numbers and four spy numbers.

Seven Secret Numbers

Could one of the secret numbers or one of the spies be

$\frac{1}{2}$? No

75 ? Yes

3.25 ? No

200 ? No

$^{-}2$? Yes

$^{-}10$? No

The seven secret numbers and the four spies are all integers larger than $^{-}4$ and smaller than 158.

5

One of the secret numbers is 0.

The six other secret numbers are:

1 ; 2 ; 4 ; 8 ; 16 ; 32 ;

or

2 ; 4 ; 8 ; 16 ; 32 ; 64 ;

or

3 ; 6 ; 12 ; 24 ; 48 ; 96 ;

or

4 ; 8 ; 16 ; 32 ; 64 ; 128 .

7

The four secret numbers who run away are

0 ; 6 ; 12 ; and 24 .

The two spies who run with them are 18 and 30 .

The three other secret numbers are

3 ; 48 ; and 96 .

9

The truck driver is 0 . Write his name in the picture.

You already know that two of the spies are 18 and 30 .

Can you explain why one of them (30) cannot be in the helicopter ? $\frac{1}{3} \times 30 = 10$ and 10 is not a secret number.

Can you explain why the other one (18) is in the helicopter ?

Write his name in the picture. It will give you a clue about one of the secret numbers in the truck.

The other two spies in the helicopter are two of these four numbers:

9 ; 36 ; 72 ; 144 .

Can you explain why these are the only possible numbers that the other two spies can be ? $\frac{1}{3} \times$ each of the above gives a secret number.

11

$^{*}\frac{1}{3} \times 18 = 6$ and 6 is a secret number. Also there are four spies and only three in the helicopter. Since we know 30 is not in the helicopter, all the other three have to be. Hence 18 must be in the helicopter.

The four secret numbers walking in the forest are

0, 24, 48, and 96.

The spy watching them with his binoculars is 72.

Two of the spies not in this picture are 18 and 30.

The fourth spy is

9 or 36 or 144.

13

The spy who comes out of the forest with his binoculars is 72

The spy who waits for him is 36 or 144.

The two secret numbers hidden behind the trees are

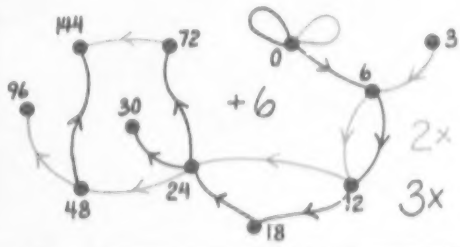
12 and 24

or

24 and 48.

15

The seven secret numbers and the four spies are in this picture.



The seven secret numbers are 0, 3, 6, 12, 24, 48, and 96.

The four spies are 18, 30, 72, and 144.

Write their names in each picture of this book. On a separate sheet of paper, write a story about some adventures these secret numbers and spy numbers could have.

16

A VARIETY OF PROBLEMS 5

This workbook has problems of varying levels of difficulty (*, **, and ***) all in one booklet — approximately the first 10 pages of which are easy (*) problems, the next 10 to 12 pages are average (**) problems and the last 10 pages are more challenging (***) problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for this workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about 20 to 25 pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Lesson 1

Give each student a copy of the workbook "A Variety of Problems 5" and let them work on their own.

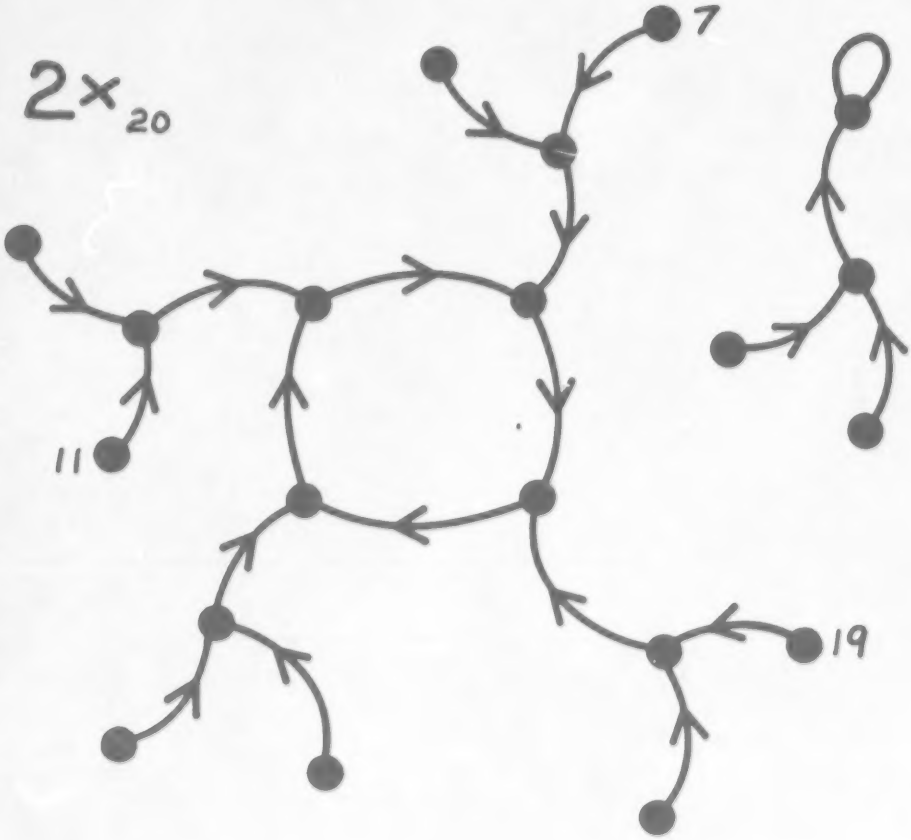
If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

14W

Lesson 2

Draw this picture with carefully spaced dots.

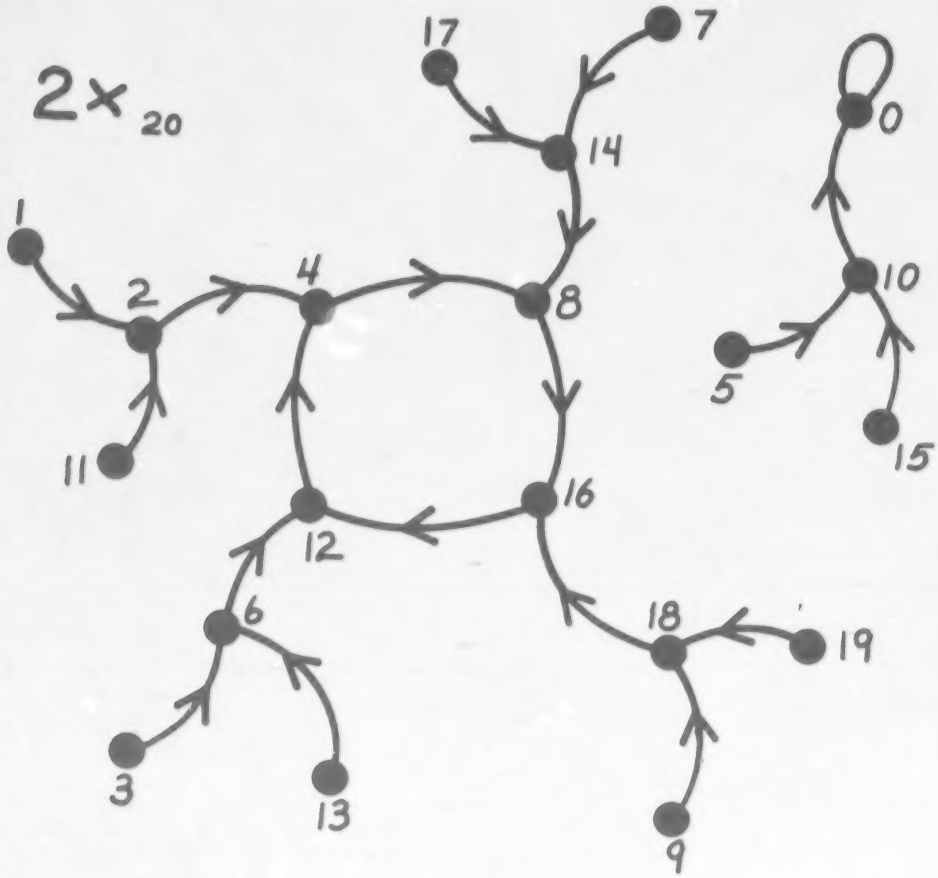


T: The red arrows are for 2 times with 20 friends. How can we calculate "$2 \times_{20} 19$"?

S: $2 \times 19 = 38$ and $38 - 20 = 18$, so $2 \times_{20} 19 = 18$.

T: Let's put all 20 friends, the numbers 0 through 19, in this picture. Who would like to put a number in the picture?

Invite students to label the dots and continue until the picture is complete. The completed picture is shown below.

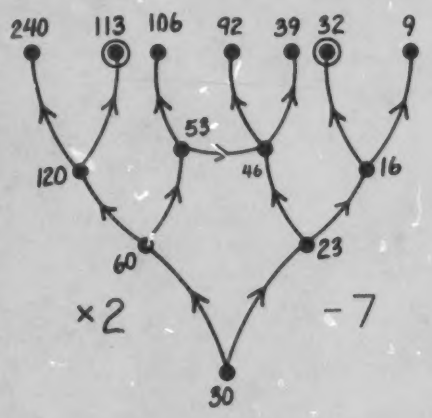


Return the students' copies of "A Variety of Problems 5". Ask them to correct their mistakes (if any), and then to continue working in the workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

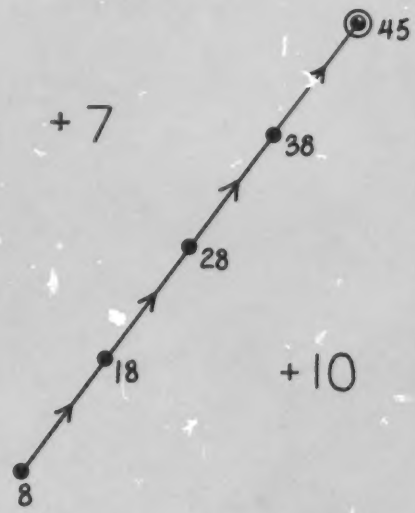
At the conclusion of this lesson, collect the workbooks for your progress check. You may wish to make this workbook available later for students to work in at their leisure.

Label the dots.
32 and 113 are in this picture. Circle their dots.



2

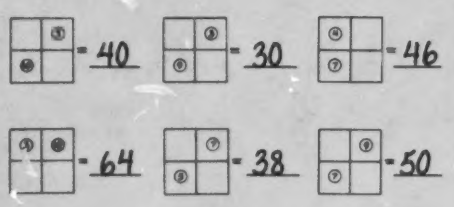
45 is the only multiple of 5 in this arrow picture.
Where is 45 ? Label all the dots.



3

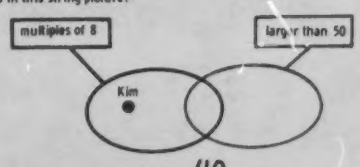
First clue

Kim is one of these numbers.



Second clue

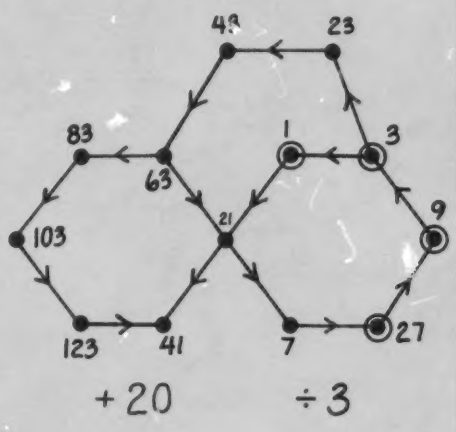
Kim is in this string picture.



Kim is 40.

4

Label the dots.
The four positive divisors of 27 are in this arrow picture. Circle their dots.

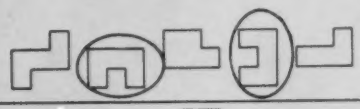


5

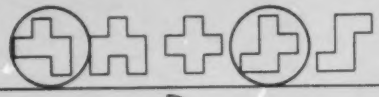
This shape is a twin to this shape

and to this shape.

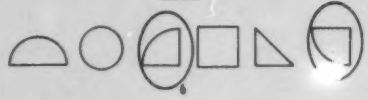
Circle the twins for this shape in the list.



Circle the twins for this shape in the list.



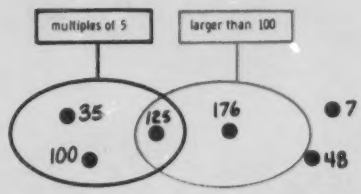
Circle the twins for this shape in the list.



6

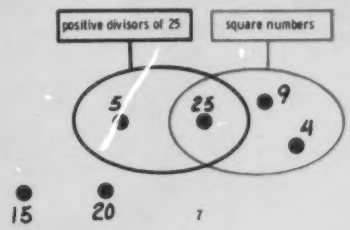
Put these numbers in this picture.

7; 35; 48; 100; 125; 176



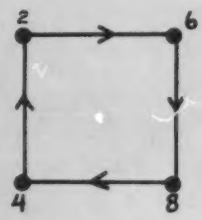
Put these numbers in this picture.

4; 5; 9; 15; 20; 25;

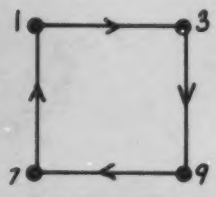


7

Put the numbers 0, 1, 3, 4, 5, 6, 7, 8, and 9 in this arrow picture.



$3 \times_{10}$

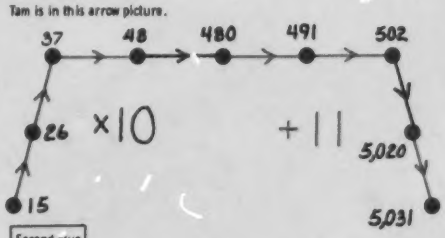


8

First clue

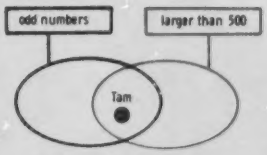
Label the dots.

Tam is in this arrow picture.



Second clue

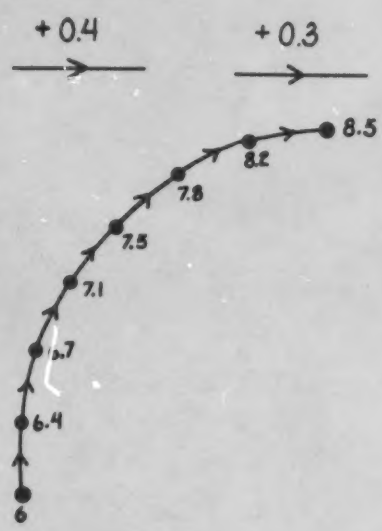
Tam is in this string picture.



Tam is 5,031.

9

Build a red-blue road from 6 to 8.5.

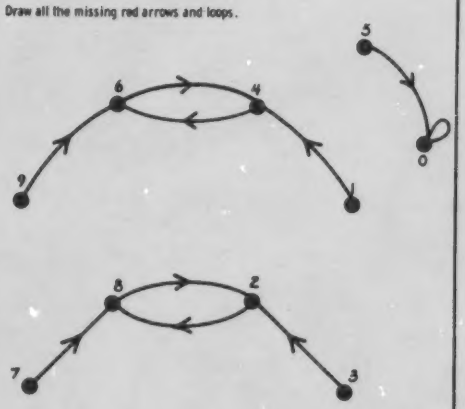


10

(Many other solutions are possible but none have less than seven arrows.)

$4 \times_{10}$

Draw all the missing red arrows and loops.

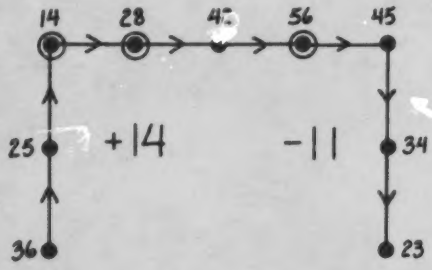


11

Mi is = 14 Fa is = 28

Sol is = 56

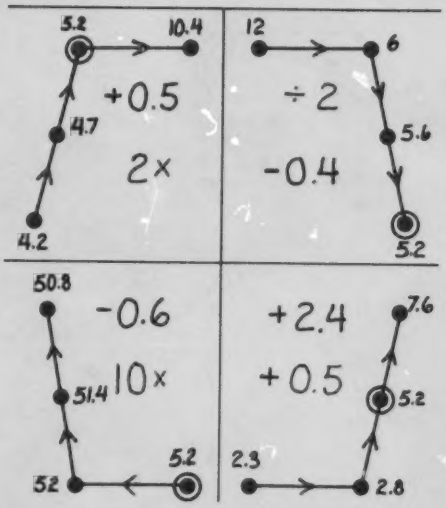
Mi, Fa, and Sol are in this arrow picture. Circle their dots. Label all the dots.



12

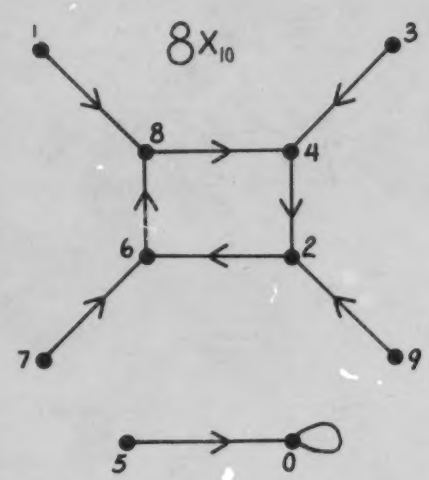
Label the dots.

5.2 is in each of these pictures. In each picture, circle its dot.



13

Put the numbers 0, 2, 3, 4, 5, 6, 7, 8, and 9 in this arrow picture.



14

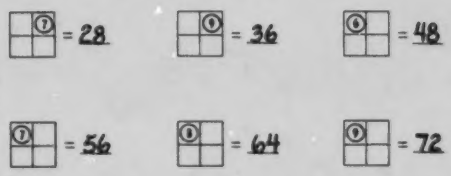
Here is a list of nine numbers.

26; 28; 36; 48; 52; 56; 64; 72; 80

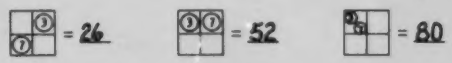
Six numbers in this list can be put on the ones' board of the Minicomputer using exactly one of these checkers:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Put the six numbers on these boards.



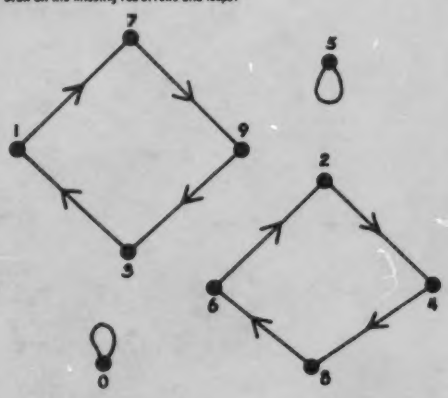
Put the other three numbers on the ones' board of the Minicomputer using exactly one ③-checker and exactly one ⑦-checker.



15

7×10

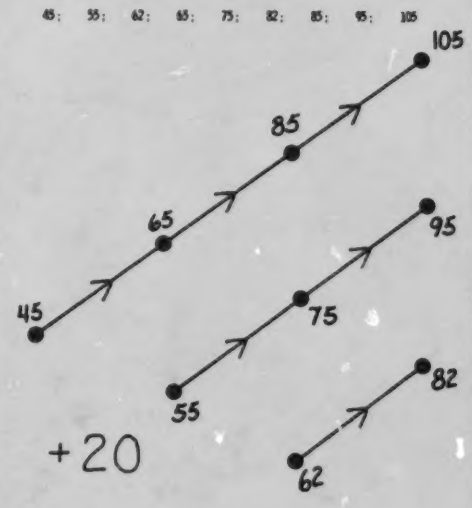
Draw all the missing red arrows and loops.



16

Put these nine numbers in this arrow picture.

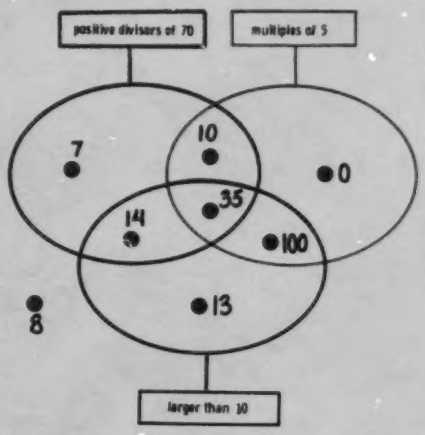
45; 55; 62; 65; 75; 82; 85; 95; 105



17

Put these numbers in this string picture.

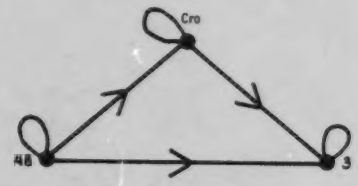
0; 7; 8; 10; 13; 14; 35; 100



18

First clue

is a positive multiple of



Cro is 24 or 12 or 6.

Second clue

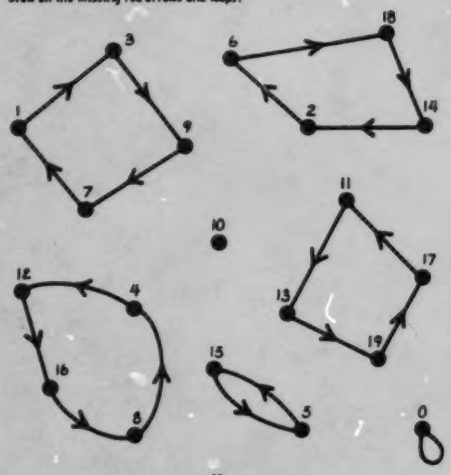
$\boxed{-}\ \boxed{5}\ \boxed{=}\ \cdots$

Cro is 24.

19

$3\times_{20}$

Draw all the missing red arrows and loops.



20

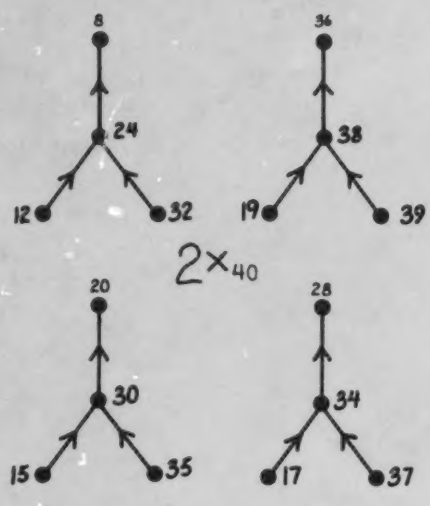
Build a red-blue road from 0.1 to 15.1.

$2\times$ $\quad$ $+0.3$

21

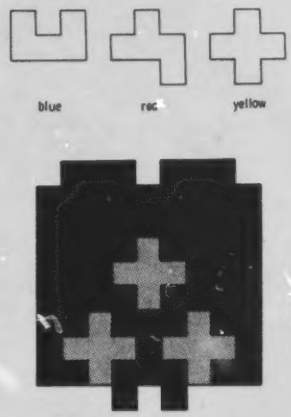
(Many other solutions are possible but none have less than nine arrows.)

Label the dots.



22

Color the twins to these shapes as indicated.



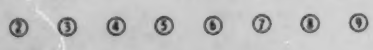
23

(In our picture, the yellow shapes are gray.)

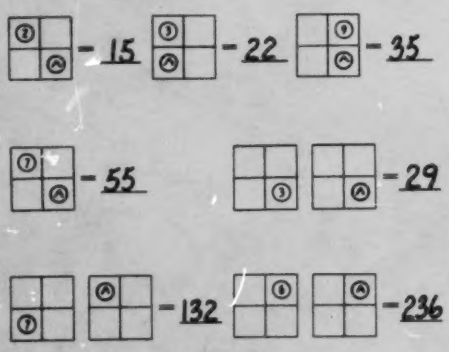
Here is a list of nine numbers.

15; 22; 29; 35; 45; 55; 132; 231; 236

Seven of the numbers in this list can be put on the Minicomputer using exactly one negative checker and exactly one of these checkers:

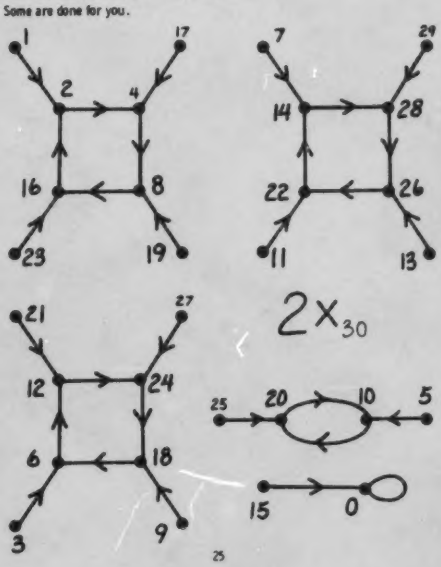


Put the seven numbers on these boards.



24

Put the numbers from 0 to 29 in this picture.
Some are done for you.

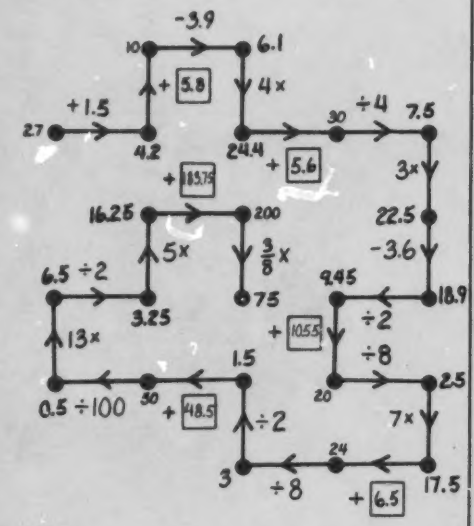


25

(In our picture, the yellow shapes are gray.)

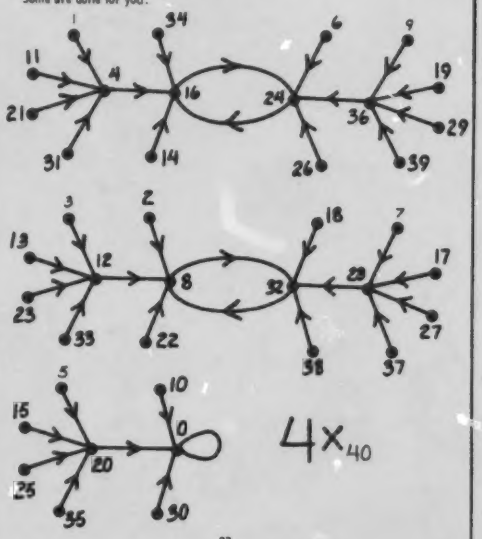
127198

Label the dots and the arrows.

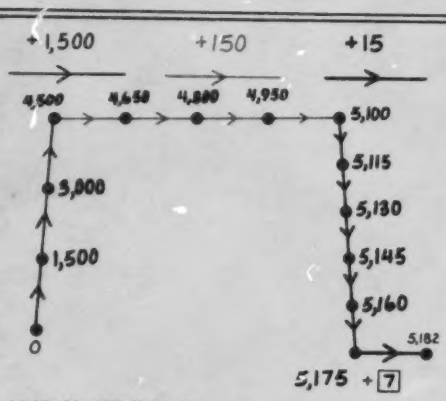


26

Put the numbers 0 to 39 in this arrow picture.
Some are done for you.



27

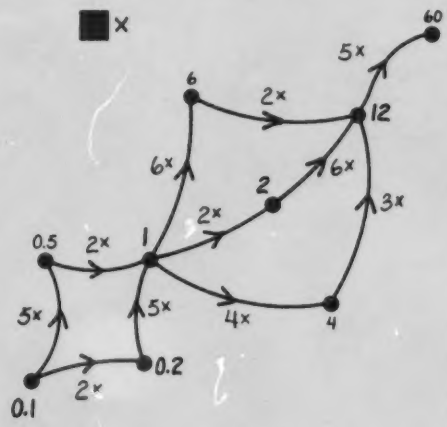


Label the dots and the black arrow.
Use the arrow picture to help solve this division problem.

$$15\overline{)5{,}182} = 345$$

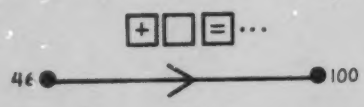
28

"■ ×" means each red arrow is for times some WHOLE number.
Label the dots. Label each arrow "□ ×". Many solutions are possible.



29

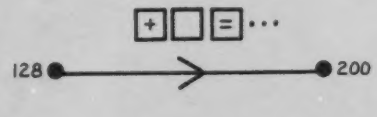
(Many solutions are possible.)



The number in the box could be 1 : 2 : 3 : 6 : 9 :

18 : 27 : 54 : 0.5 : 0.1 : 0.2 : 0.25 : 0.4 : and so on.

Your list should include as many whole numbers as possible.



The number in the box could be 1 : 2 : 3 : 4 : 6 :

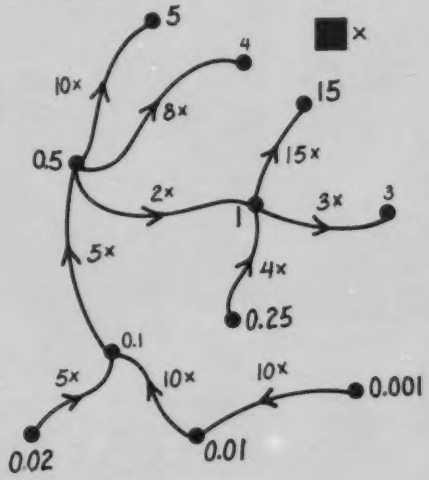
8 : 9 : 12 : 18 : 24 : 36 : 72 : 1.2 : and so on.

Your list should include as many whole numbers as possible.

30

"■ ×" means each red arrow is for times some WHOLE number.

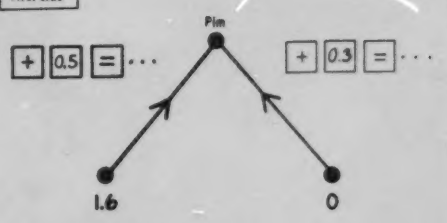
Label the dots. Label each arrow "□ ×". Many solutions are possible.



31

(Many solutions are possible.)

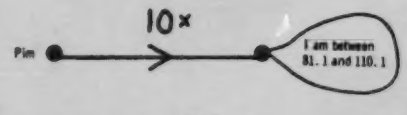
First clue



Pim could be 2.1 : 3.6 : 5.1 : 6.6 : 8.1 : 9.6 : 11.1 : 12.6 : 14.1 :

15.6 : 17.1 : 18.6 : 20.1 : 21.6 : and so on.

Second clue



Pim is 9.6.

32

A VARIETY OF PROBLEMS 6

This workbook has problems of varying levels of difficulty (*, **, and ***) all in one booklet — approximately the first 10 pages of which are easy (*) problems, the next 10 to 12 pages are average (**) problems and the last 10 pages are more challenging (***) problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for this workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about 20 to 25 pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Lesson 1

Give each student a copy of the workbook "A Variety of Problems 6" and let them work on their own.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

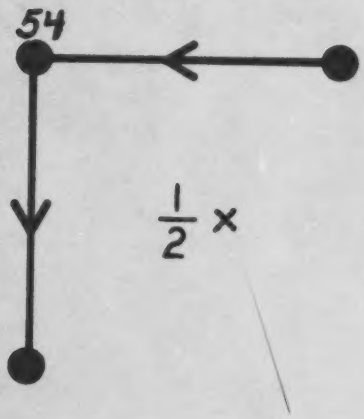
At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

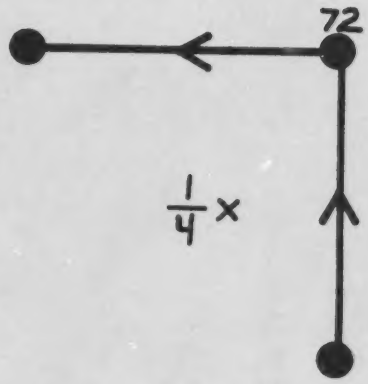
17W

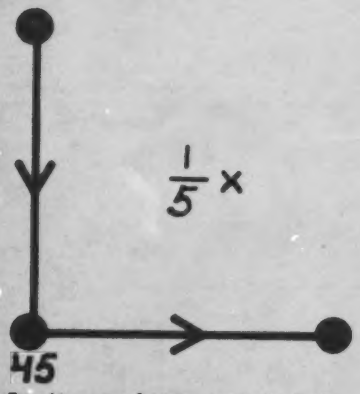
Lesson 2

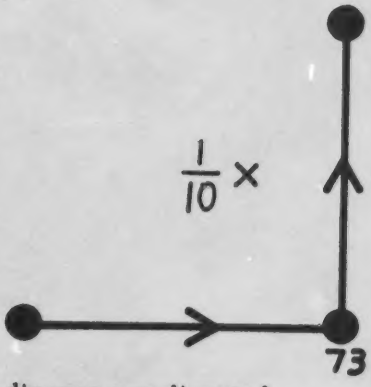
Exercise 1

Draw these five arrow pictures on the board.



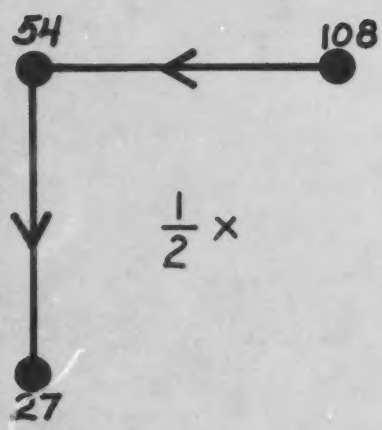


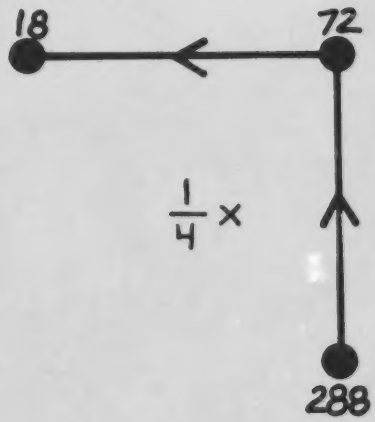


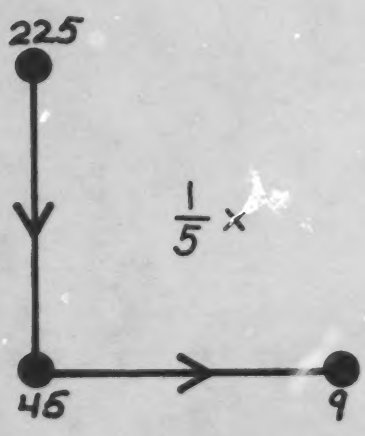


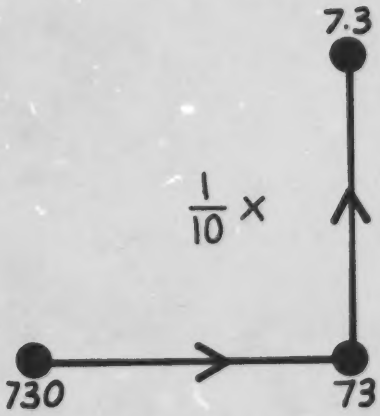
Invite students to label the dots. Be prepared to discuss any disputed

answers. Completed arrow pictures are shown below.









17W

Exercise 2

Draw these arrow pictures on the board.

T: Fill in the box for the red arrows and label the dots.

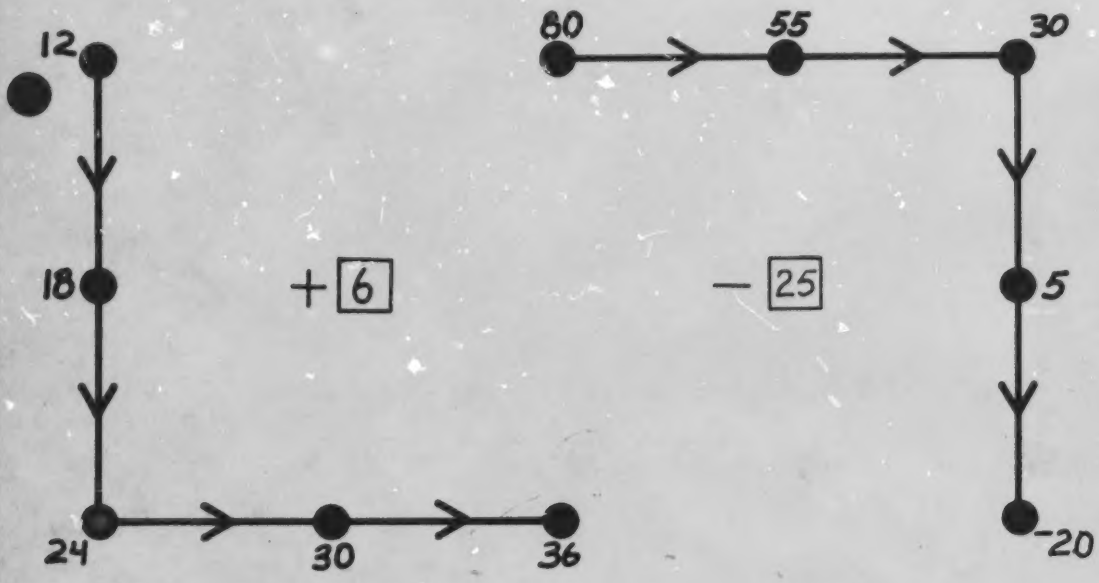
Let the students have a minute to think about the problem.

T: What could the red arrows be for?

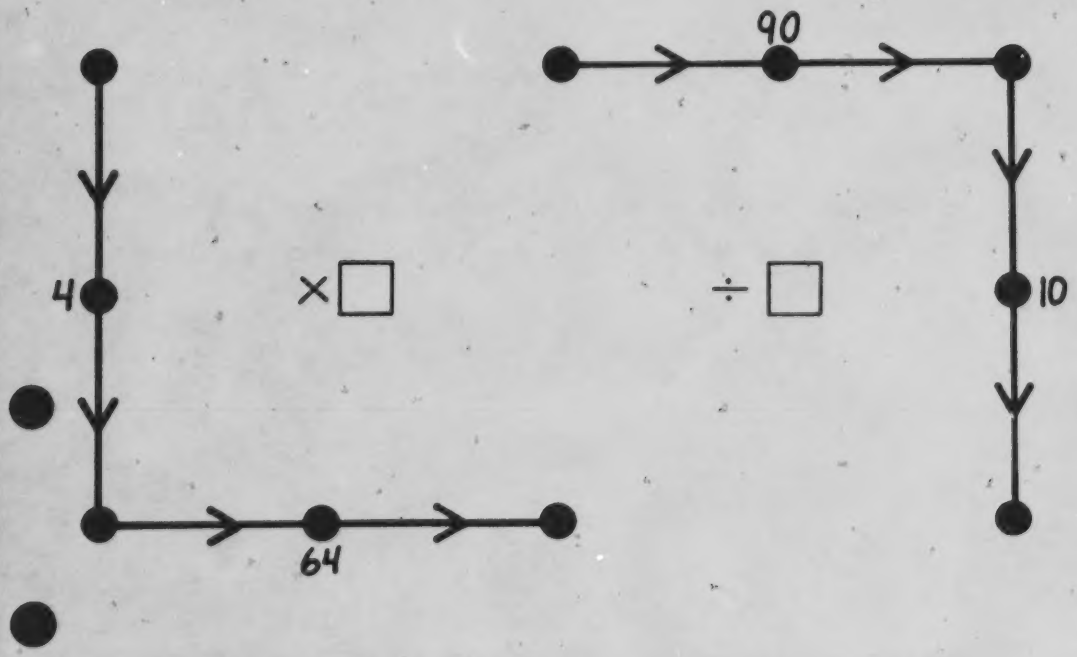
S: + 6.

T: Good. If the red arrows are for + 6, then who can label the dots?

Ask a student to label the dots. In a similar manner, fill in the box for the blue arrows and label the dots. Completed arrow pictures are shown below.

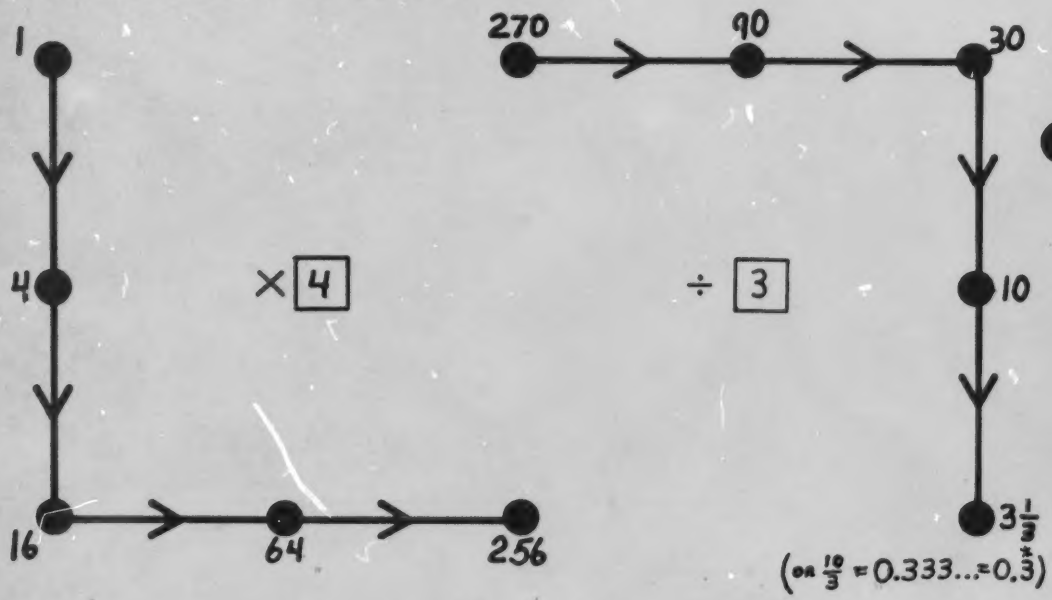


Erase the labels and relabel the arrow pictures as shown below.



17W

Repeat the activity, filling in the boxes for the arrows and labeling the dots. Completed arrow pictures are shown below.



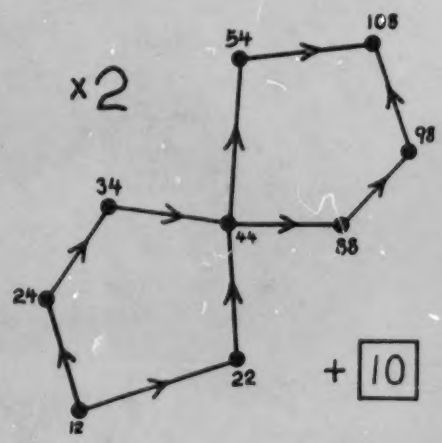
Return the students' copies of "A Variety of Problems 6". Ask them to correct their mistakes (if any), and then continue working in the workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the conclusion of this lesson, collect the workbooks for your progress check. You may wish to make this workbook available later for students to work in at their leisure.

A Variety of Problems 6

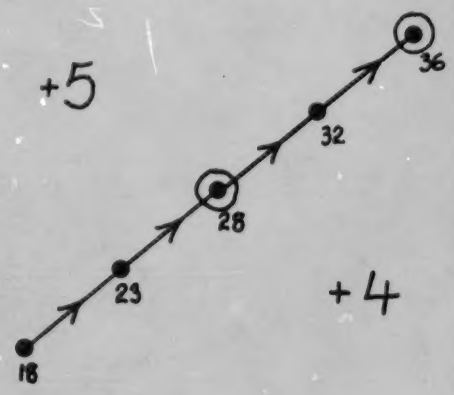
Label the dots and fill in the box for the blue arrows.



2

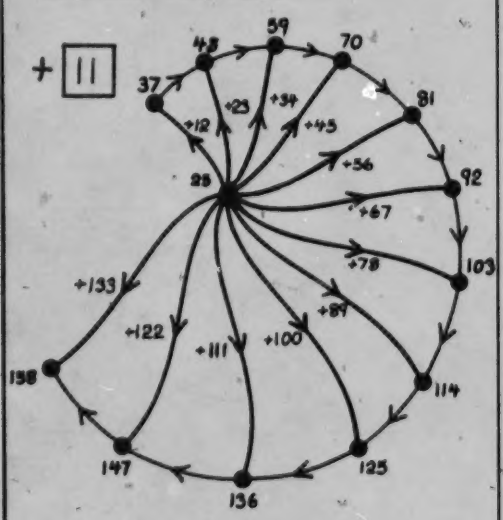
Flip = = 28 Flop = = 36

Flip and Flop are in this arrow picture. Circle their dots. Label all the dots.



3

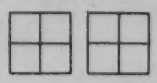
Label each dot and fill in the box for the blue arrows.



4

First clue

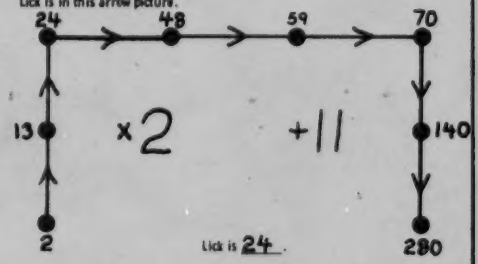
Lick can be put on these Minicomputer boards using exactly one ③-checker.



Lick is 3 or 6 or 12 or 24 or 30 or 60 or 120 or 240.

Second clue

Lick is in this arrow picture.

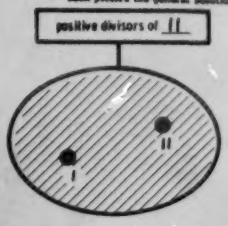


Lick is 24.

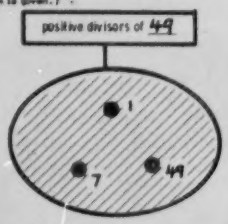
5

207

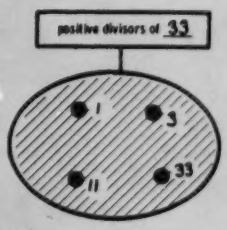
Fill in the blanks. Label the dots. Many answers are possible.
(For each of these one answer is given and below each picture the general solution is given.)



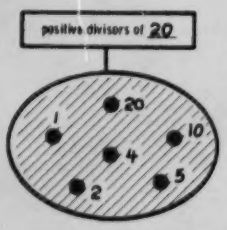
Any prime number.



The square of any prime number.



The product of two different prime numbers or the cube of a prime number.

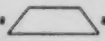


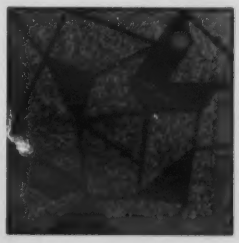
The fifth power of a prime number or any number in the form $p \cdot q^4$ where p and q are prime numbers and $p \neq q$.

6

This crazy quilt is made of patches of many sizes and many shapes.

Find four triangles exactly like and color them red.

Find three shapes exactly like  and color them blue.



Color the other patches with different colors.

7

One regular positive checker is missing on EACH of these Minicomputer boards. On each board, draw the missing checker.

= 18

= 26

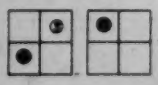
= 37

= 50

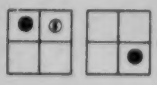
= 36

= 60

Be sure to draw a regular positive checker on the ones' board and on the tens' board.



= 148

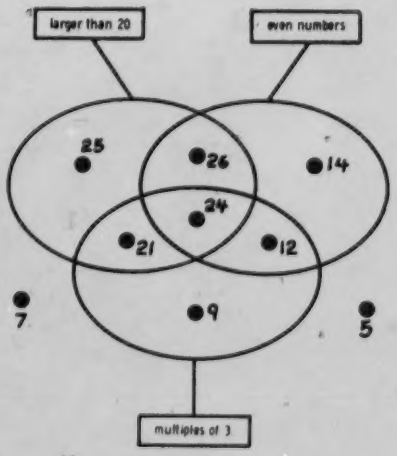


= 281

8

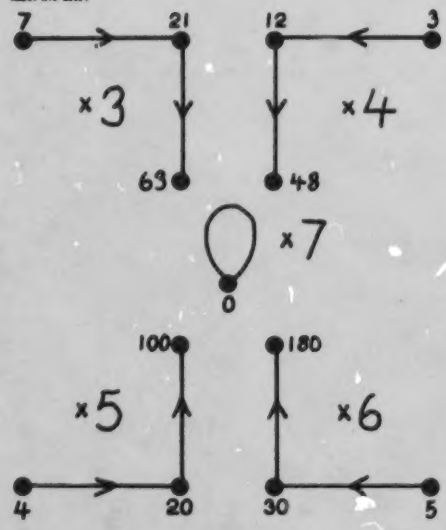
Put these numbers in this picture.

5; 7; 9; 12; 14; 21; 24; 25; 26



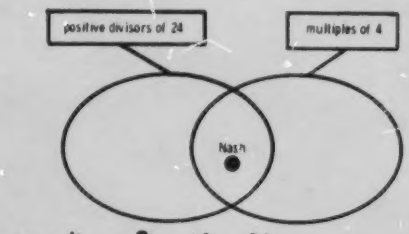
9

Label the dots.



10

First clue



Nash is 4 or 8 or 12 or 24.

Second clue

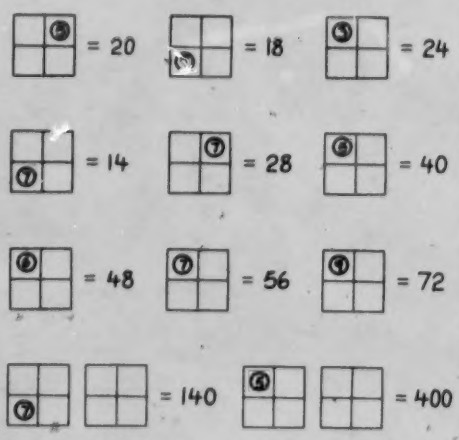
$\boxed{-}\ \boxed{3}\ \boxed{=}\ \cdots$

Nash is 4.

11

Put each number on the Minicomputer using exactly one of these checkers:

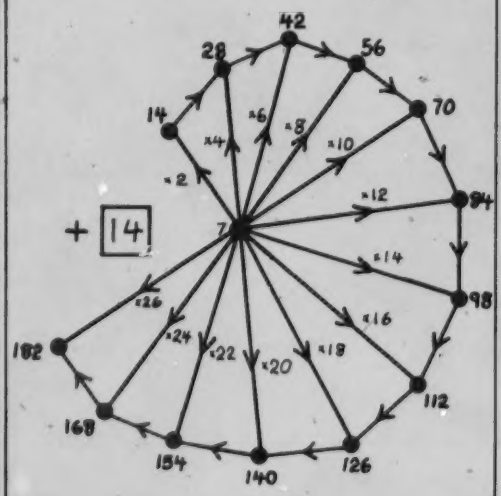
② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨



(Other solutions are possible.)

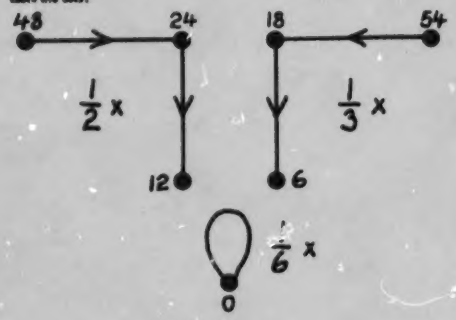
12

Label each dot and fill in the box for the blue arrows.



13

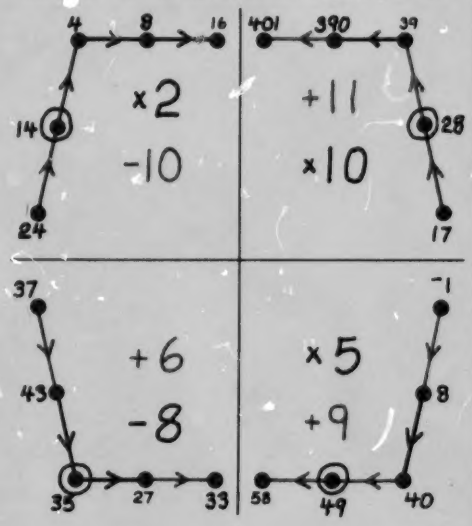
Label the dots.



14

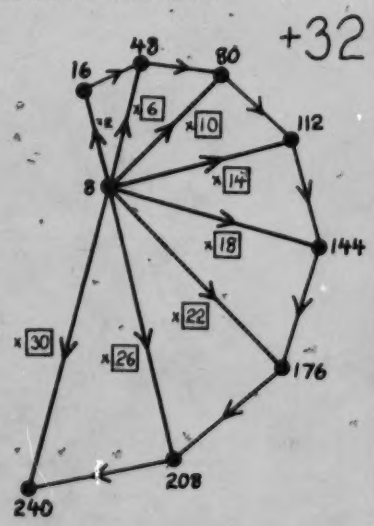
Label the dots.

A multiple of 7 is in each of these pictures. In each picture, circle its dot.



15

Label the dots and fill in the box for EACH red arrow.



16

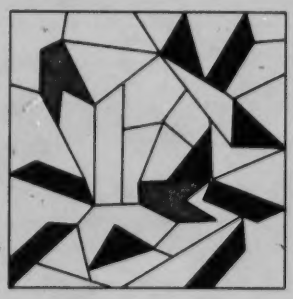
Here is another crazy quilt for you to color.

Color blue three shapes exactly like

Color red four shapes exactly like

Use another color for the two shapes exactly like

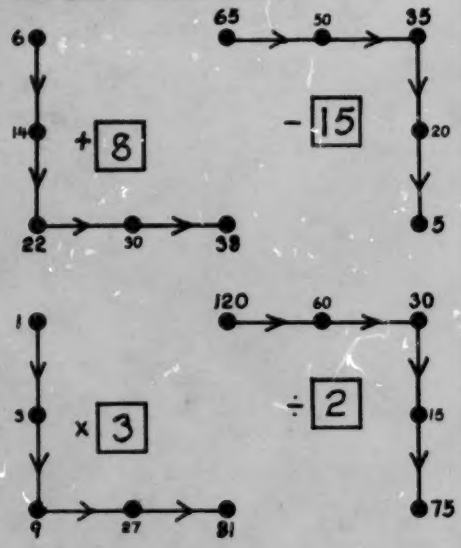
Use yet another color for the two shapes exactly like



Color the other patches as you like with different colors.

17

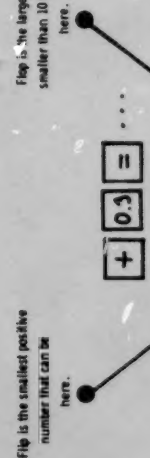
Label the dots and fill in the boxes for the arrows.



18

Flip is the largest number smaller than 10 that can be here.

Flip is 9.7.



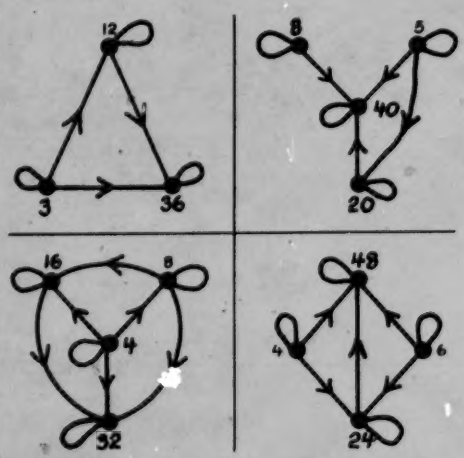
Flip is the smallest positive number that can be here.

Flip is 0.1.

19

Label the dots. Draw all the missing arrows and loops. Many answers are possible.

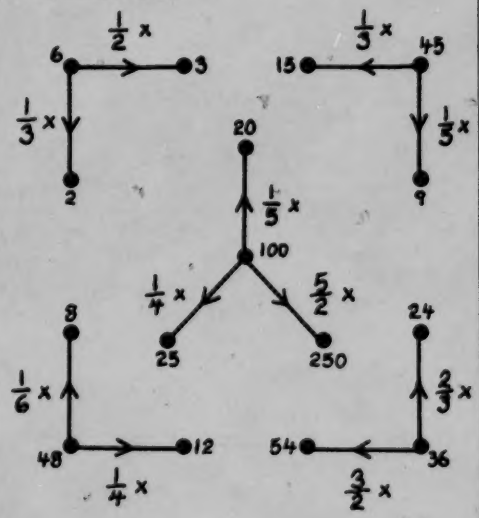
is a positive divisor of →



20

(Many solutions are possible for each arrow picture above.)

All the dots are for WHOLE numbers. Label the dots as you wish. Many answers are possible.



21

(Many other solutions are possible.)

One of the symbols + , − , × , ÷ , is hidden in each blank of the hand-calculator sentence. A symbol may be used more than once.

[2.5] [ ] [1] [ ] [2] [=] [KO]

Which is the largest number that Ko could be? 7

Which is the smallest number that Ko could be? −0.5

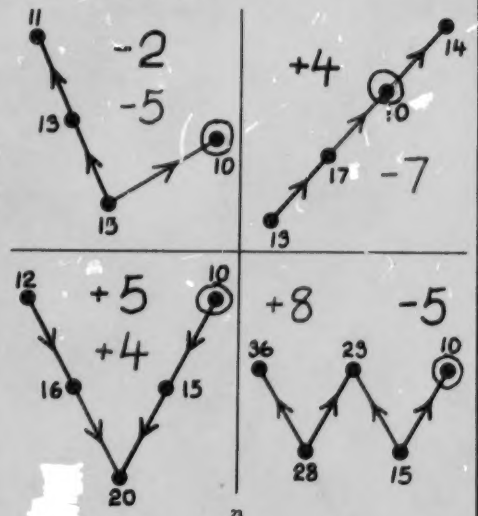
[480] [ ] [10] [ ] [10] [=] [KU]

Which is the largest number that Ku could be? 48,000

Which is the smallest number that Ku could be? 4.8

22

10 is the smallest number in each of these pictures. In each picture, circle its dot. Label all the dots.

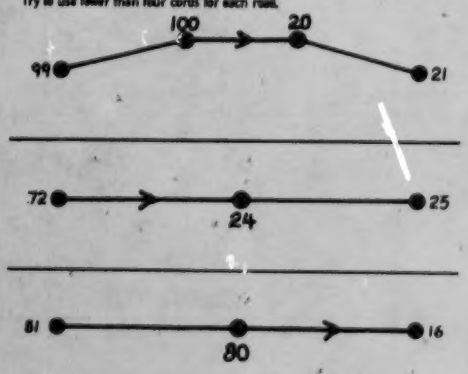


23

is a multiple of →

+1 or −1 ——

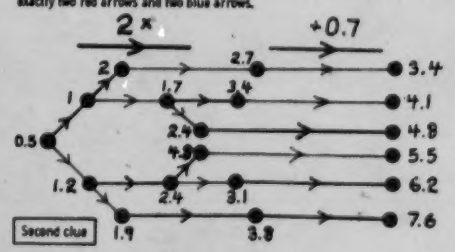
Build the shortest red - blue road possible between these pairs of numbers. Try to use fewer than four cards for each road.



24

First clue

Shig is the ending number of a red - blue road starting at 0.5 and using exactly two red arrows and two blue arrows.



Second clue

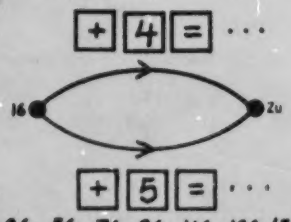
Shig can be put on the Minicomputer using exactly one of these checkers:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Shig is = 4.8

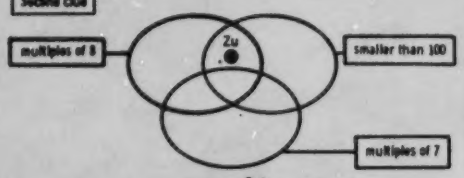
25

First clue



Zu could be 36 ; 56 ; 76 ; 96 ; 116 ; 136 ; 156 ; 176 ; 196 ; 216 ; 236 ; 256 ; and so on.

Second clue

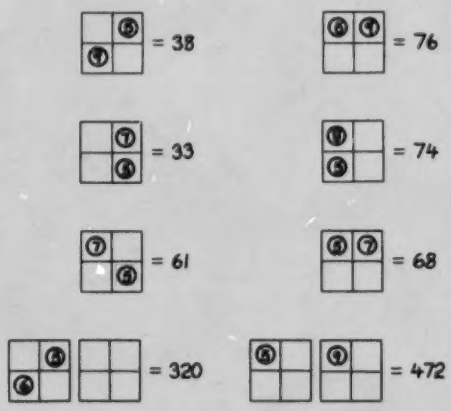


Zu is 96.

26

Put each number on the Minicomputer using a ⑤ - checker and exactly one of these checkers:

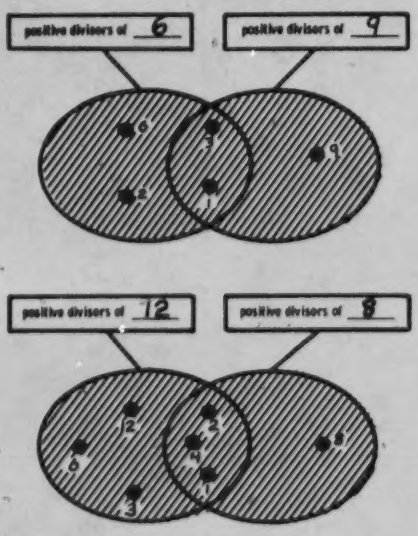
② ③ ④ ⑥ ⑦ ⑧ ⑨



27

(Other solutions are possible.)

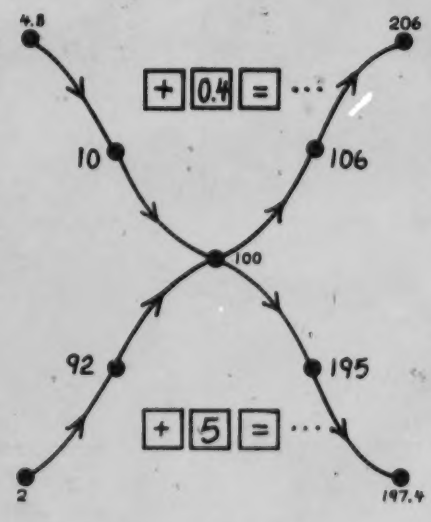
Fill in the blanks. Label the dots. Many answers are possible.



28

(Many other solutions are possible for each string picture.)

Label the dots. Many answers are possible.

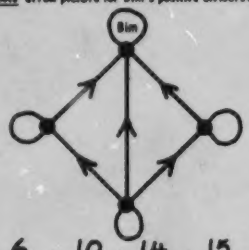


29

(Many solutions are possible.)

is a positive divisor of

Here is the complete arrow picture for Bim's positive divisors.



Bim could be 6 ; 10 ; 14 ; 15 ; 21 ; 22 ; 26 ; 33 ; 35 ; 39 ; and so on.

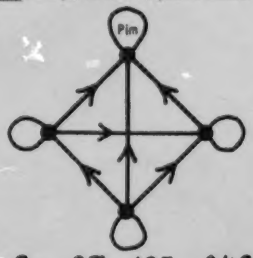
What do you notice?

Bim is the product of two different prime numbers.

30

is a positive divisor of

Here is the complete arrow picture for Pim's positive divisors.

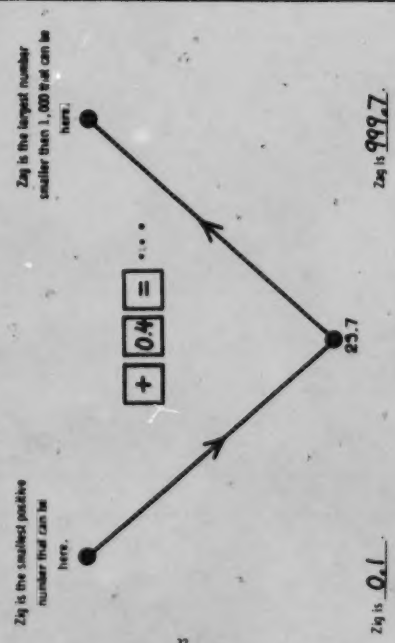


Pim could be 8 ; 27 ; 125 ; 343 ; 1,331 ; 2,197 ; and so on.

What do you notice?

Pim is the cube of a prime number ; i.e., Pim = p x p x p where p is some prime number.

31



Zig is 0.1

Zig is 999.7

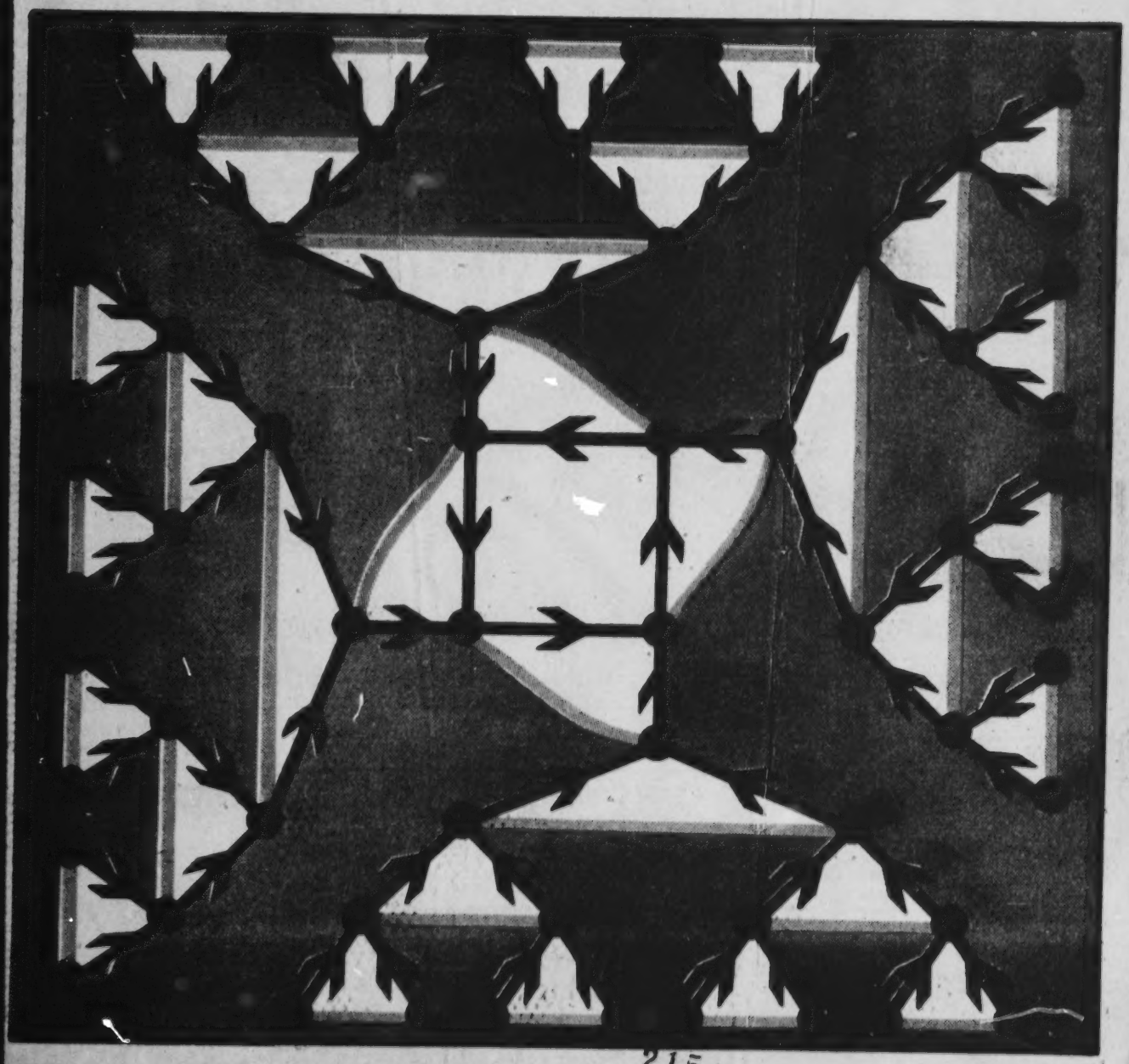
32

Comprehensive School Mathematics Program
CEMREL, Inc.
5-25601

CSMP Mathematics for the Intermediate Grades Part III

Name

Worksheets



WHAT'S IN THIS BOOK ?

This book contains all the worksheets you will need this semester. They are in the following order:

1. All the "N" worksheets: N01 through N91
2. All the "L" worksheets: L01 through L30
3. All the "G" worksheets: G01 through G26

HOW DO YOU REMOVE PAGES ?

Step 1: Flip through until you find the page you want. The number of each worksheet is in the top right corner of the page.

Step 2: Fold and crease the page along the perforation line.

Step 3: With your left hand index finger and thumb, press book firmly down left of the perforation line and along center fold, starting at the top of the book.

Step 4: With your right hand holding top right corner of page, tear page along the perforation line. Follow the tear with your left hand holding down center fold of book.

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Comprehensive School Mathematics Program
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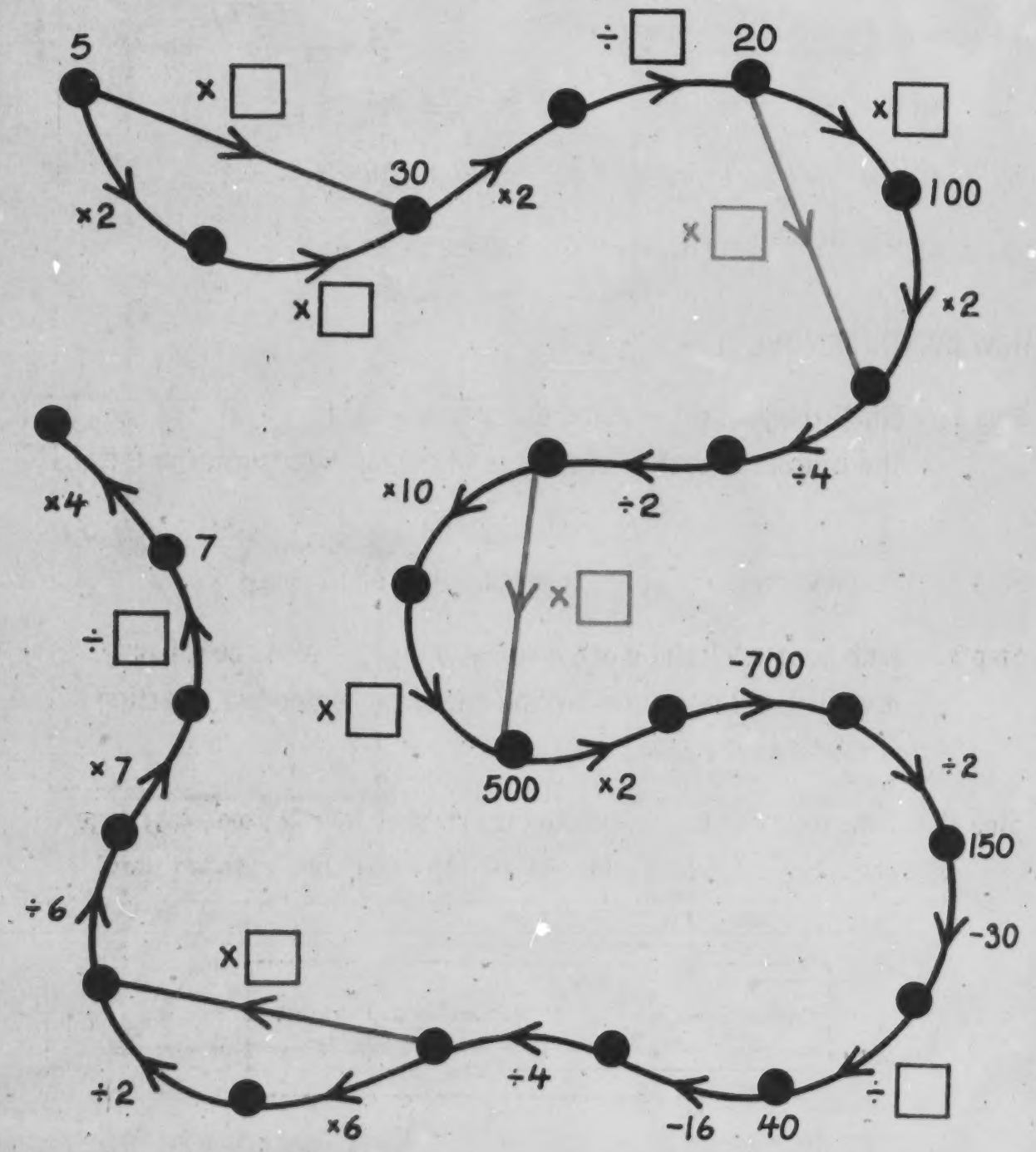
Name _____ **N 01** *

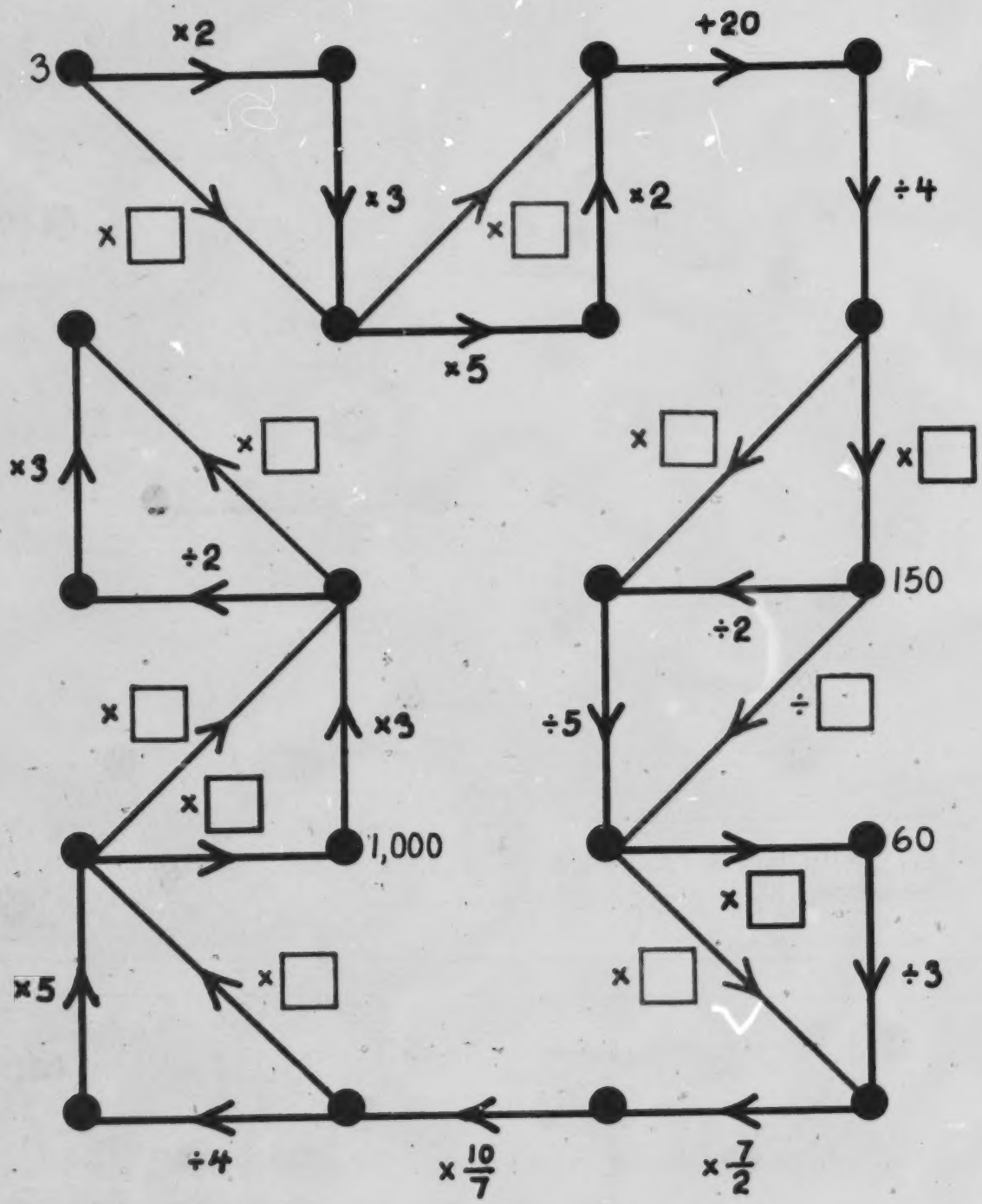
Label the dots and fill in the boxes for the arrows.



Name _____

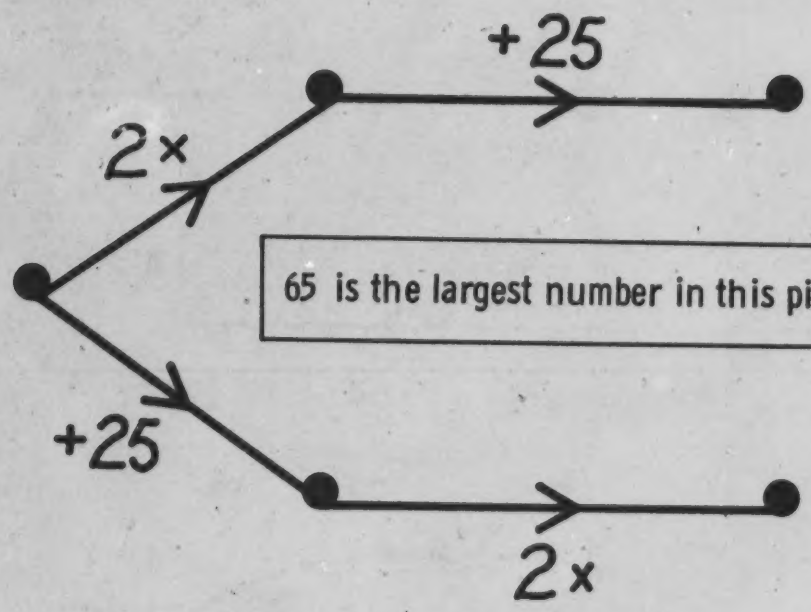
N 02 **

Label the dots and fill in the boxes for the arrows.

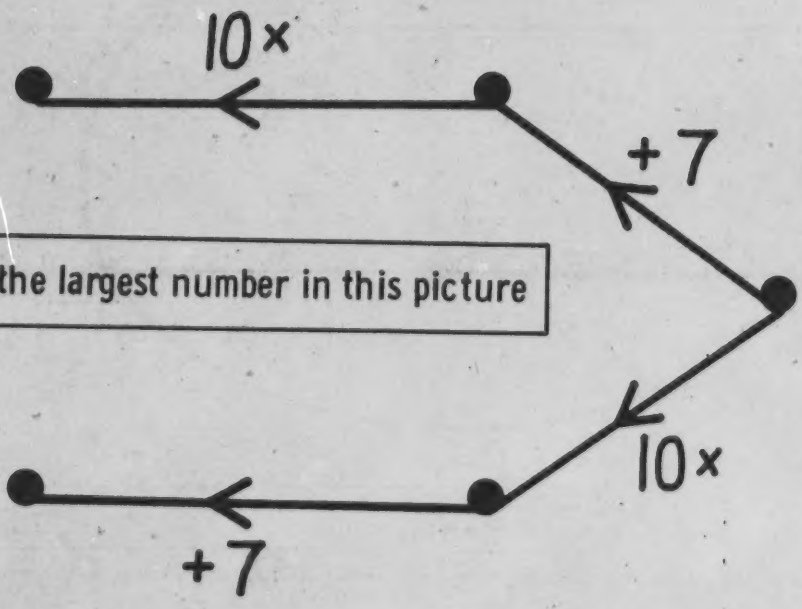


Name _____ **N 03** ***

All the dots are for positive numbers.
Label all the dots.



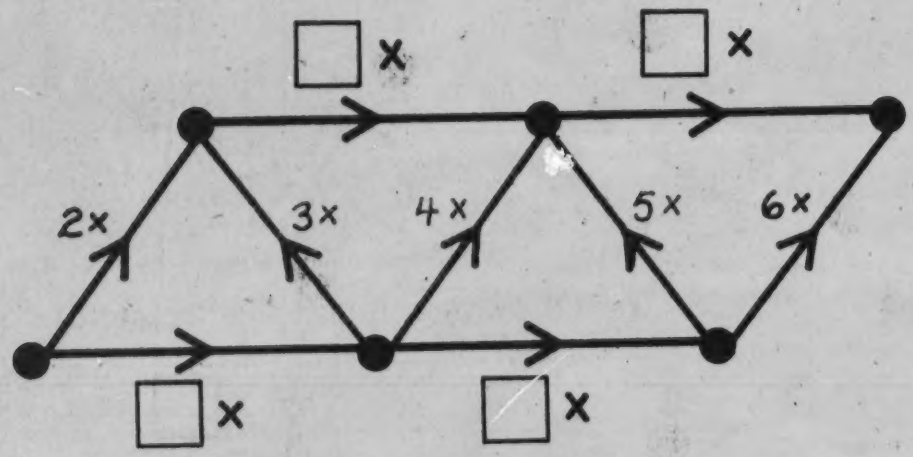
65 is the largest number in this picture

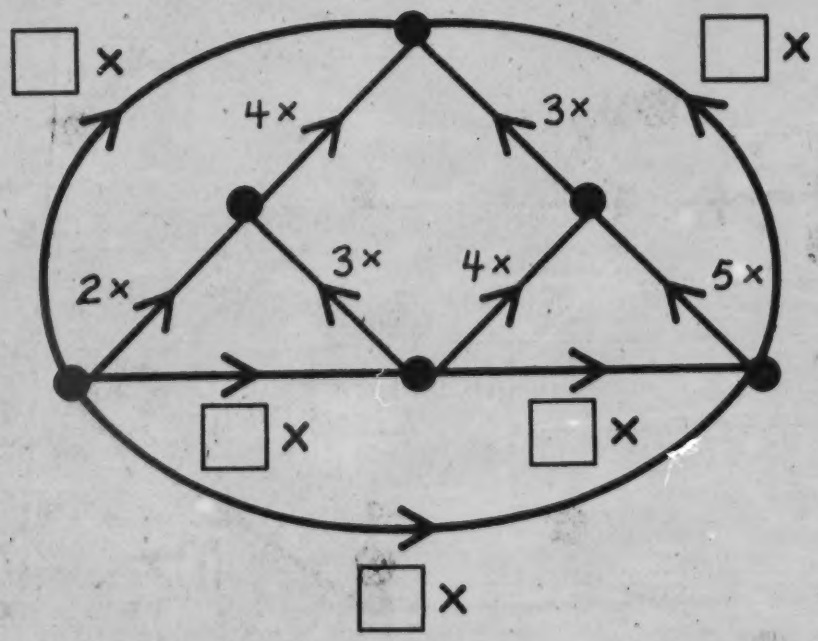


86 is the largest number in this picture

Name _____ **N 04 ******

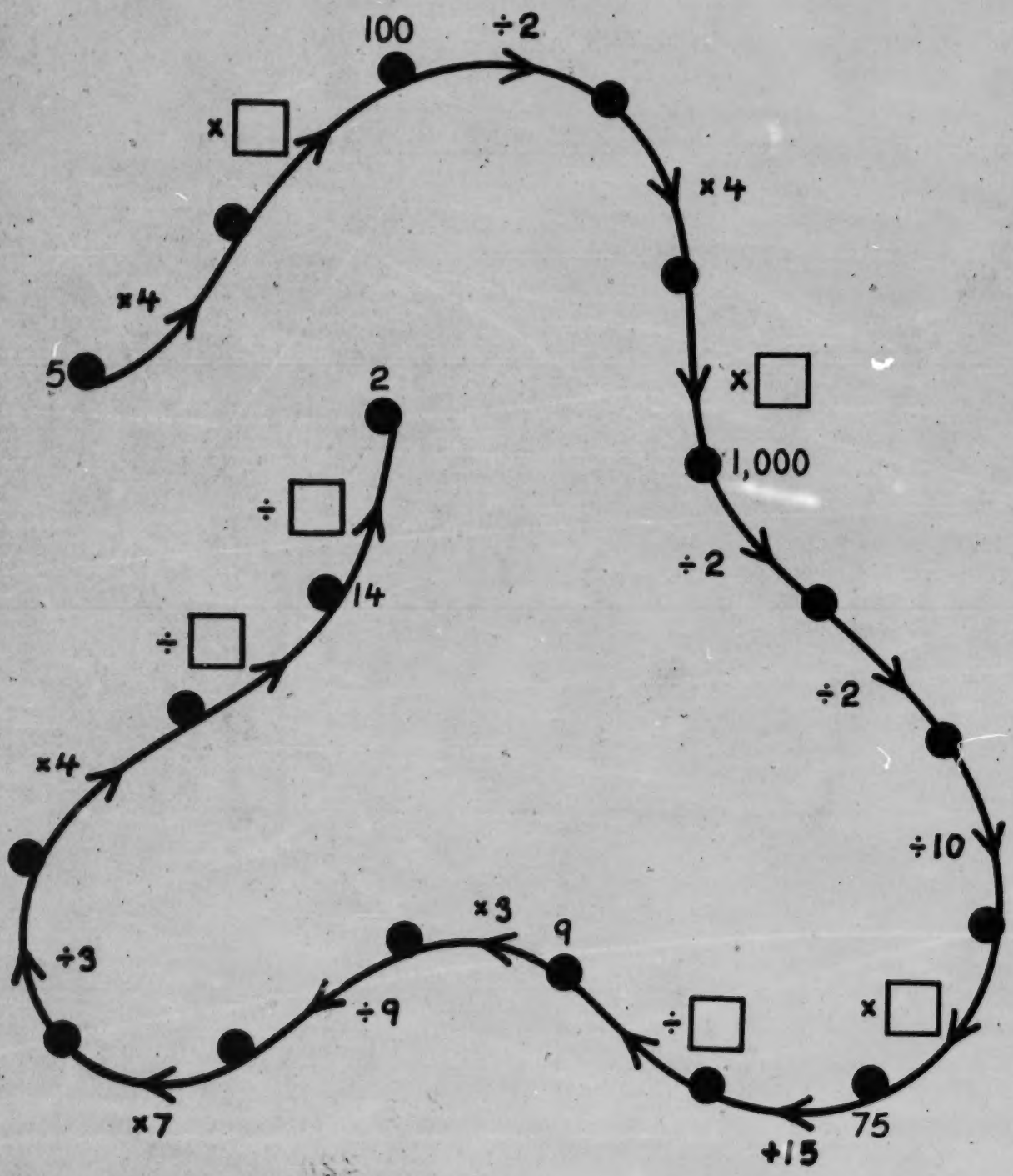
Fill in the box for each blue arrow.





Name _____ **N 05** *

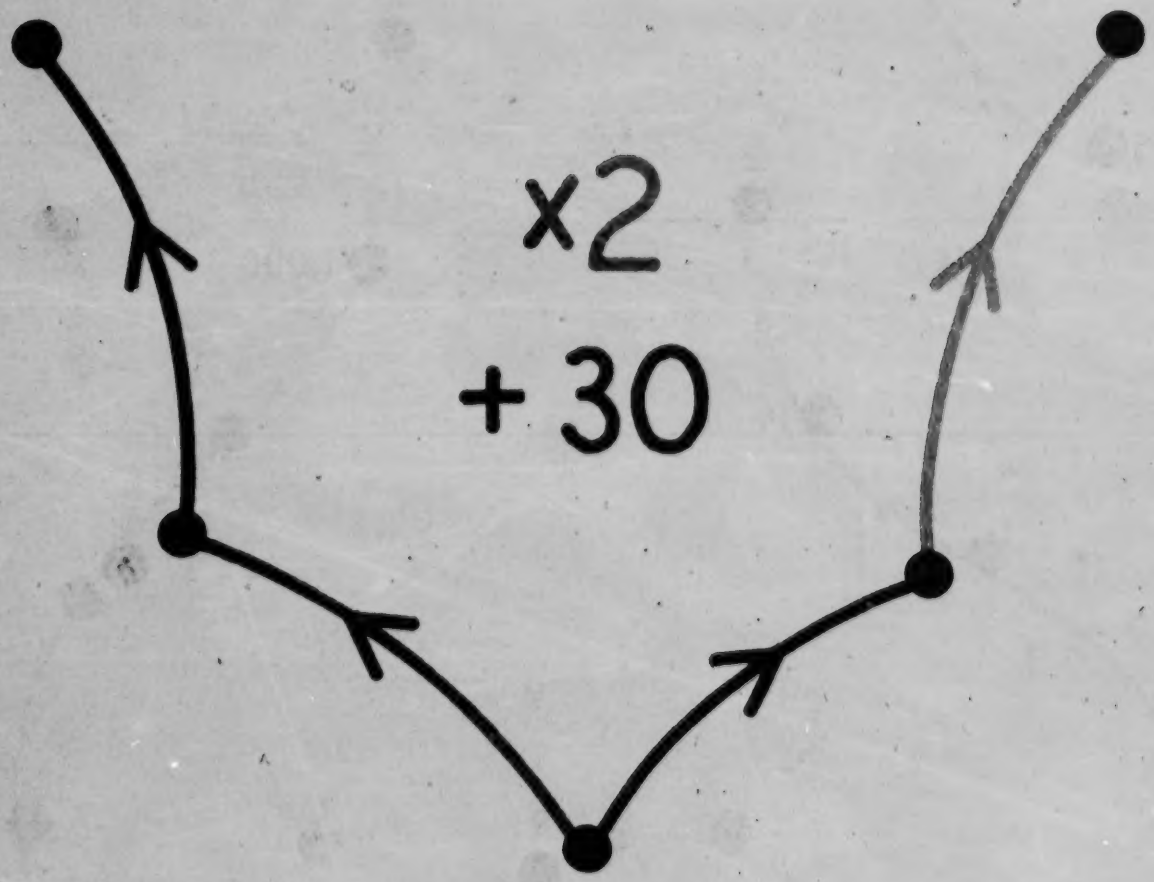
Label the dots and fill in the boxes for the arrows.



Name _____ **N 06** **

In this picture, all the dots are for POSITIVE numbers and 100 is the largest number.

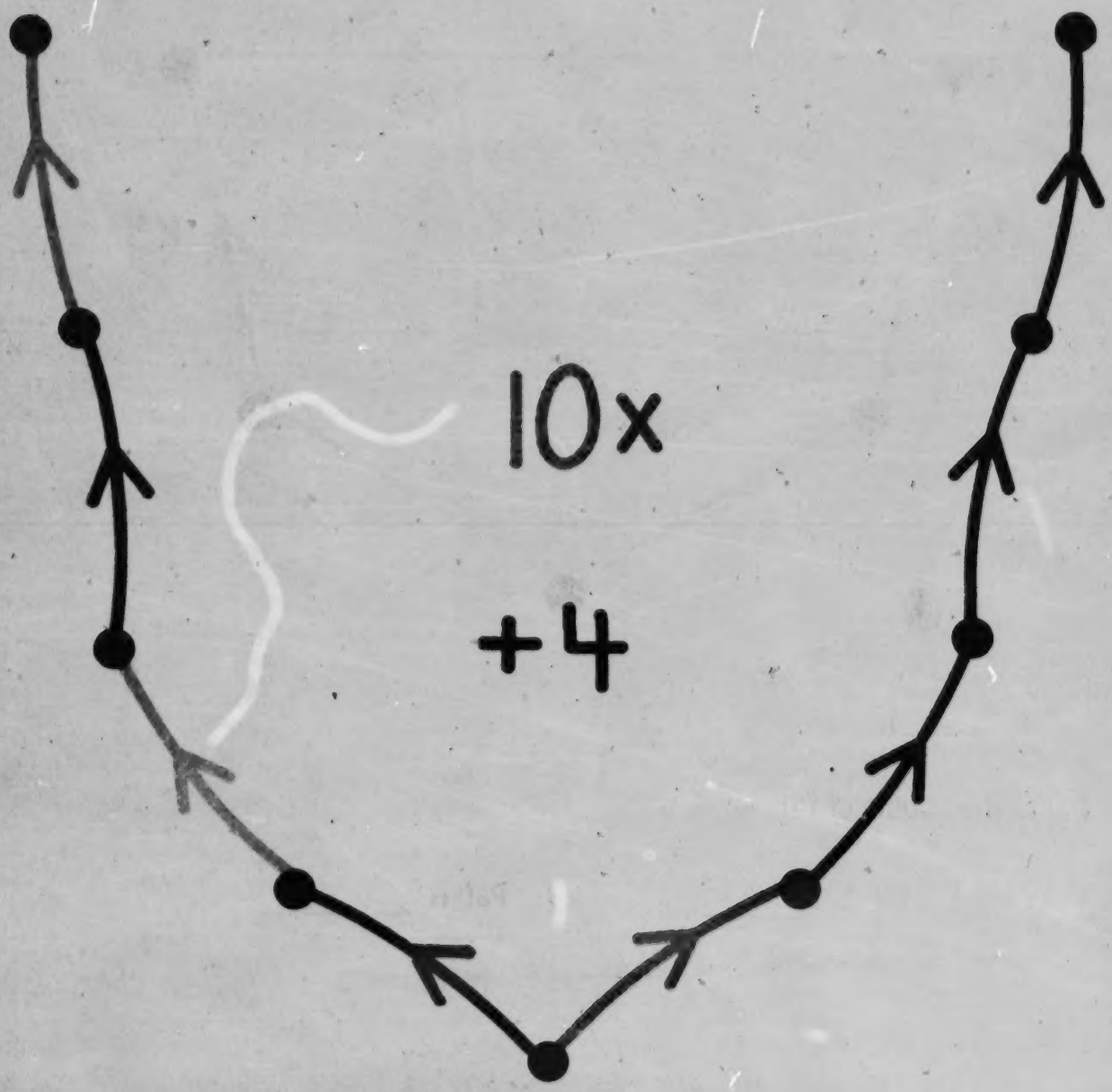
Put 100 in the picture and label the dots.



Name _____ **N 07** ***

In this picture, all the dots are for POSITIVE numbers and 685 is the largest number.

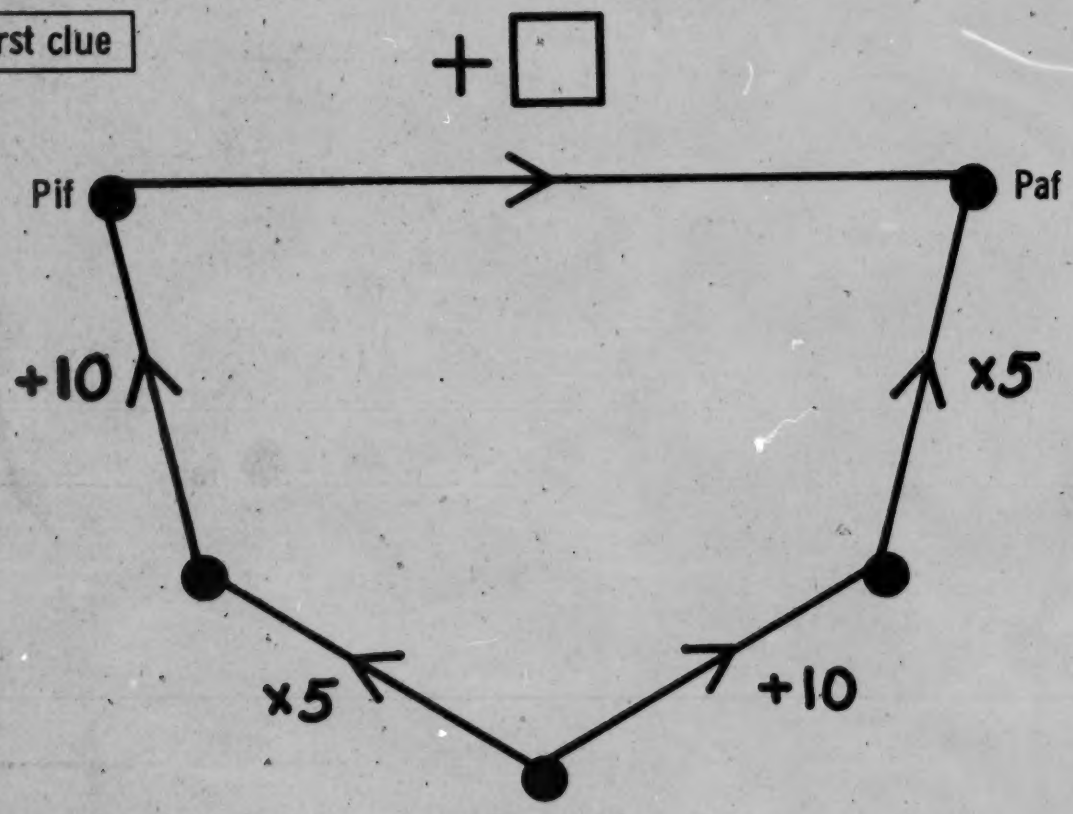
Put 685 in the picture and label the dots.



Name _____ **N 08 ******

Pif and Paf are two secret numbers.

First clue

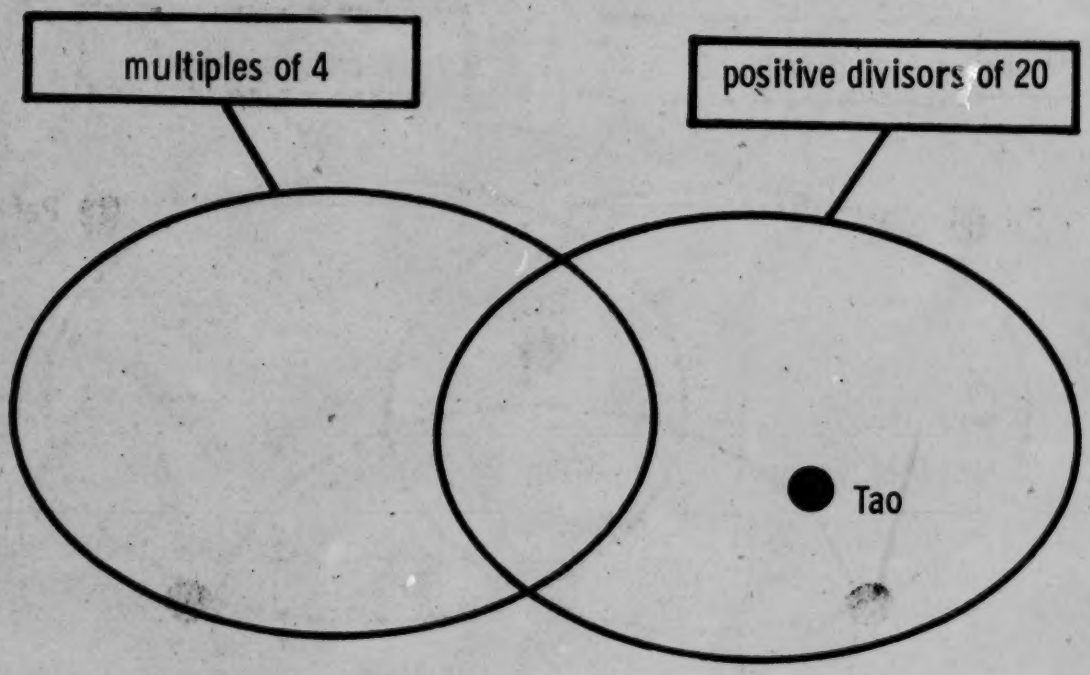


Second clue

Paf is the double of Pif.

Pif is _____. Paf is _____.

Name _____ **N 09** *



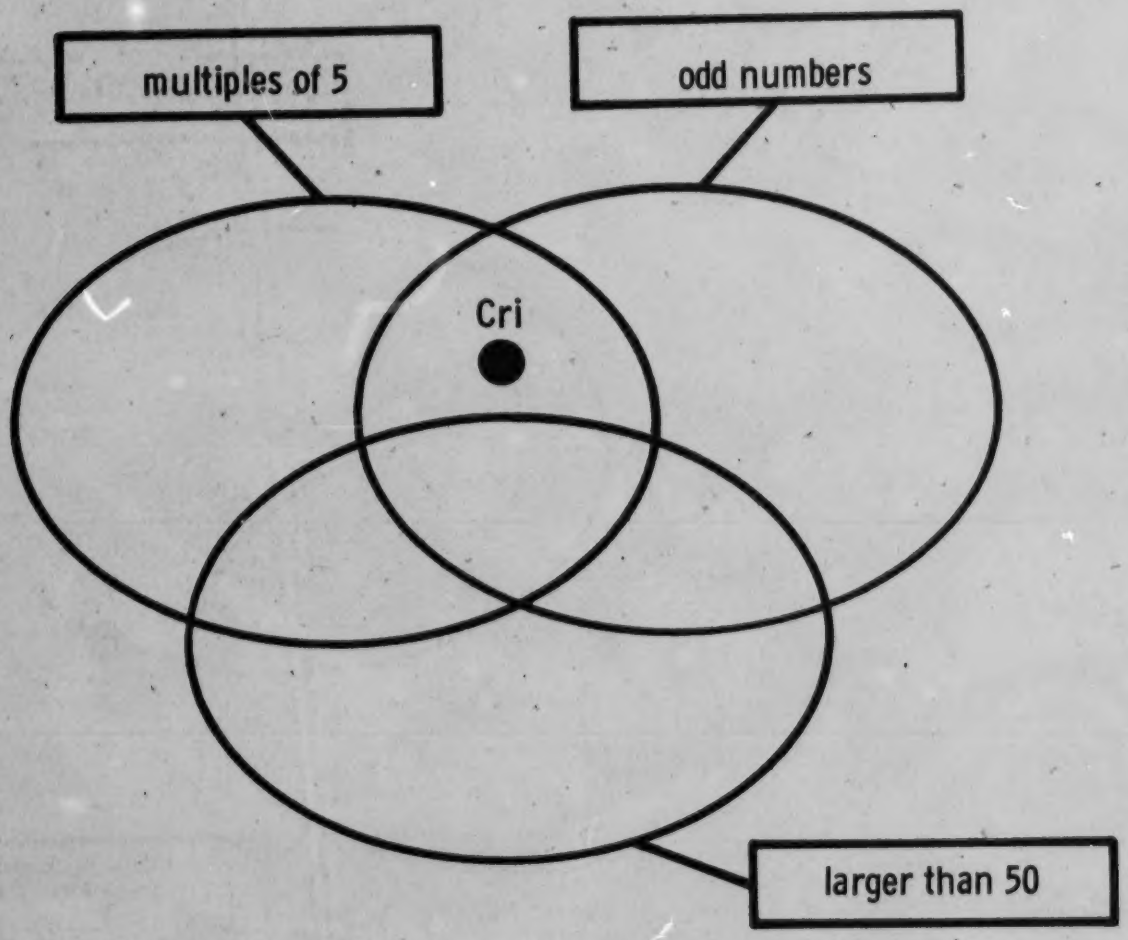
Can Tao be 10? _____; 4? _____; 8? _____; 20? _____;

6? _____; 5? _____; 2? _____; 7? _____

Tao is larger than 5. Who is Tao? _____

Name _____

N 10 **



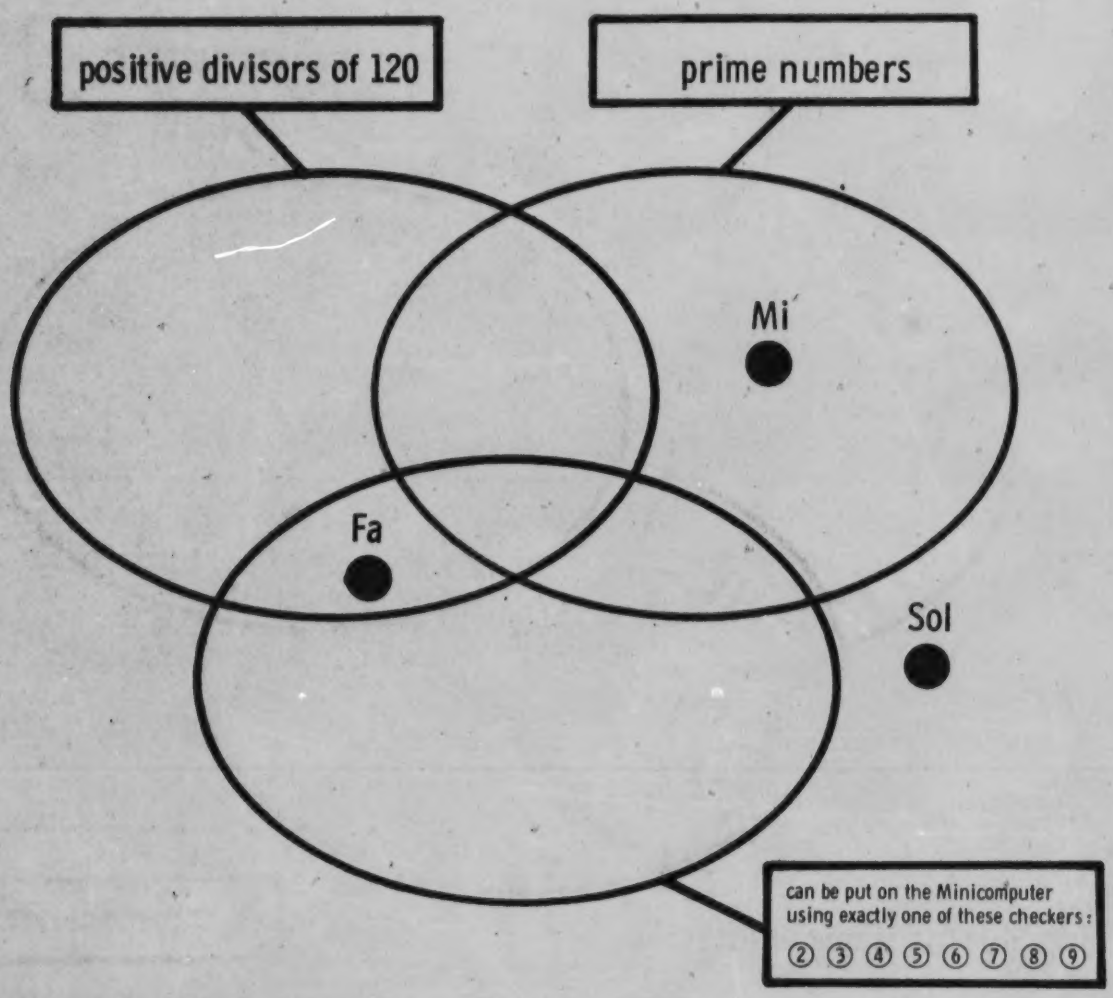
Can Cri be 75 ? _____ ; 20 ? _____ ; 15 ? _____ ; 50 ? _____ ;

8 ? _____ ; 35 ? _____ ; 105 ? _____ ; 72 ? _____

Cri is larger than 37. Who is Cri ? _____

Name _____

N 11 ***



Mi, Fa and Sol are three of these numbers:

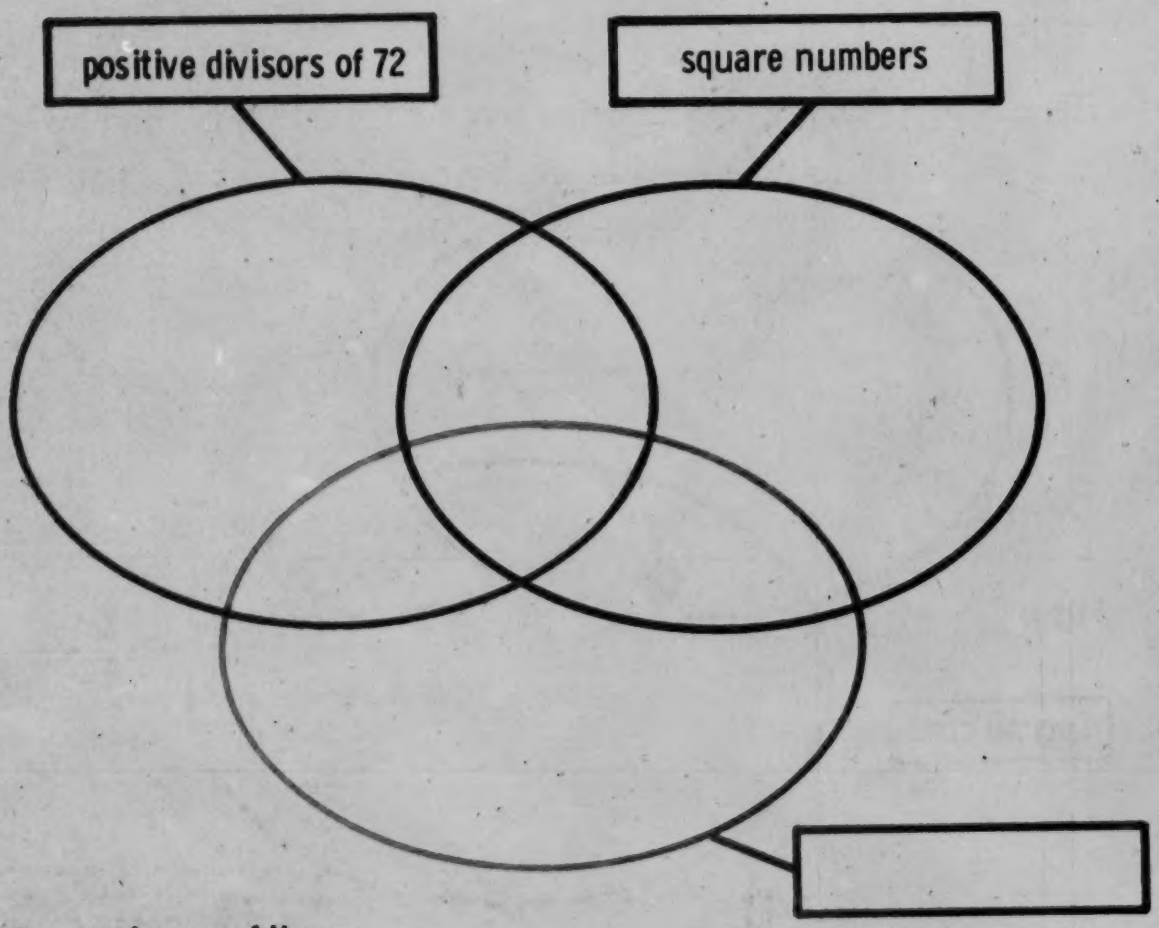
3; 5; 7; 15; 35; 36; 37; 60

Mi is _____; Fa is _____; Sol is _____.

One region of this diagram should be hatched. Hatch it.

Name _____

N 12 ****



The blue tag is one of these:

larger than 50

multiples of 4

With this information you should be able to place seven of these numbers in the string picture.

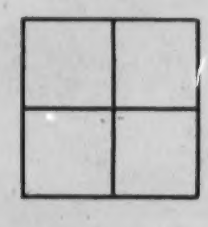
2; 4; 9; 15; 20; 49; 60; 64; 72.

Place them.

Name _____ **N 13** *

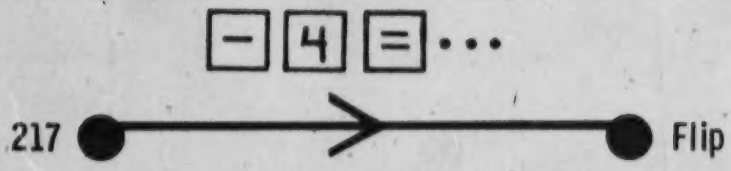
First clue

Flip can be put on this Minicomputer board using exactly two positive checkers.



Flip is ___ or ___ or ___ or ___ or ___ or ___ or ___ or ___ or ___ or ___.

Second clue



Flip is ___ or ___.

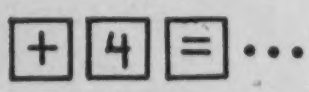
Third clue

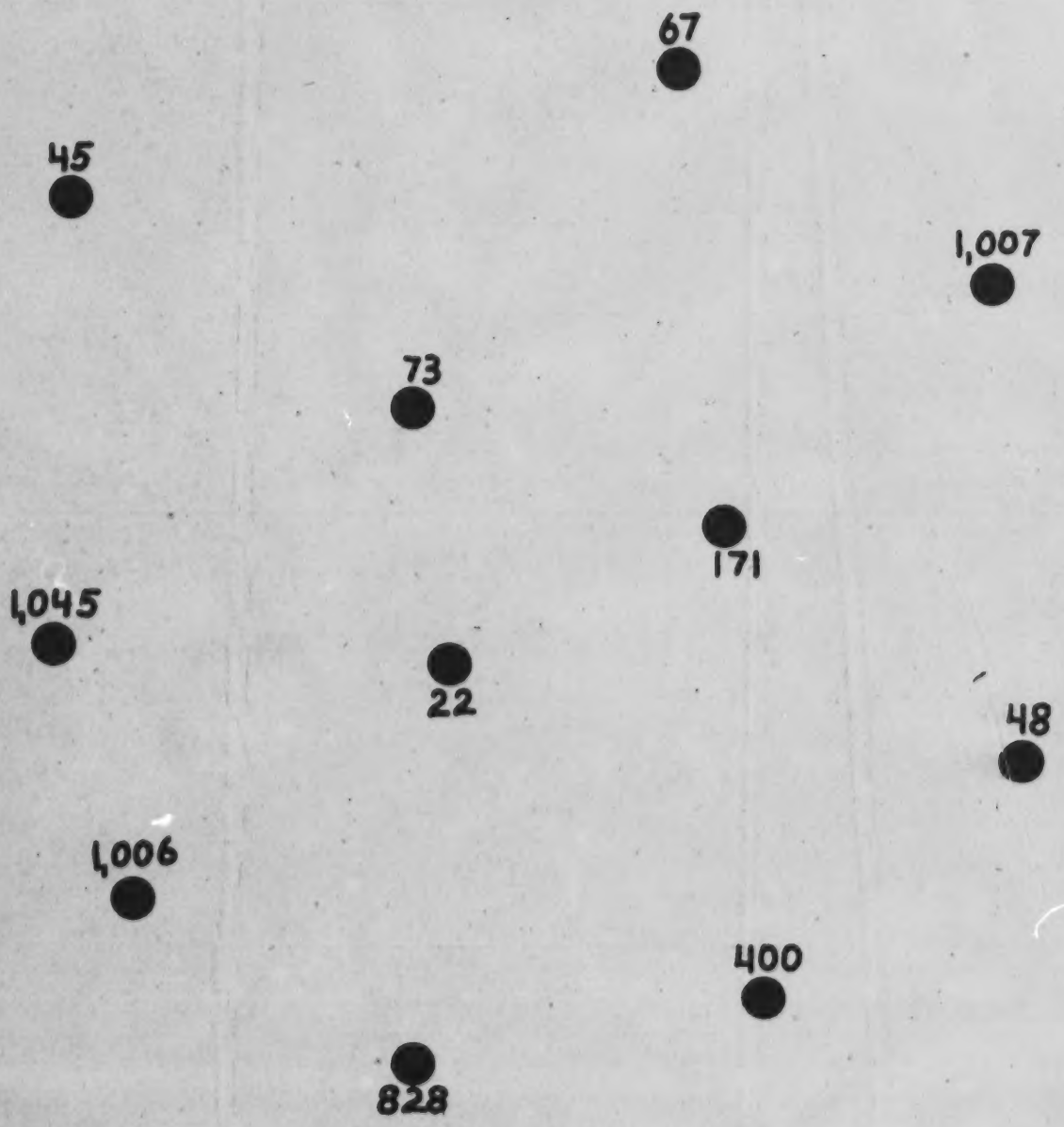
Flip is a prime number. Flip is _____.

Name _____

N 14 **

Draw all the missing arrows.

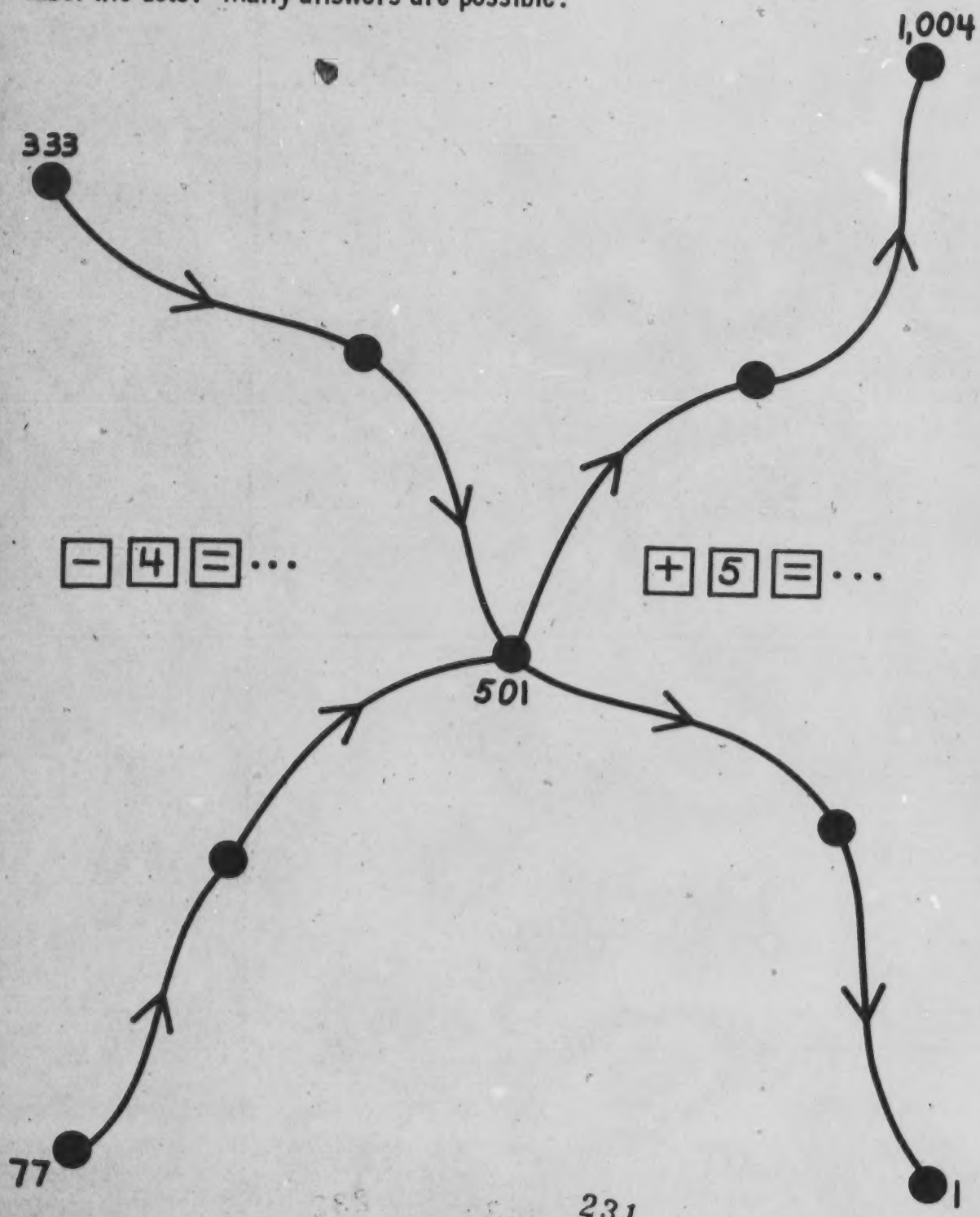




Name _____

N 15 ***

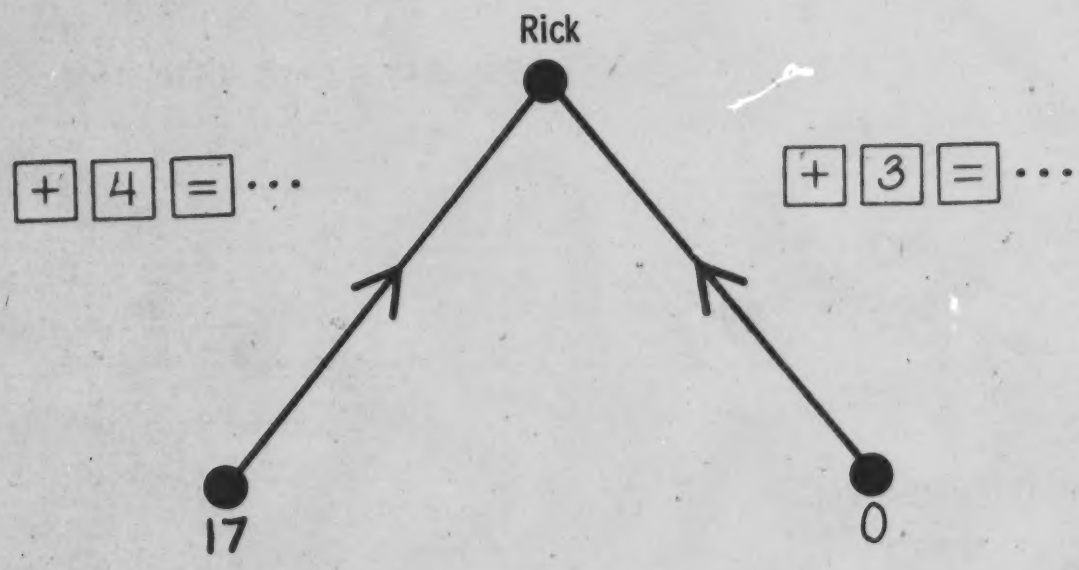
Label the dots. Many answers are possible.



Name _____

N 16 ****

First clue



Rick could be ____; ____; ____; ____; ____; ____; ____; ____; ____;

____; ____; ____; ____; ____; and so on.

Second clue

Rick is between 1,200 and 1,220.

Rick is _____.

Name _____ N 17 _____ *

Jo is a secret number.

First clue

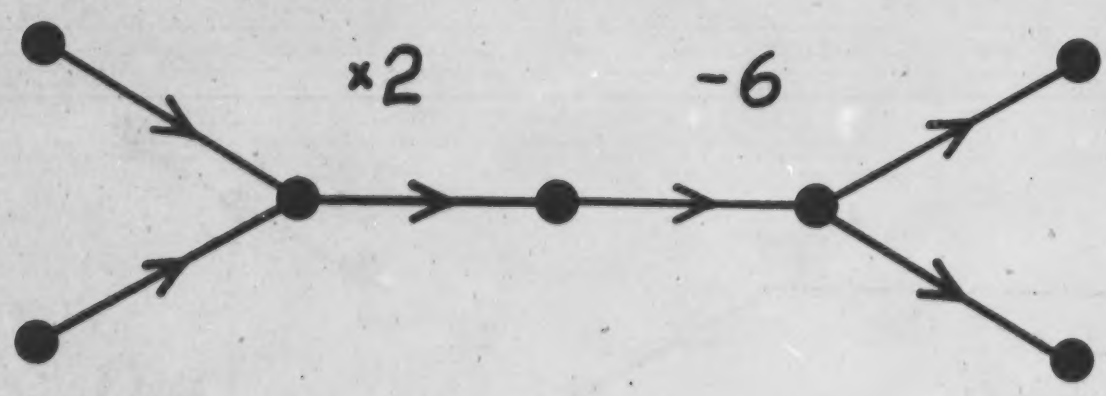
One of the symbols: + , − , x is hidden in each blank of this hand-calculator sentence. A symbol may be used twice.

7		3		2	=	Jo

Jo is ____ or ____ or ____ or ____ or ____ or ____ or ____ or ____.

Second clue

Jo is in this arrow picture.



Jo is _____ or _____.

Third clue

If you add 1 to Jo, you will get a square number.

Jo is _____.

Name _____ **N 18** **

Lou is a secret number.

First clue

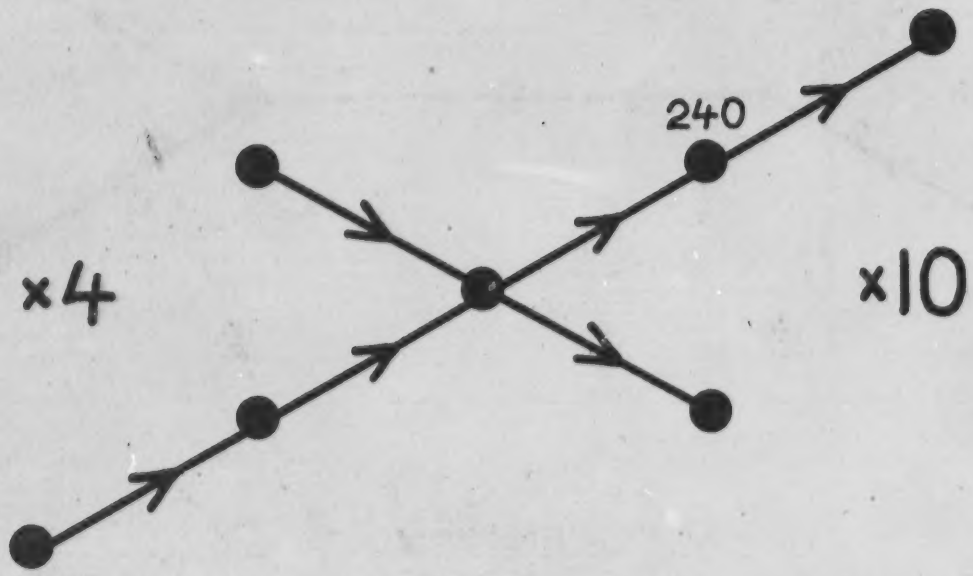
One of the symbols: +, ×, ÷ is hidden in each blank of this hand-calculator sentence. Each symbol may be used only once.

9		3		2		10	=	Lou

Lou is _____ or _____ or _____ or _____ or _____ or _____.

Second clue

Lou is in this arrow picture.



Lou is _____.

Name _____ **N 19** ***

Kir is a secret number.

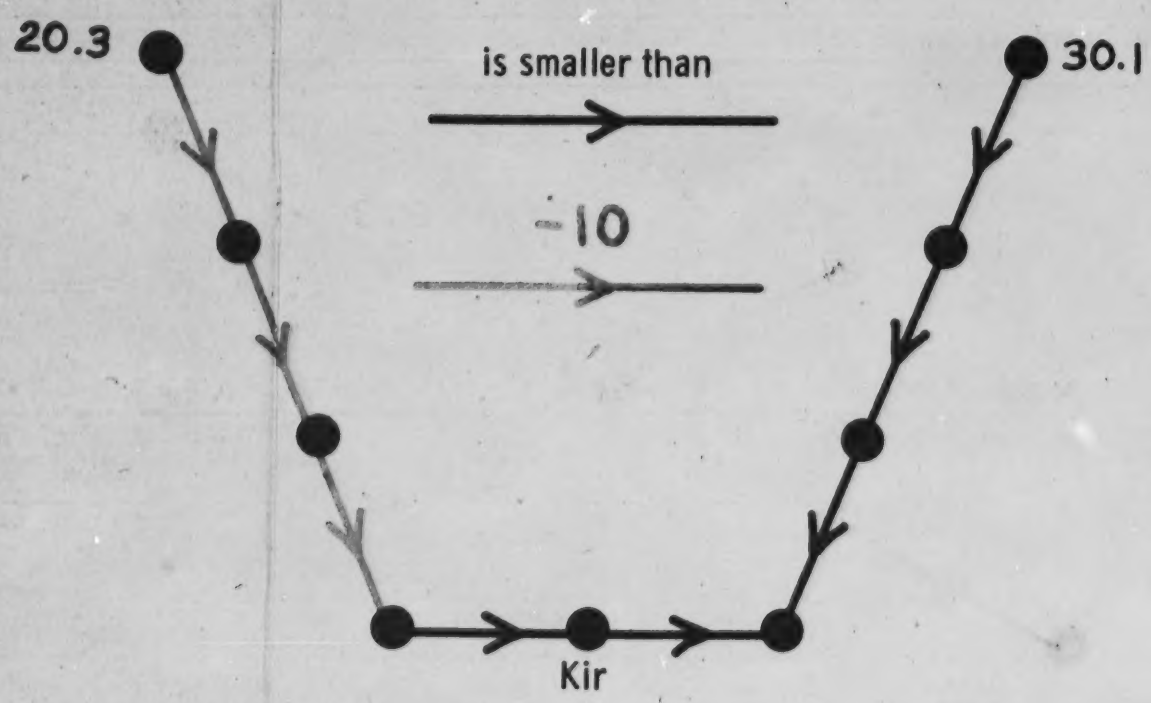
First clue

One of the symbols: +, ÷ is hidden in each blank of this hand-calculator sentence. The same symbol may be used in all three blanks.

8		2		10		10	=	Kir

Kir is ____ or ____ or ____ or ____ or ____ or ____ or ____ or ____.

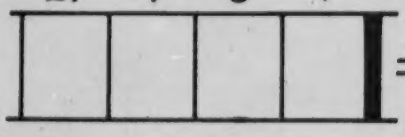
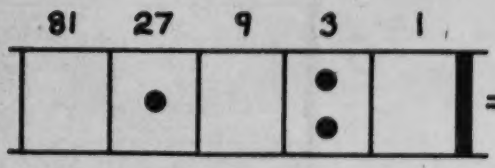
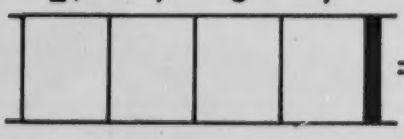
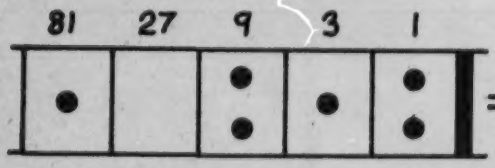
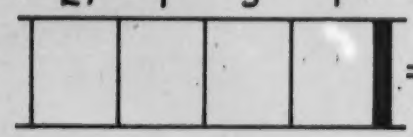
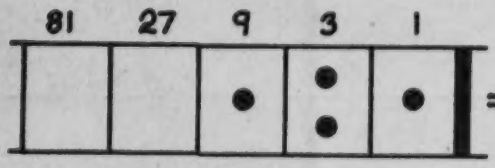
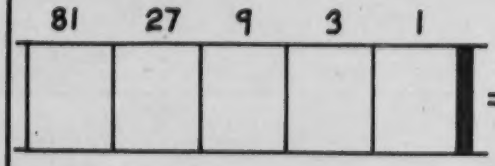
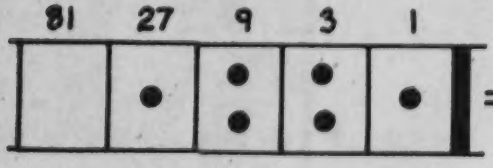
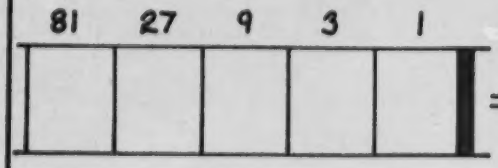
Second clue



Kir is _____.

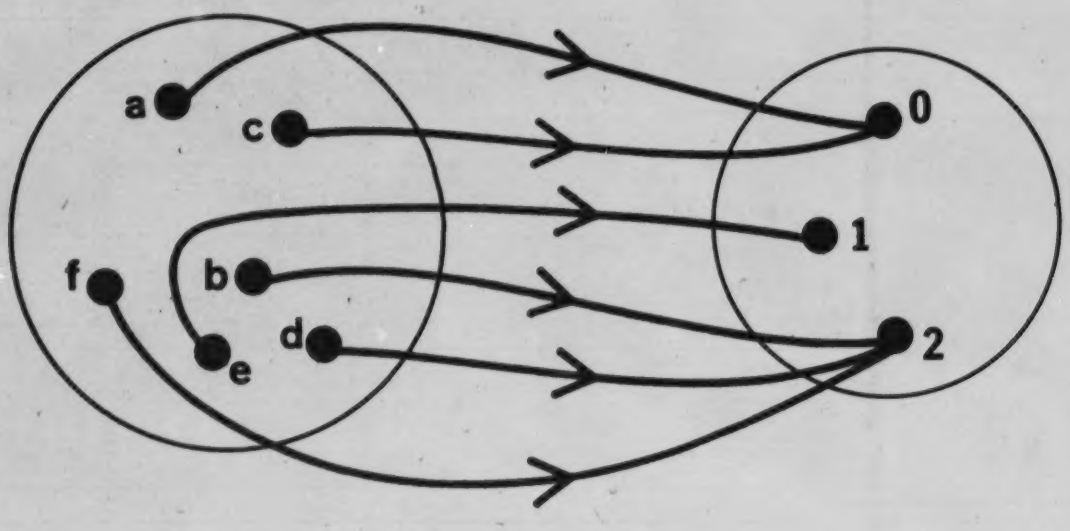
Name _____ **N 20** *

Base Three Abacus

What number is on the abacus?	Put the number on the abacus. Use at most two checkers on each board.
81 27 9 3 1 = ____	27 9 3 1  = 15
81 27 9 3 1  = ____	27 9 3 1  = 25
81 27 9 3 1  = ____	27 9 3 1  = 55
81 27 9 3 1  = ____	81 27 9 3 1  = 80
81 27 9 3 1  = ____	81 27 9 3 1  = 110

Name _____ **N 21** **

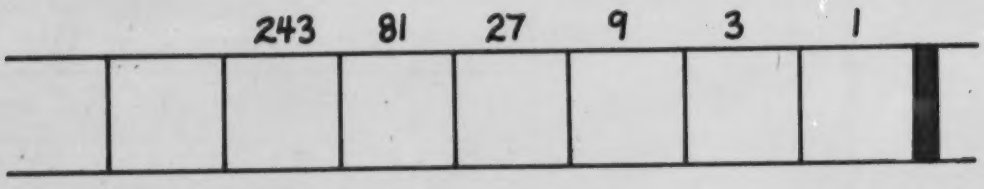
This arrow picture shows how the spy Boris assigned his six helpers to observe three bridges.



Write the secret message that he will send to communicate this information.

_____ (Code 3)

Explain your answer below with the help of the Base Three abacus.



Name _____

N 22 ***

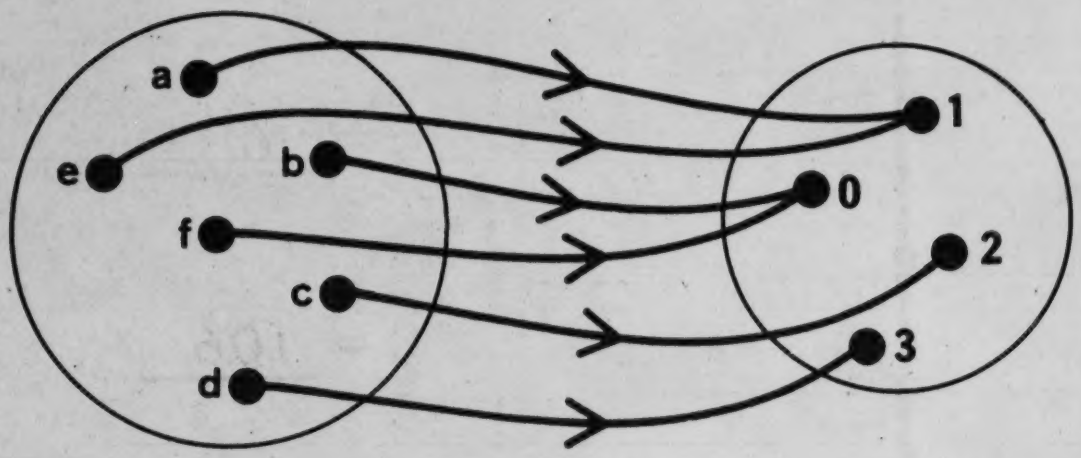
Boris sent this secret message to headquarters.

583 (Code 3)

Can you decode his message? Use the Base Three abacus and an arrow picture.

Name _____ **N 23 ******

Today the spy Boris has FOUR bridges to observe. This arrow picture shows how he assigned his six helpers.



Can you invent a new code to send this message secretly?
Explain it below.

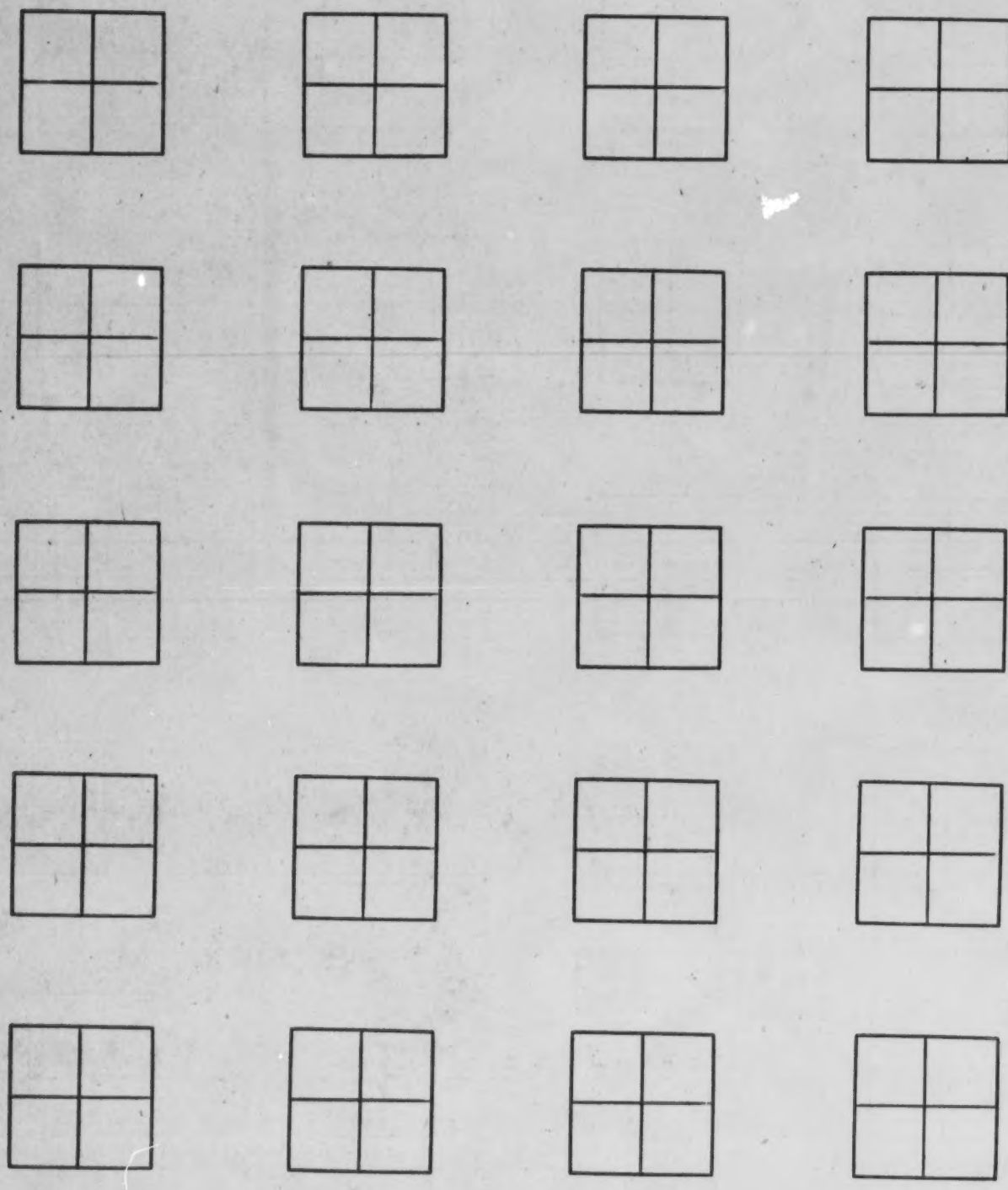
Name _____ **N 24** *

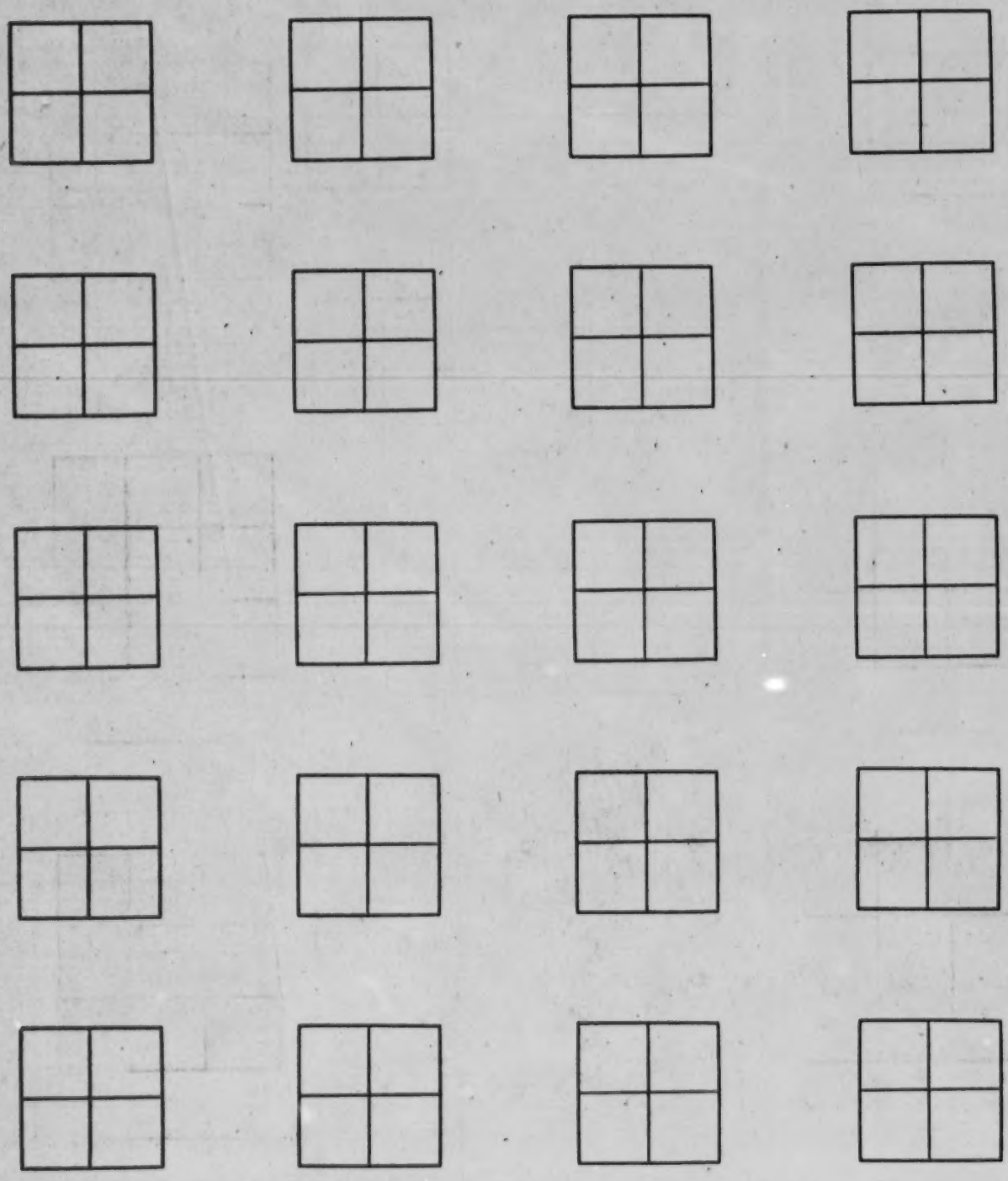
Put the numbers on the Base Five abacus. Use at most four checkers on each board.

3,125	625	125	25	5	1	
						= 17
						= 61
						= 106
						= 417
						= 1,236
						= 5,075
						= 20,084

Name _____

N 25 *





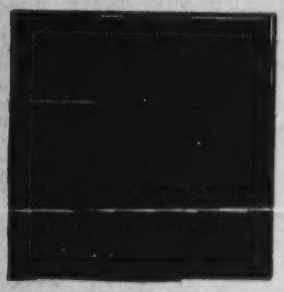
Name _____

N 26 **

Label each picture with its code number.

_____

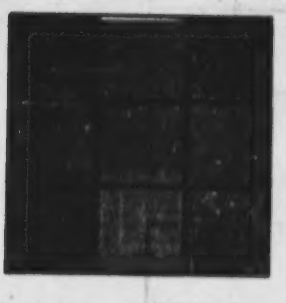
_____



_____

_____

_____

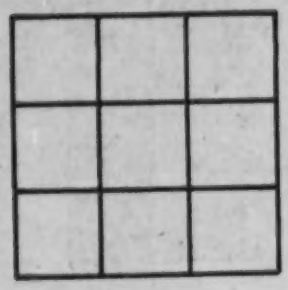


_____

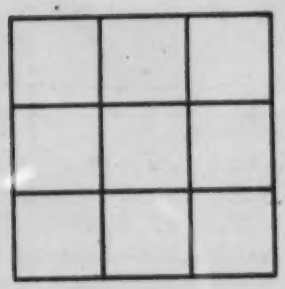
Name _____

N 27 ***

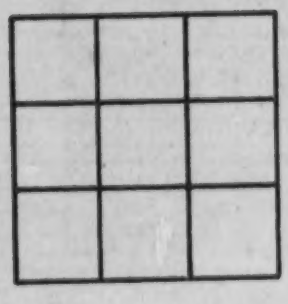
Color the pictures which have these code numbers.



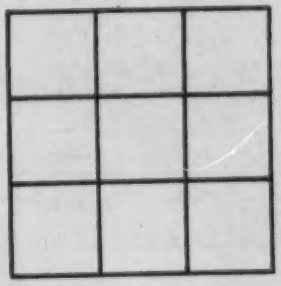
12



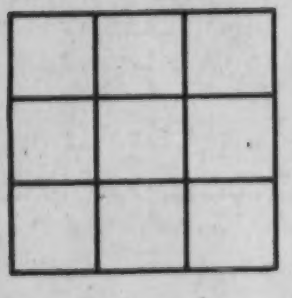
50



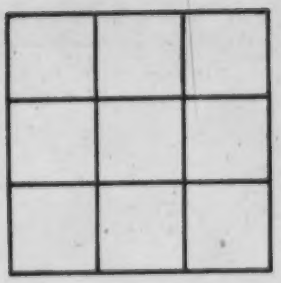
20



100



75



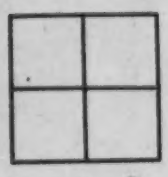
300

Name _____

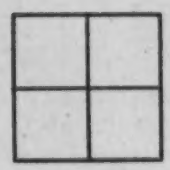
N 28 *

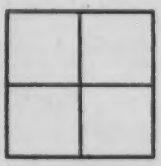
Put any number you wish on the Minicomputer using exactly one negative checker and exactly one of these checkers:

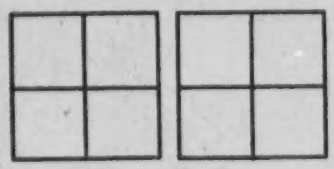
② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

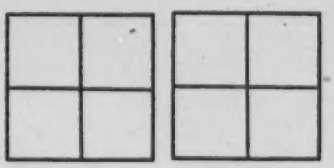
 = _____

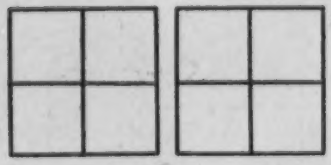
= _____

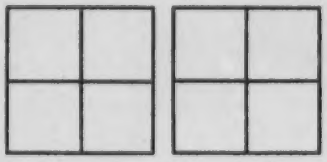
 = _____

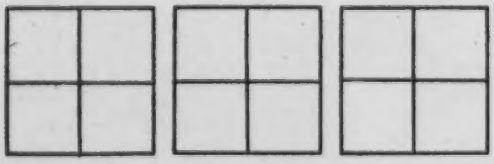
 = _____

 = _____

 = _____

 = _____

 = _____

 = _____

Name _____ **N 29** **

Put each number on the Minicomputer using one ⑩ – checker and exactly one of these checkers:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

= 28

= 56

= 92

= 100

= 128

= 240

= 520

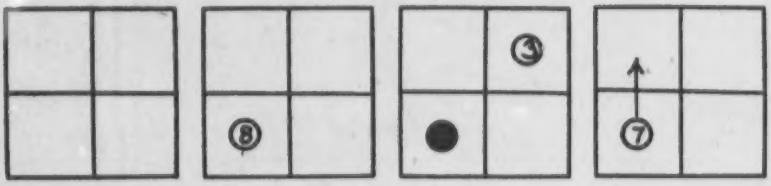
= 836

= 1,000

= 960

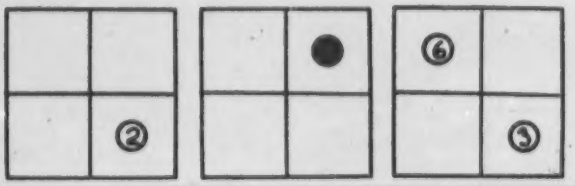
Name _____ **N 30** ***

By moving the ⑦ - checker from the 2 - square to the 8 - square, you make this number _____ larger.

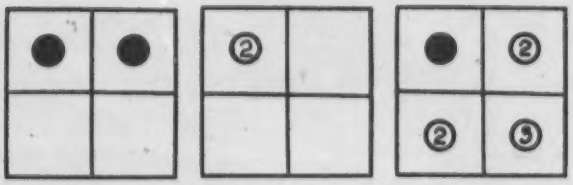


By moving exactly one checker, make the number

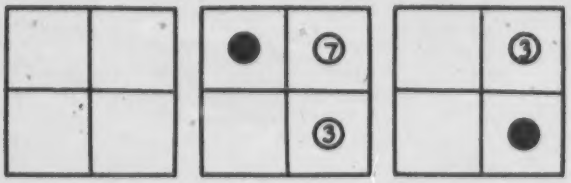
24 smaller.



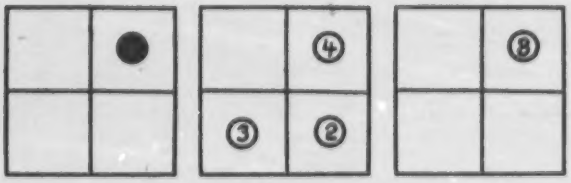
36 larger.



27 smaller.



240 larger.

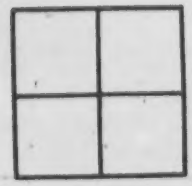


Name _____ **N 31** *

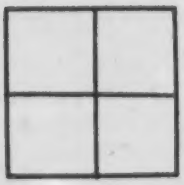
14; 15; 24; 28; 30; 40; 48; 54; 64

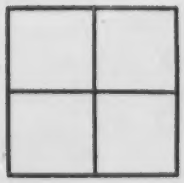
Six of these numbers can be put on the ones' board of the Minicomputer using exactly one of these checkers. Put them on.

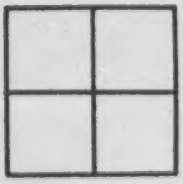
② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

 = _____

= _____

 = _____

 = _____

 = _____

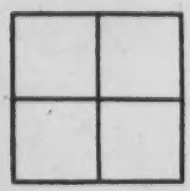
= _____

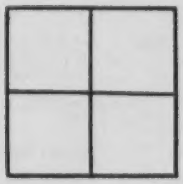
Name _____ **N 32** **

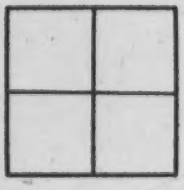
11; 21; 30; 38; 42; 47; 54; 112; 115

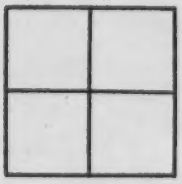
Six of these numbers can be put on these Minicomputer boards using exactly one negative checker and exactly one of these checkers. Put them on.

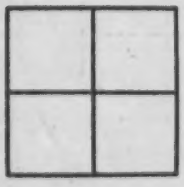
② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

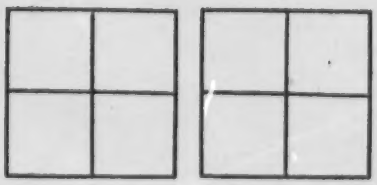
 = _____

 = _____

 = _____

 = _____

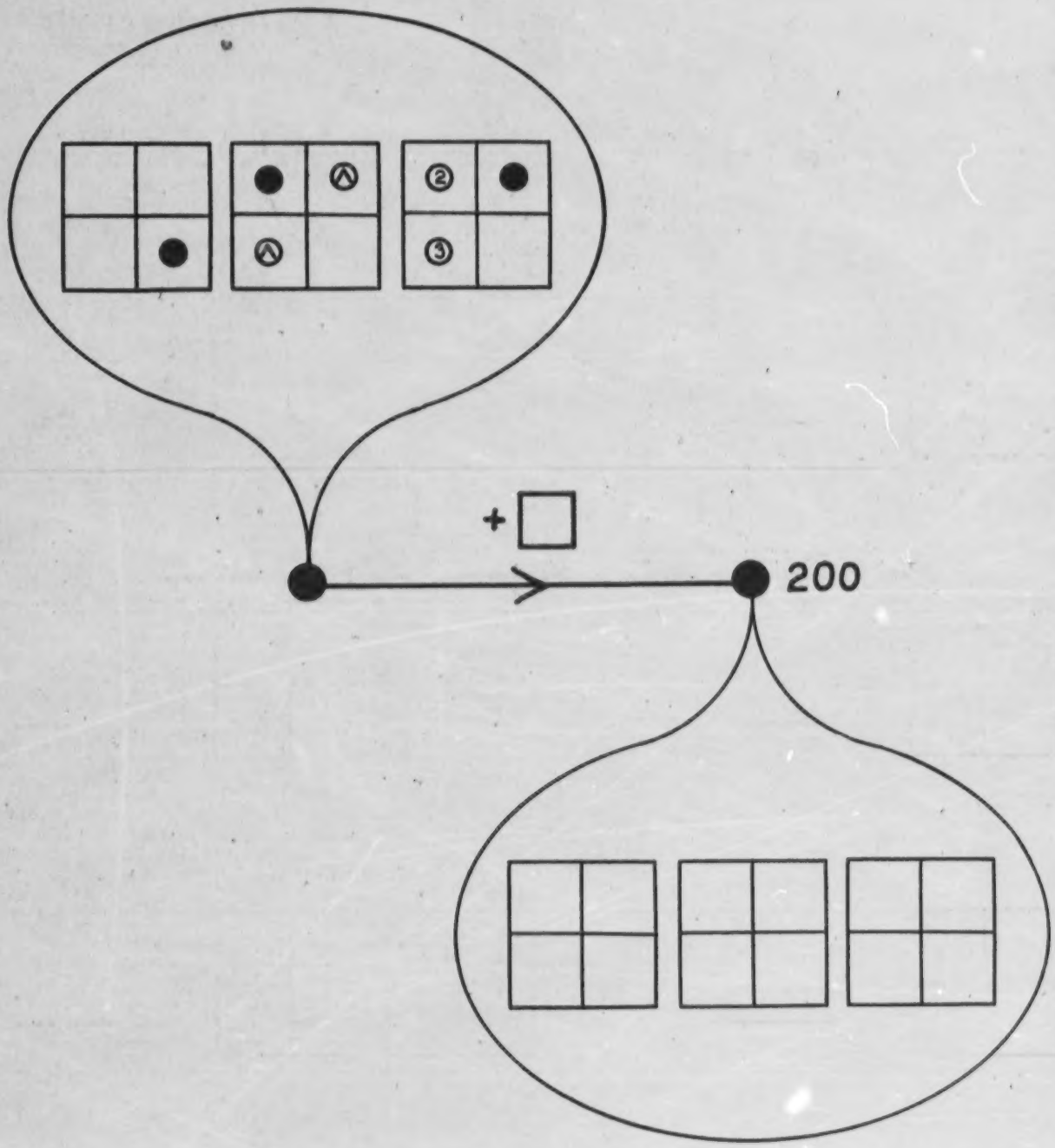
 = _____

 = _____

Name _____

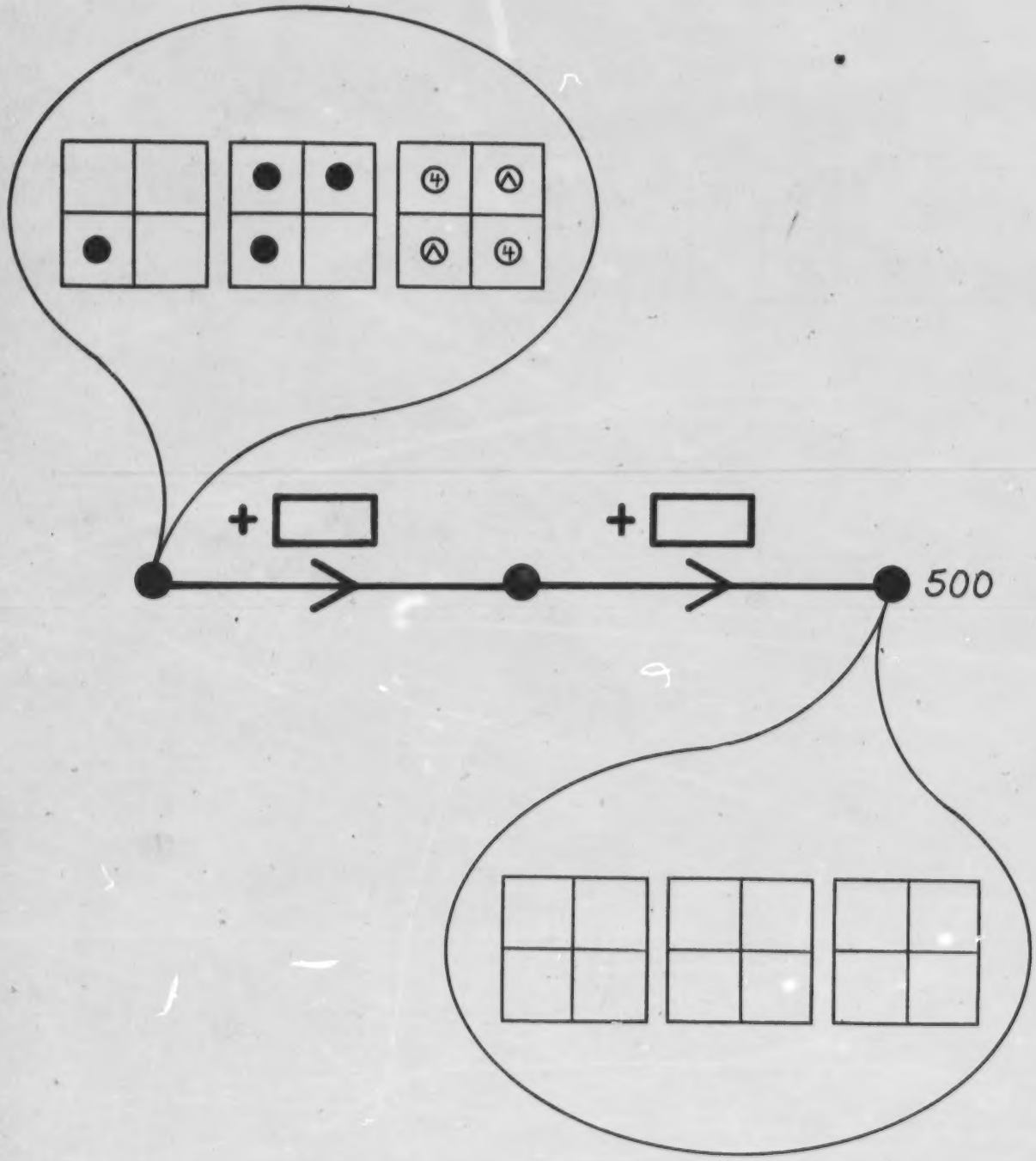
N 33 ***

Solve this puzzle by moving exactly one checker.



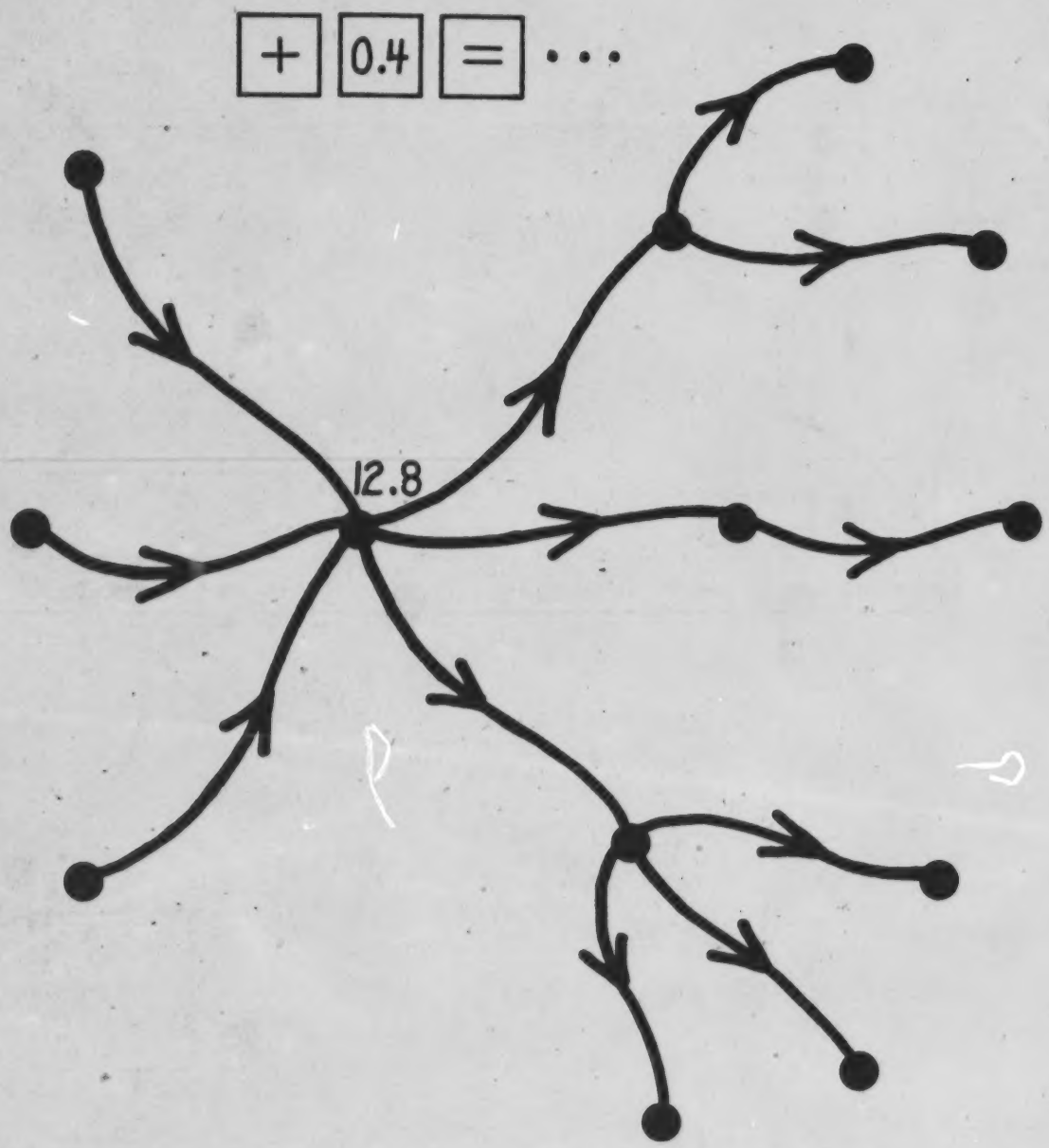
Name _____ **N 34** ****

Solve this puzzle by moving exactly two checkers, one for EACH arrow.



Name _____ N 35 *

Label the dots. Many answers are possible.

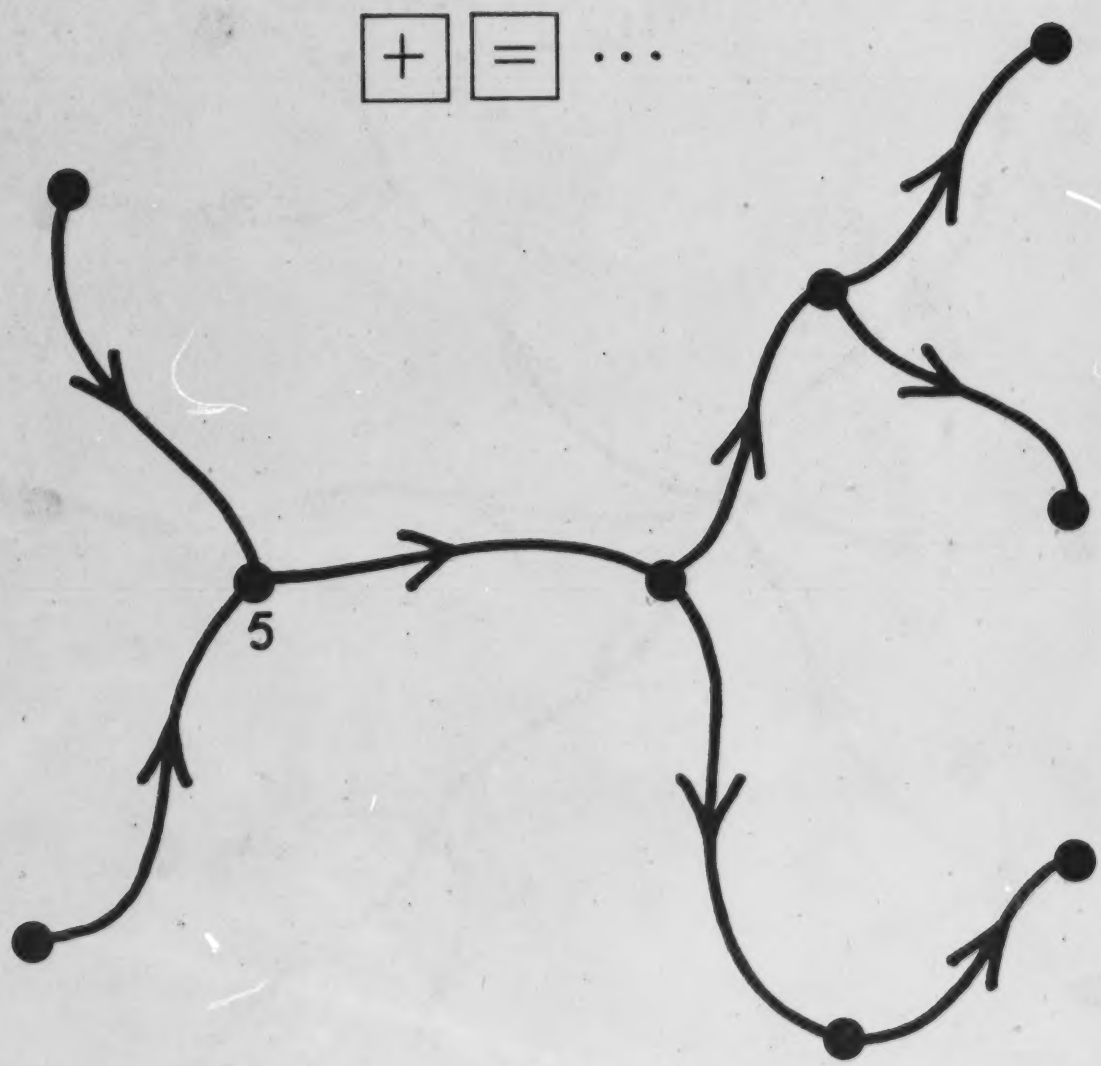


Name _____

N 36 **

Label the dots. Label each arrow "□ x".

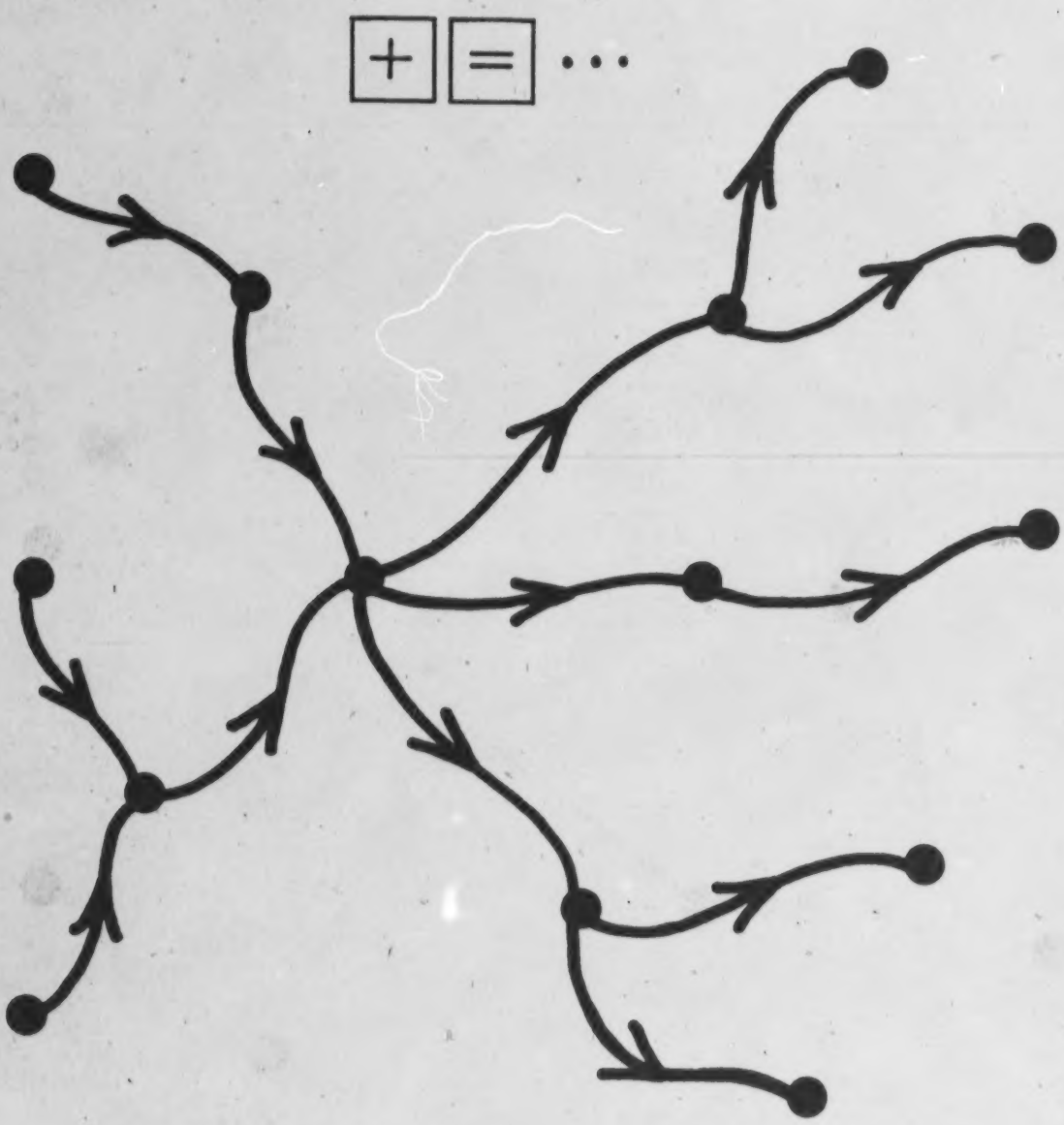
Draw two more red arrows and label each of them "□ x".



Name _____ **N 37** ***

Label the dots. Label each arrow "□ x".

Draw three more red arrows and label each of them "□ x".

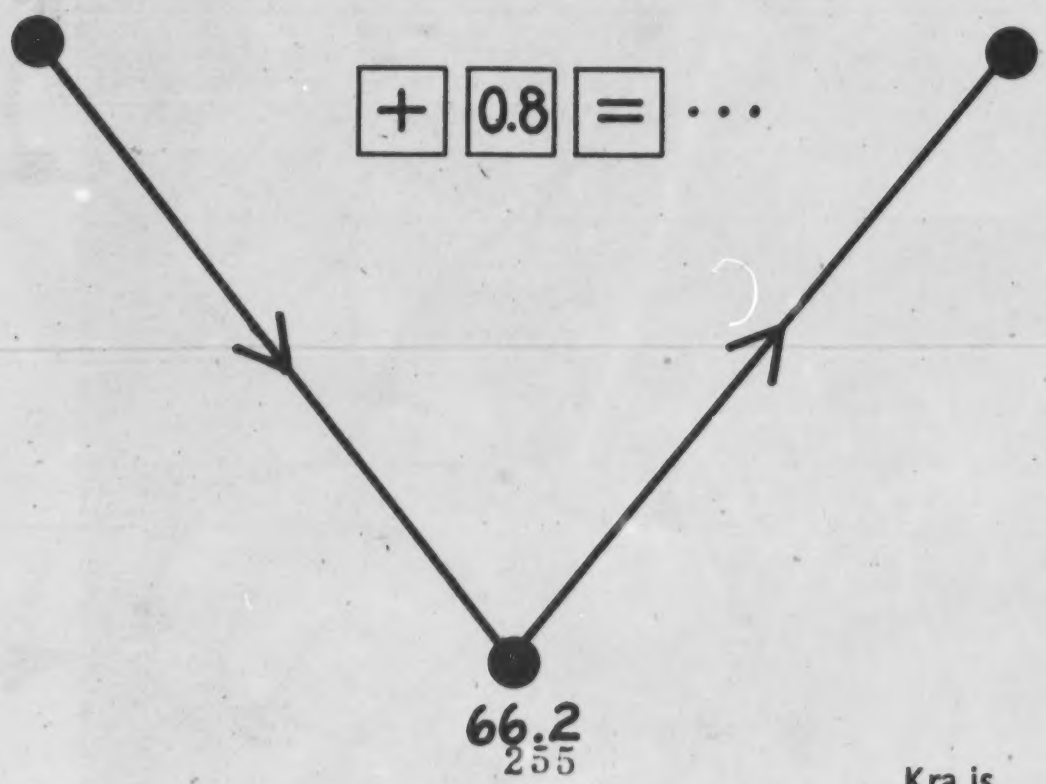


Name _____

N 38 ****

Kri is the smallest positive number which could be here.

Kra is the smallest number larger than 1,000 which could be here.



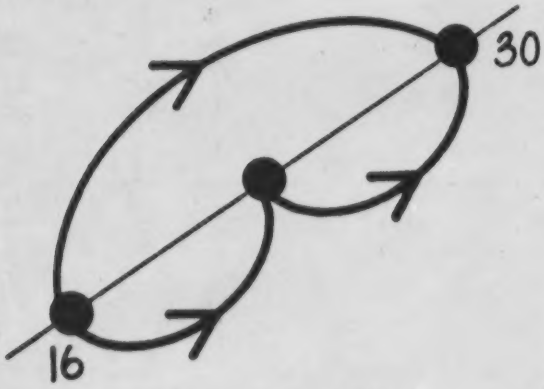
Kri is _____.

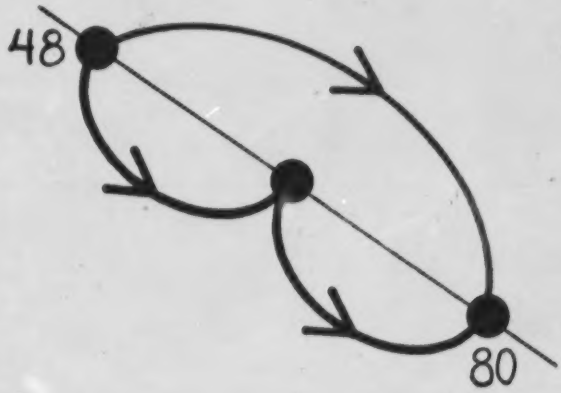
Kra is _____.

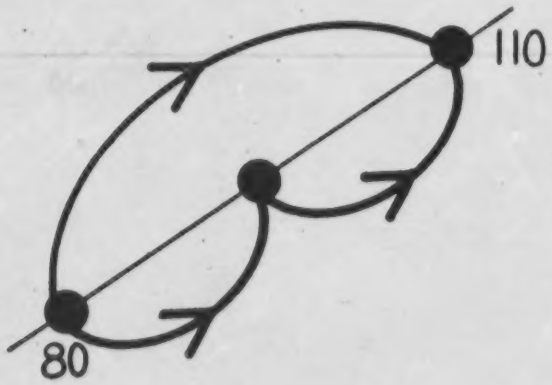
Name _____

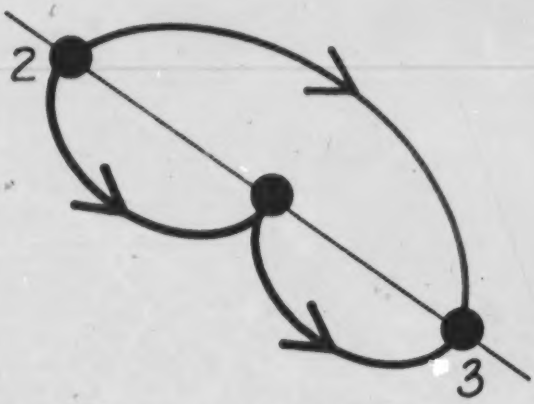
N 39 *

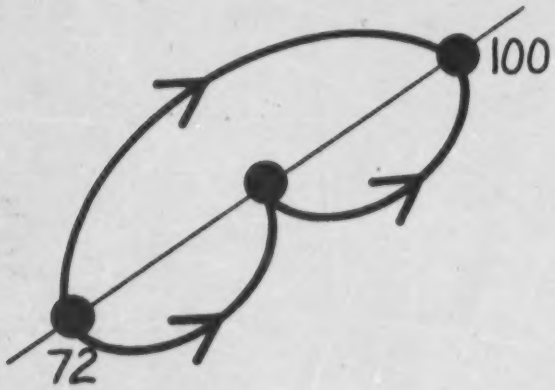
Label the midpoints. Label each arrow " + □ ".

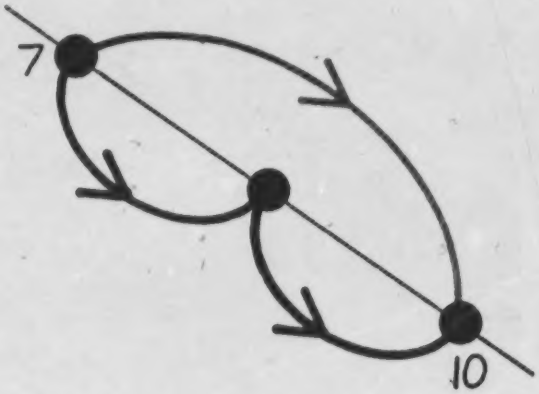










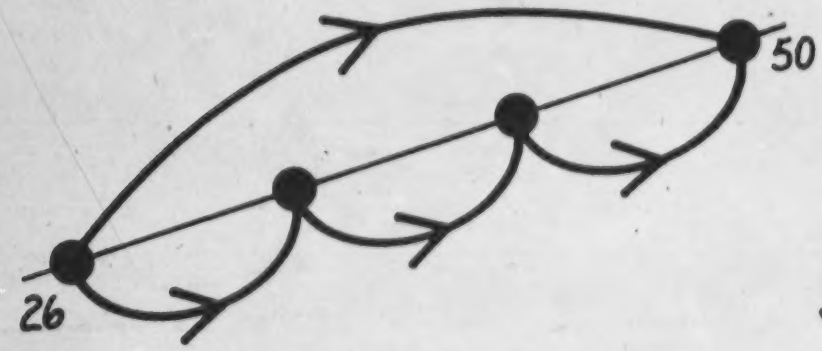


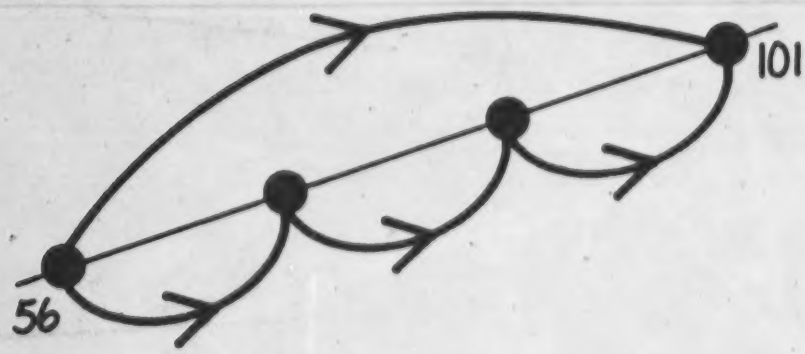
Name _____

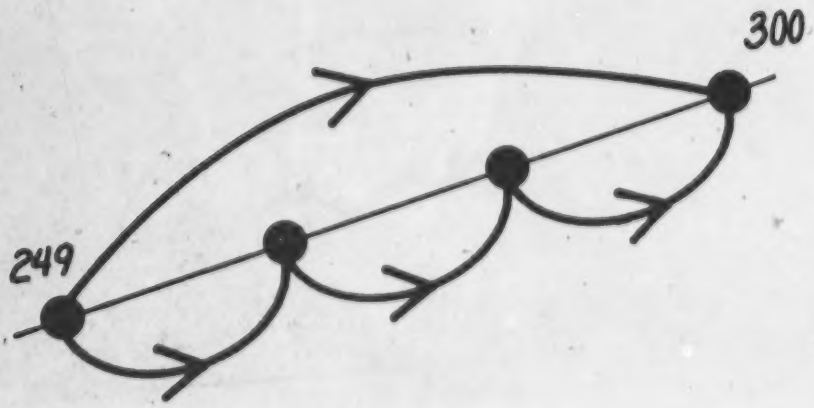
N 40 **

Label the dots. Label each arrow " + □ ".

Note: In each separate number line picture, the red arrows all have the same label.



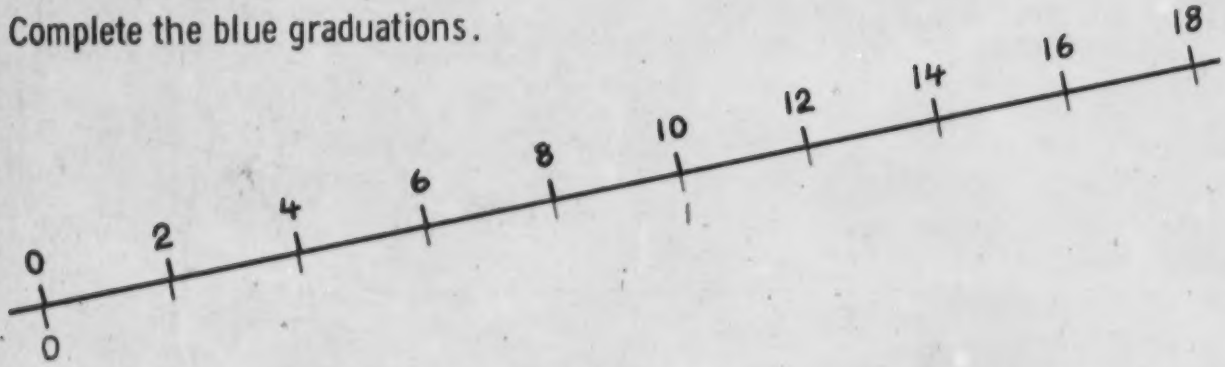


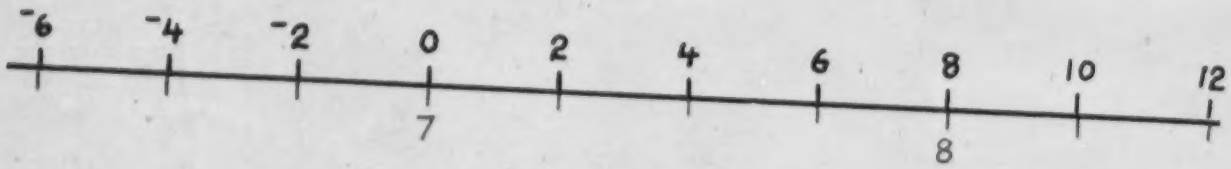


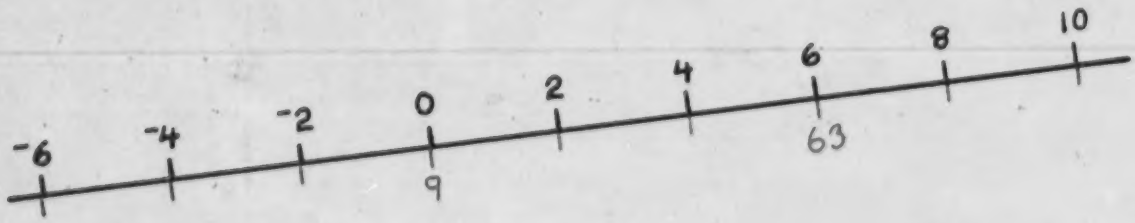
Name _____

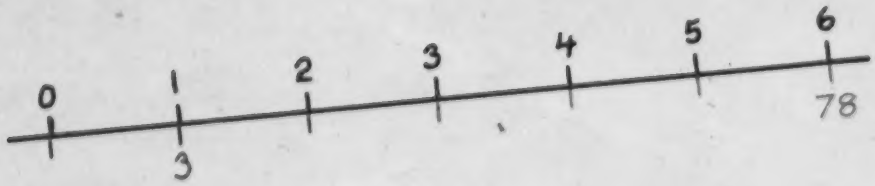
N 41 ***

Complete the blue graduations.



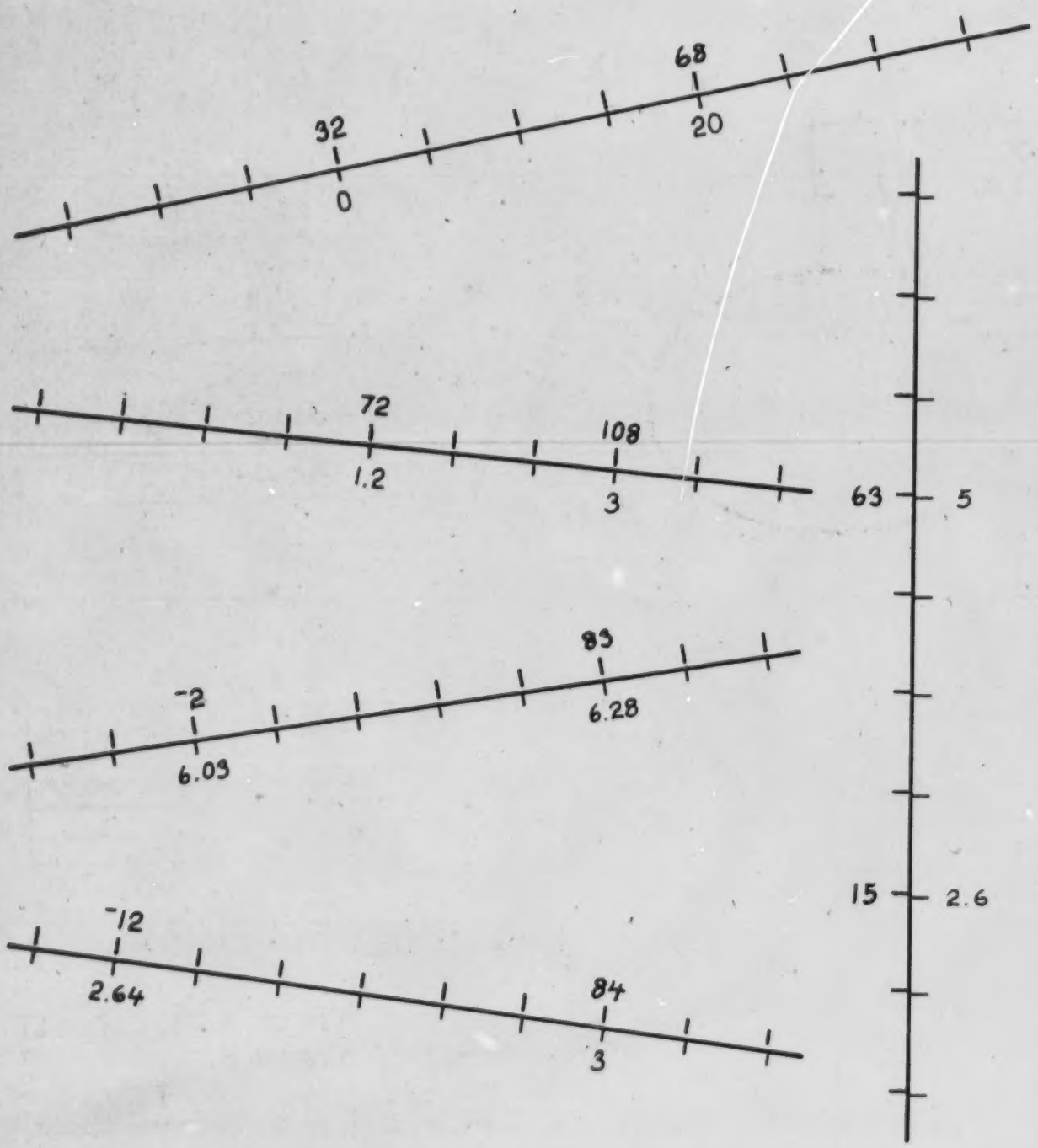






Name _____ **N 42 ******

Complete the red graduation and the blue graduation.



Name _____ **N 43** *

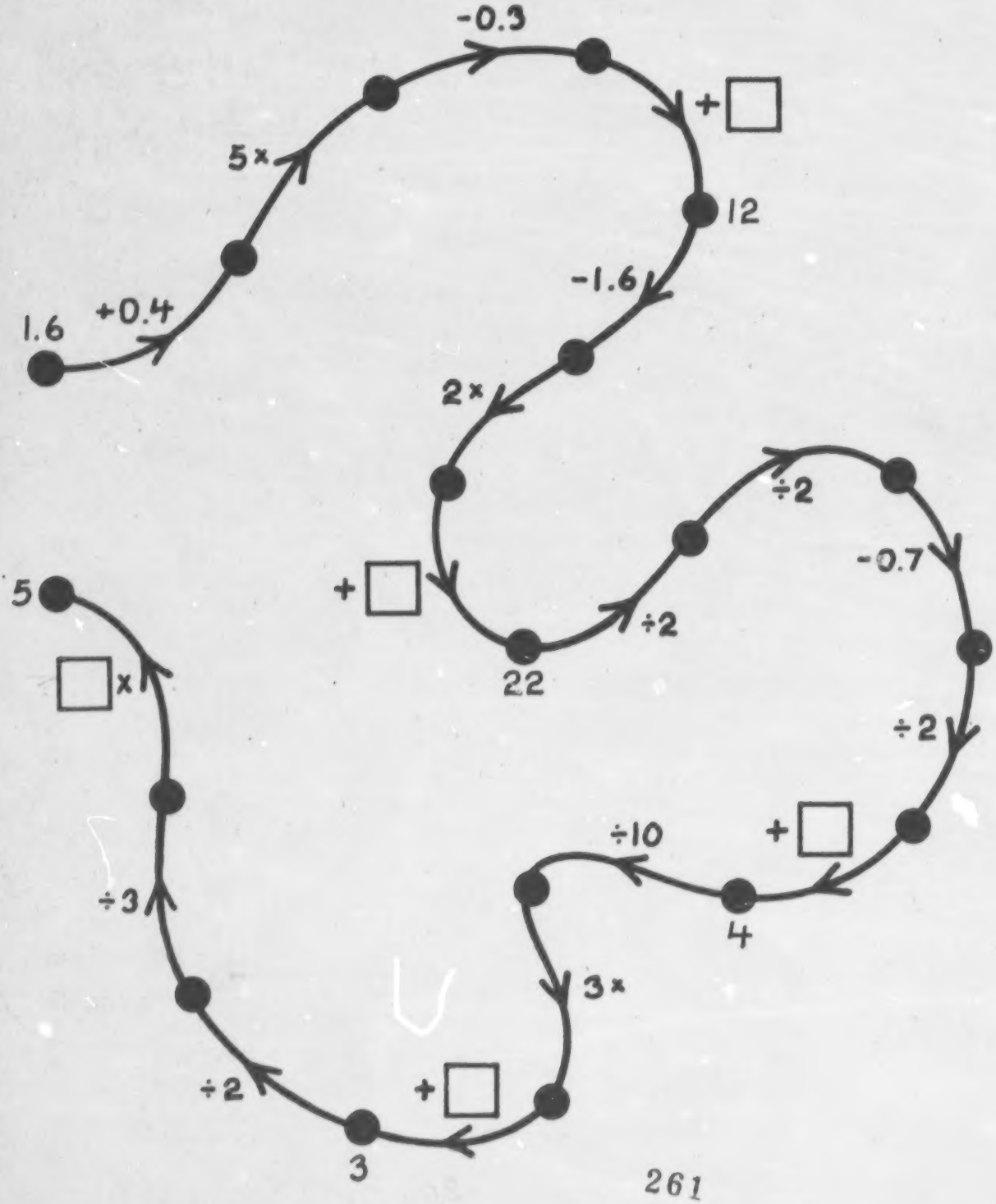
Build a blue - red road from 10 to 1.

Try to use less than seven arrows.

Name _____

N 44 **

Label the dots and fill in the boxes for the arrows.



Name _____ **N 45** ***

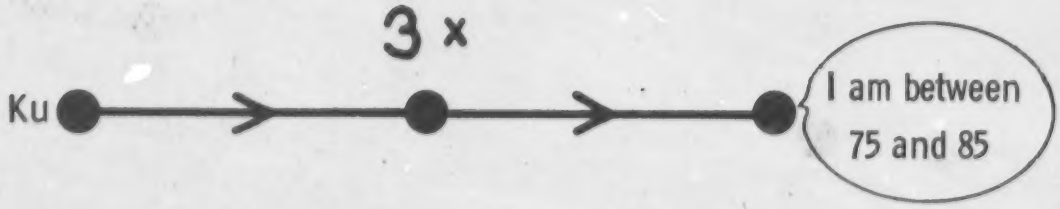
First clue

Ku is the ending number of a red-blue road starting at 1.5 and using exactly two red arrows and two blue arrows.

+0.7 2 ×

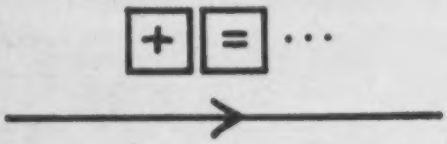
1.5 ●

Second clue



Ku is _____.

Name _____ **N 46** ****



+0.1 or -0.1

Use at most three arrows or cords to build a red-blue road

from 0.7 to 20.

0.7 ● ● 20

from 0.8 to 10.8.

0.8 ● ● 10.8

from 0.15 to 13.1.

0.15 ● ● 13.1

Name _____ **N 47** *

Put one of the symbols: +, ×, −, ÷ in each blank box to make the hand-calculator sentence true.

12		7		8	=	40
3		7		19	=	40
18		3		7	=	42
4		7		0	=	28
35		5		8	=	56
750		10		1	=	75

Name _____

N 48 **

Put one of the symbols: +, ×, −, ÷ in each blank box to make the hand-calculator sentence true. A symbol may be used twice in one sentence.

1.8		10		9	=	2
10		4		0.7	=	1.8
2.6		3.9		2	=	3.25
5.1		2.3		10	=	28
4.4		5		5	=	110
5		6		100	=	0.3
0.7		0.8		5	=	7.5

Name _____ **N 49** ***

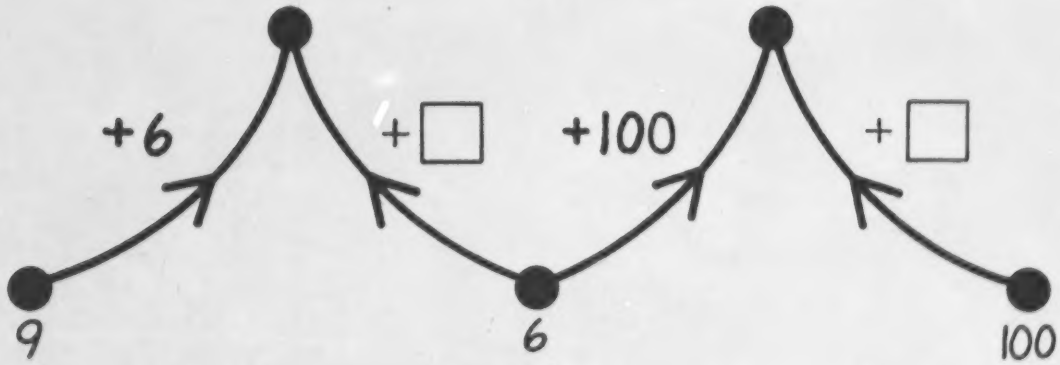
Put one of the symbols: + , × , − , ÷ in each blank box to make the hand-calculator sentence true. A symbol may be used more than once in one sentence.

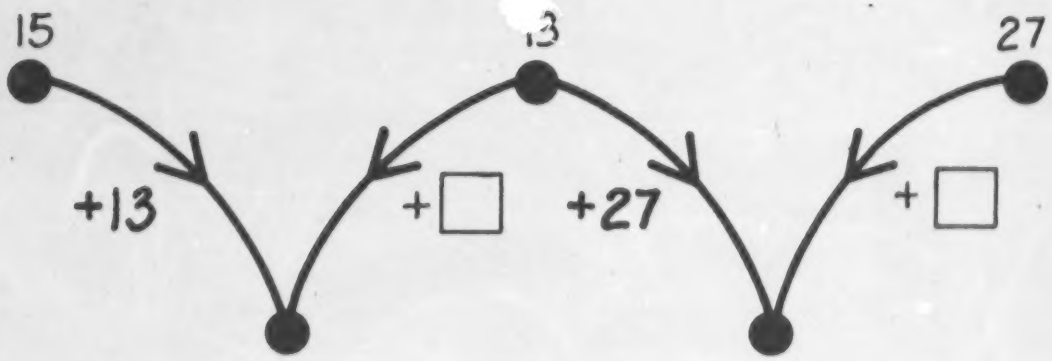
5		8		20		5	=	0.4
1.7		4.3		7		4	=	10.5
3		6		2		8	=	2
0.4		0.6		10		0.1	=	0
0		387		25		50	=	0.5
2		1.2		50		0	=	40
8		8		8		8	=	7
5		5		5		5	=	1.2

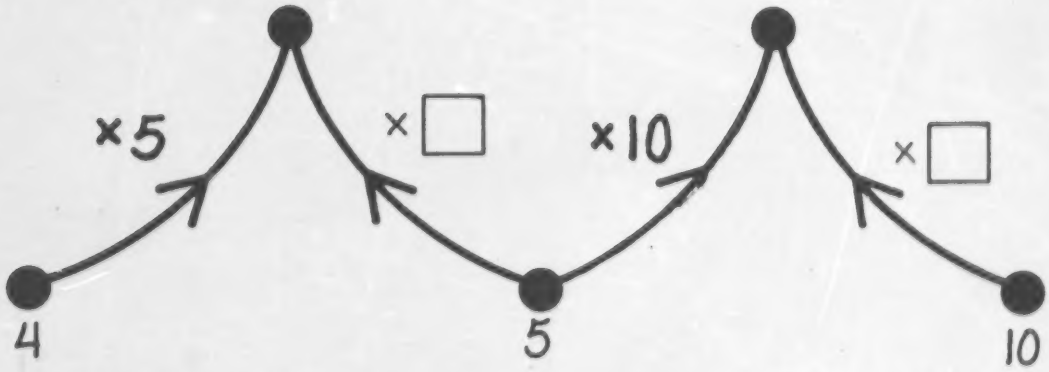
Name _____

N 50 *

Label the dots and fill in the boxes for the arrows.



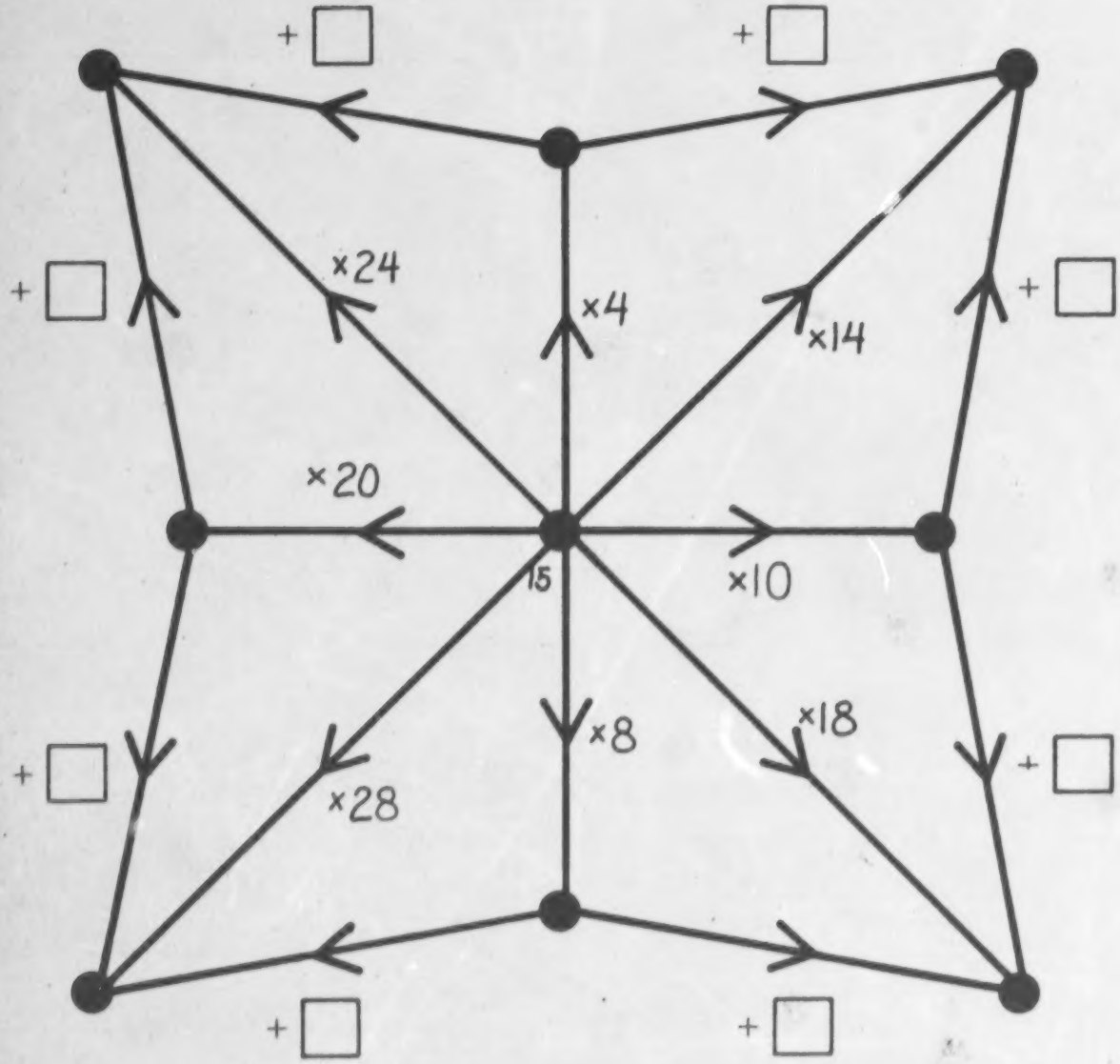




Name _____

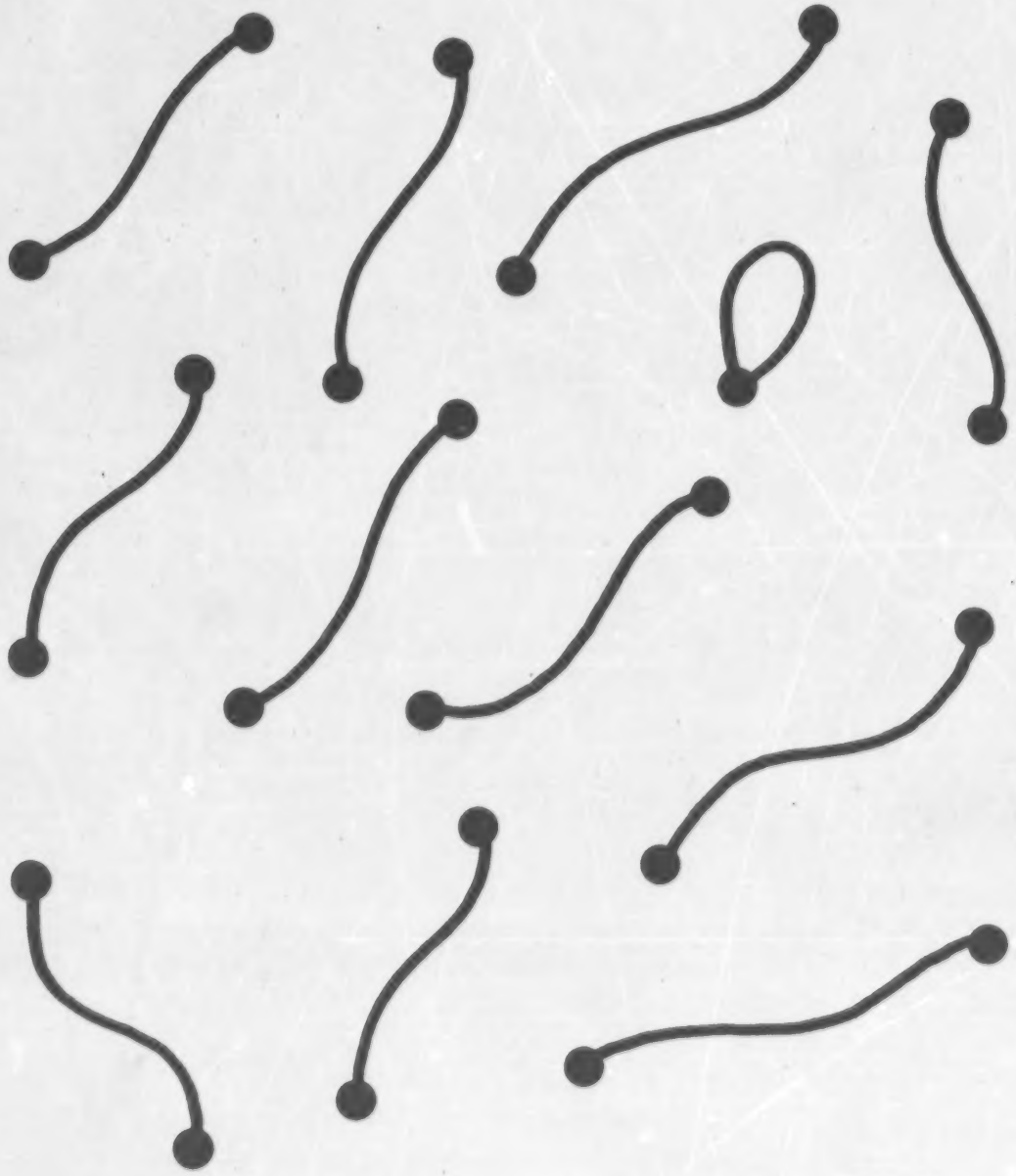
N 51 **

Label the dots and fill in the boxes for the arrows.



Name _____ N 52 ***

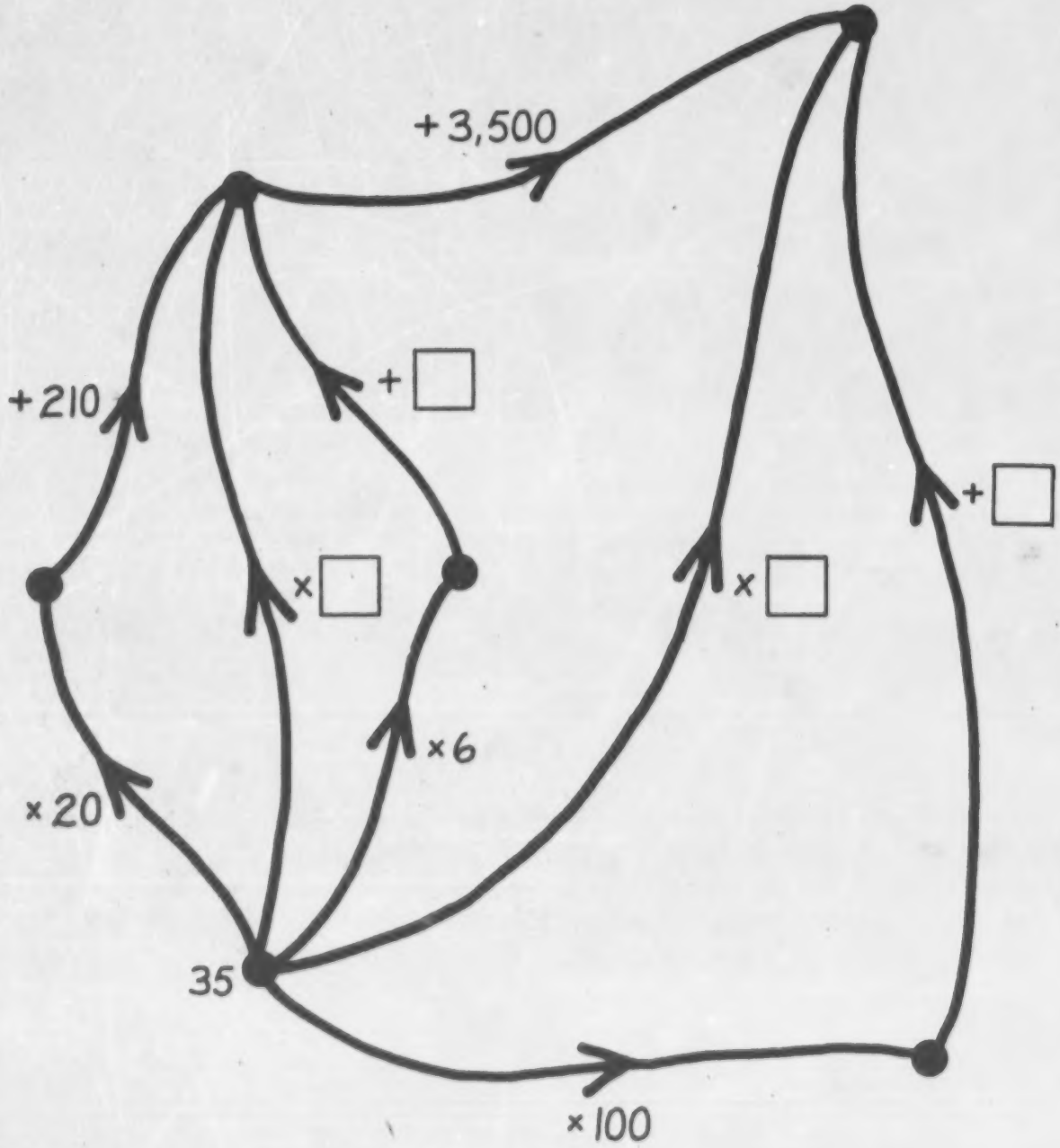
Two numbers are joined by a red cord if and only if their product equals 36.
Label the dots. Many answers are possible.



Name _____

N 53 ****

Label the dots and fill in the boxes for the arrows.

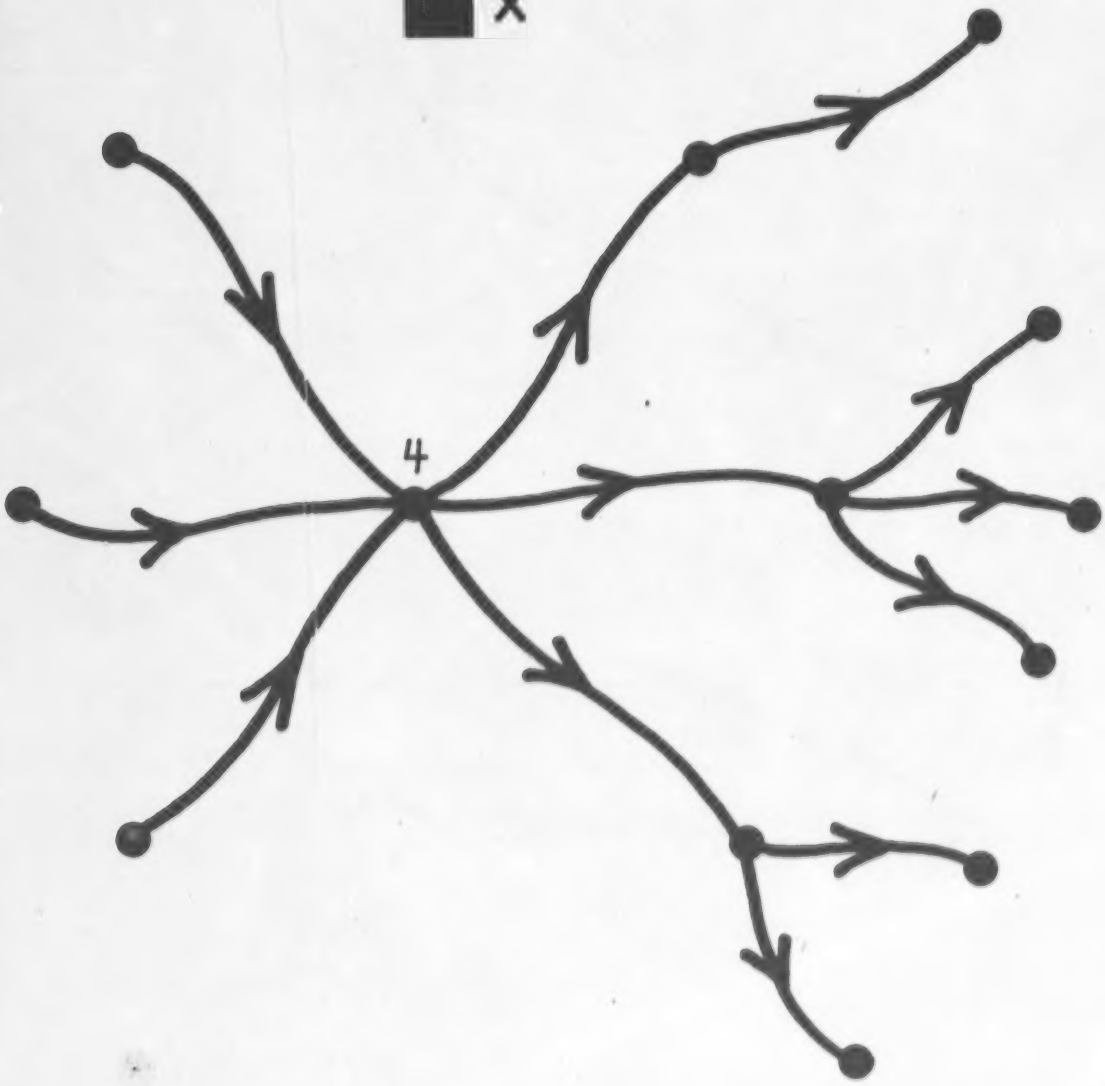


Name _____ N 54 *

" ■ × " means each red arrow is for times some WHOLE number.

Label the dots. Label each arrow " □ × ". Many solutions are possible.

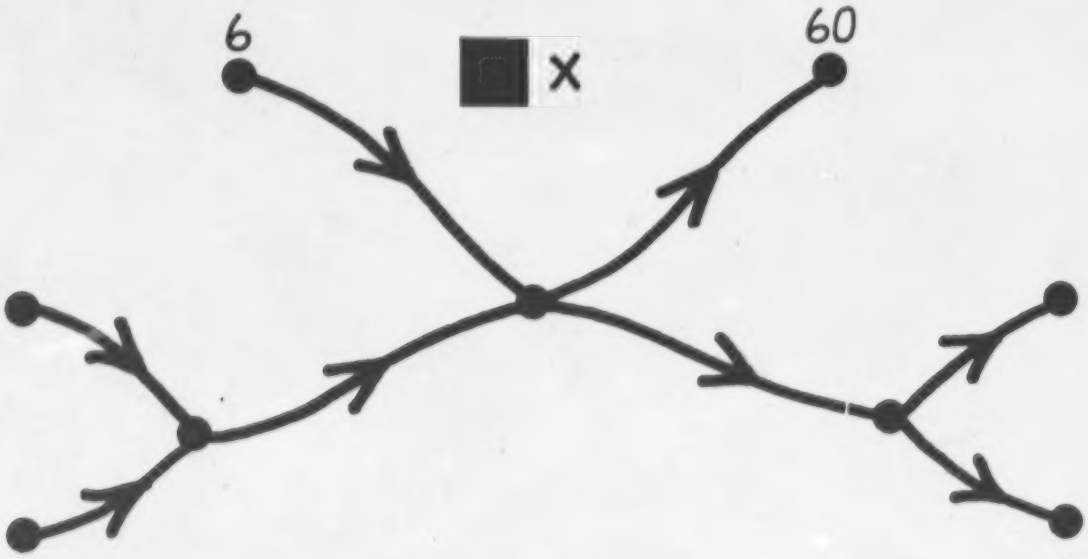
■ ×

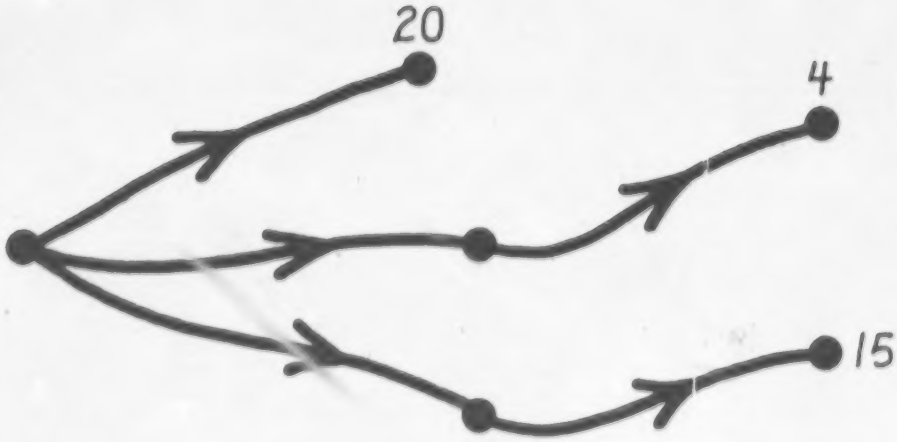


Name _____ **N 55** **

" ■ x " means each red arrow is for times some WHOLE number.

Label the dots. Label each arrow " □ x ". Many solutions are possible.

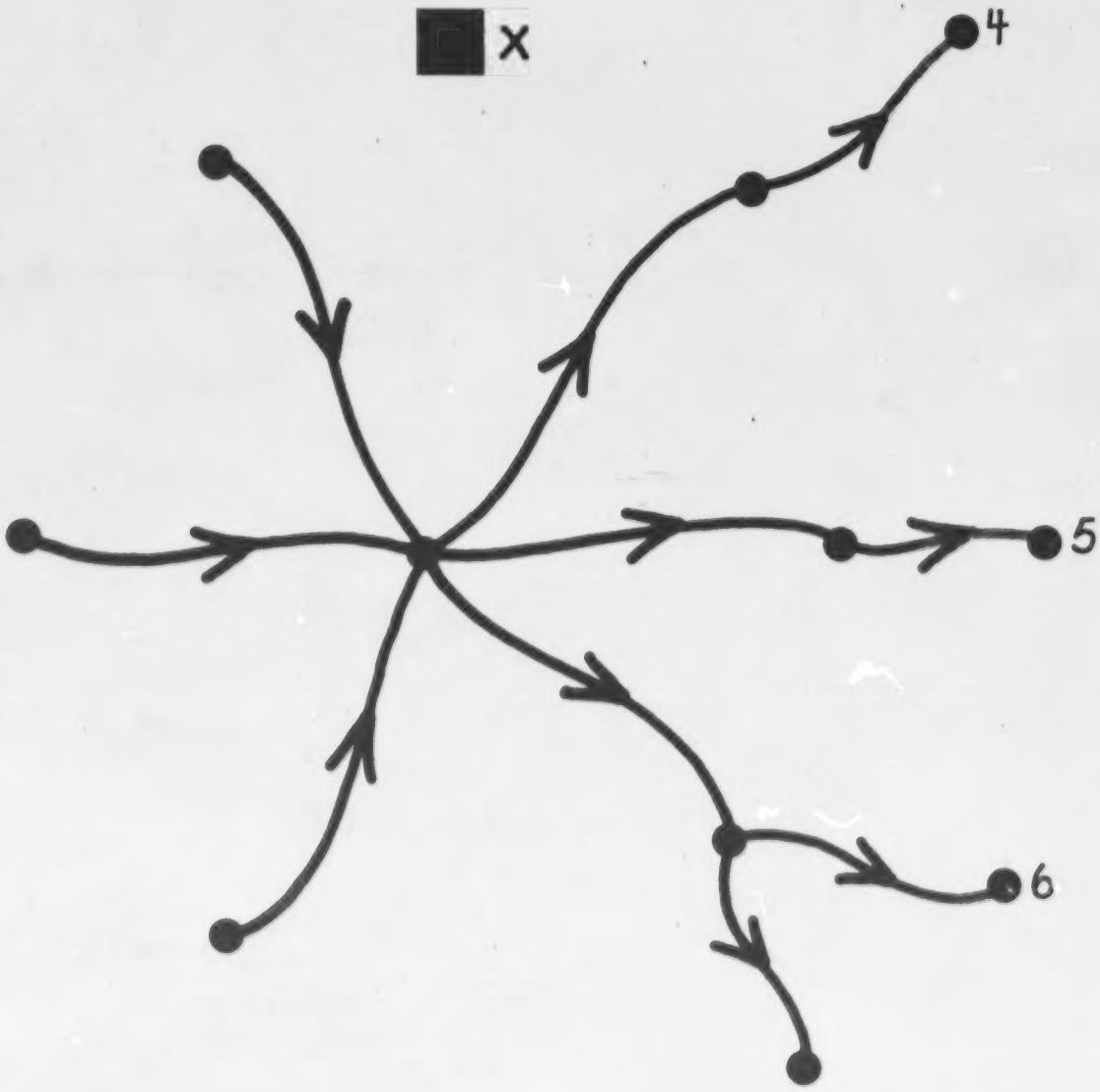




Name _____ **N 56** ***

" ■ x " means each red arrow is for times some WHOLE number.

Label the dots. Label each arrow " □ x ". Many solutions are possible.



Name _____

N 57 ****

First clue

" ■ x " means each red arrow is for times some WHOLE number.

■ x

Click is _____ or _____ or _____ or _____.

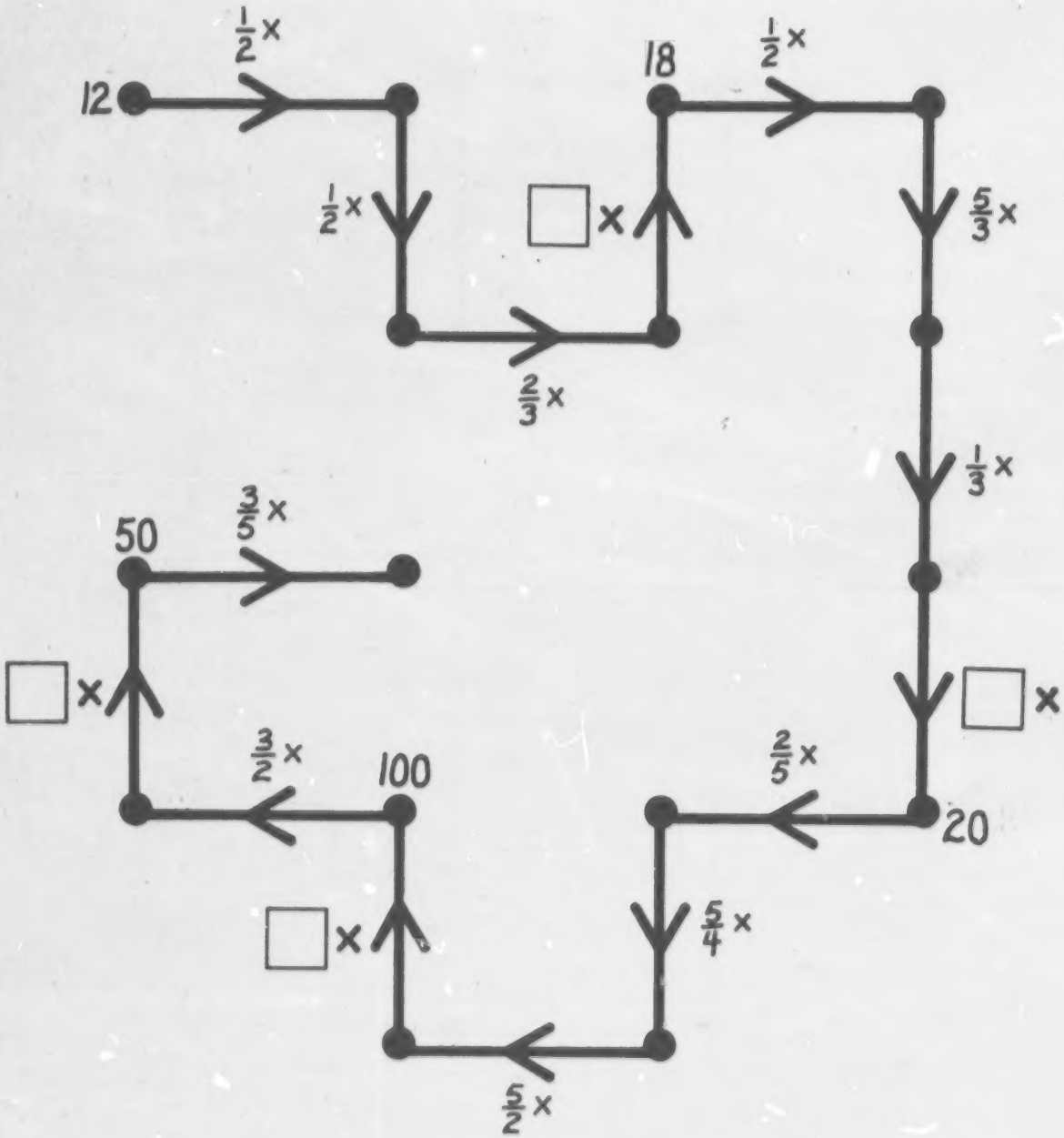
Second clue

If you multiply Click by 20, you will get a square number.

Click is _____.

Name _____ **N 58** *

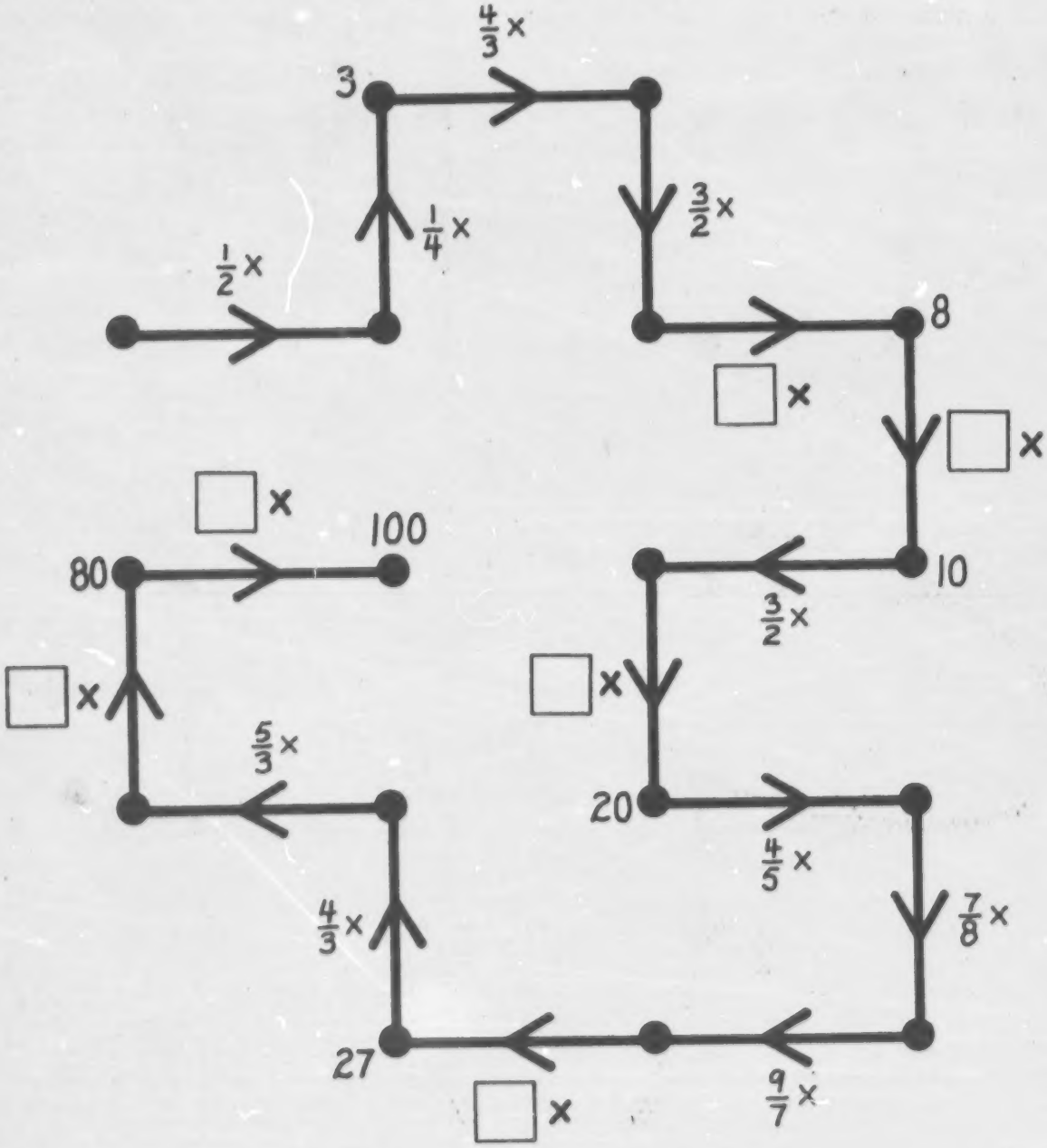
Label the dots and fill in the boxes for the arrows.



Name _____

N 59 **

Label the dots and fill in the boxes for the arrows.

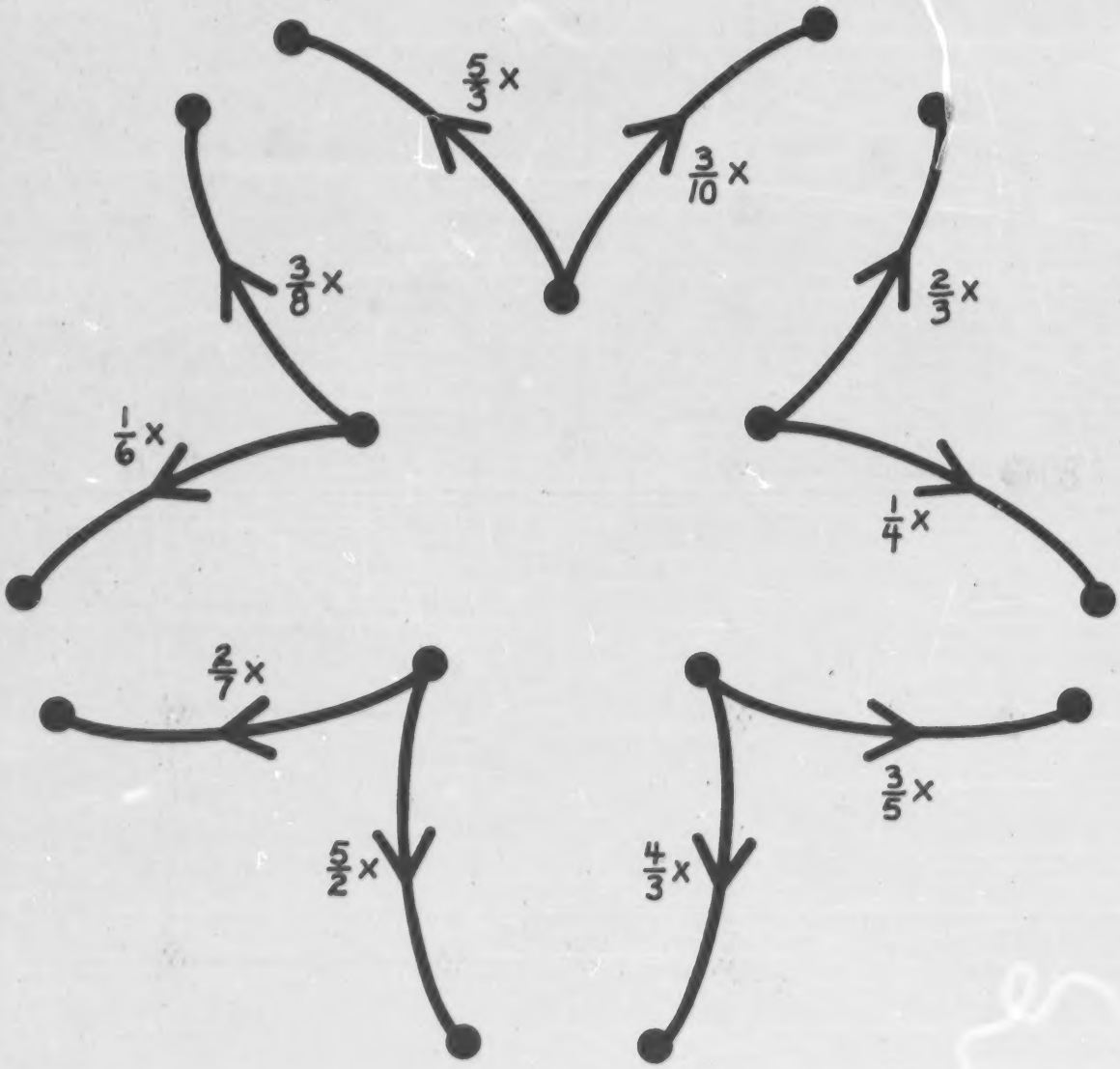


Name _____

N 60 ***

All the dots are for positive WHOLE numbers.

Label the dots. Many answers are possible.

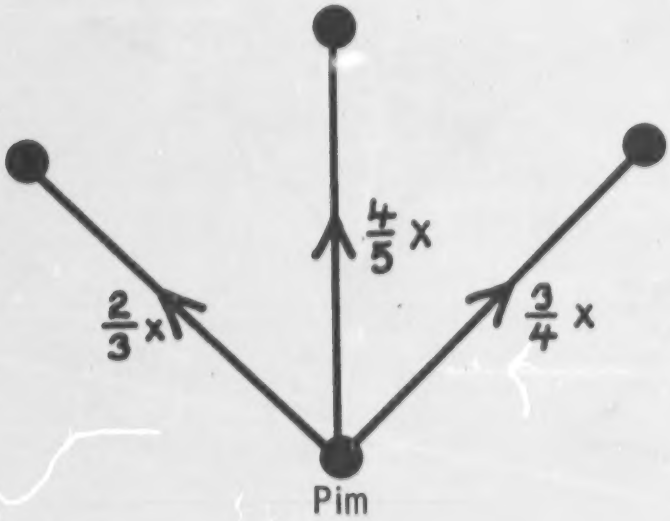


Name _____

N 61 ****

First clue

In this picture, all the dots are for positive WHOLE numbers.



Pim could be _____; _____; _____; _____; _____; _____; _____;

_____; _____; _____; and so on.

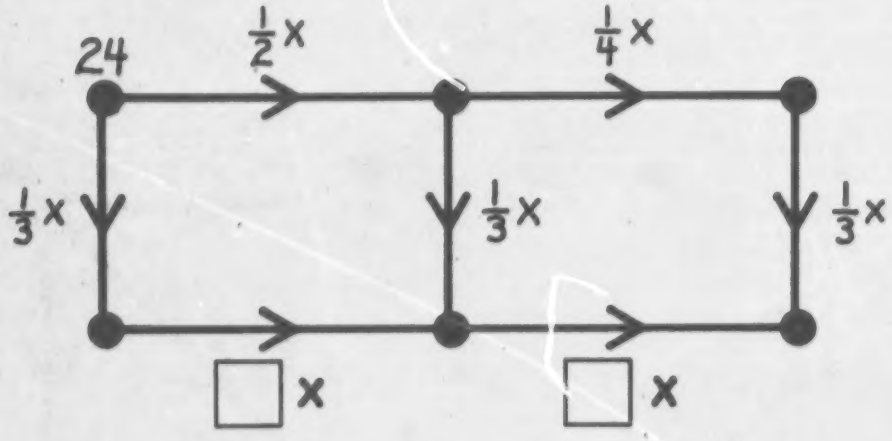
Second clue

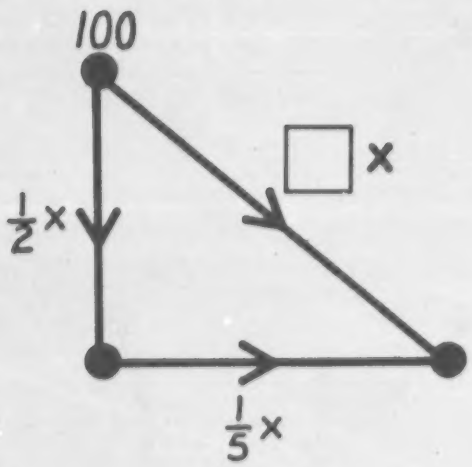
Pim is a square number smaller than 1,000.

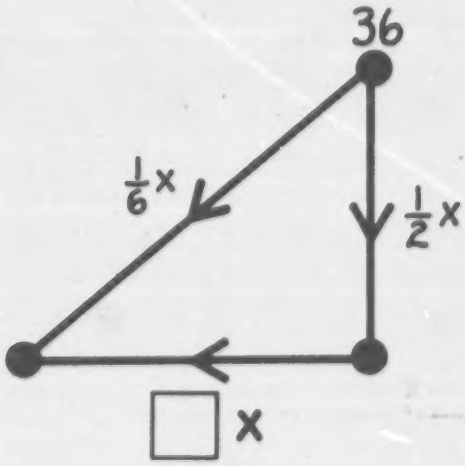
Pim is _____.

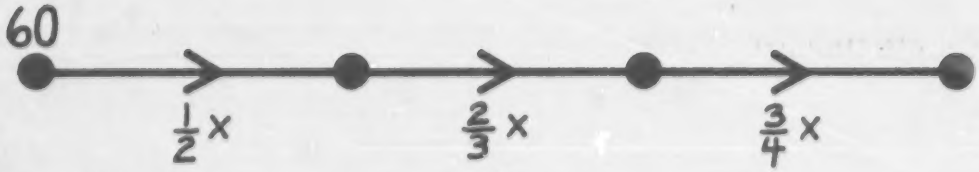
Name _____ **N 62** *

Label the dots and fill in the boxes for the arrows.







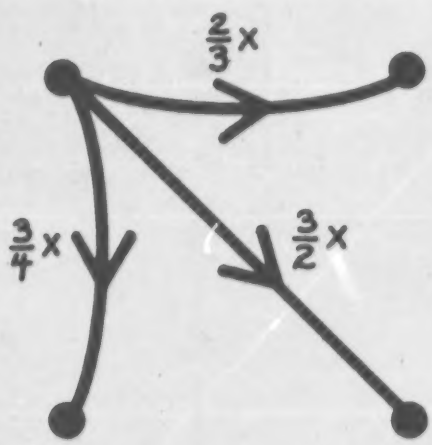


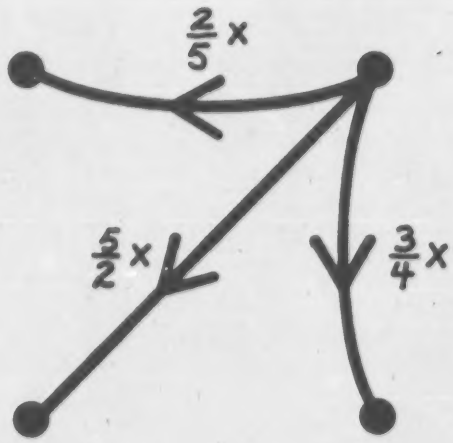
Name _____

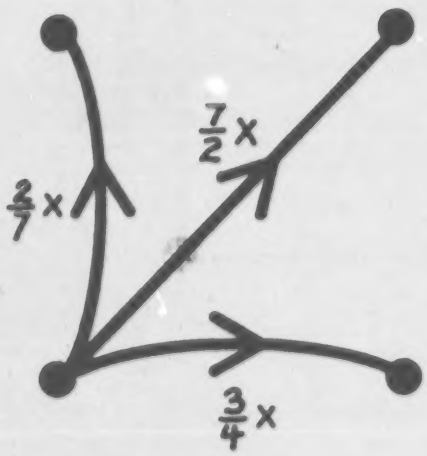
N 63 **

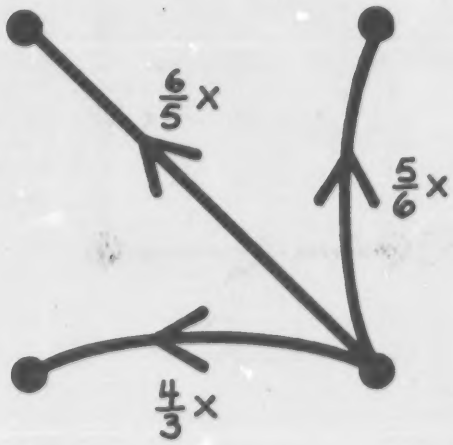
All the dots are for WHOLE numbers.

Label the dots. Many answers are possible.



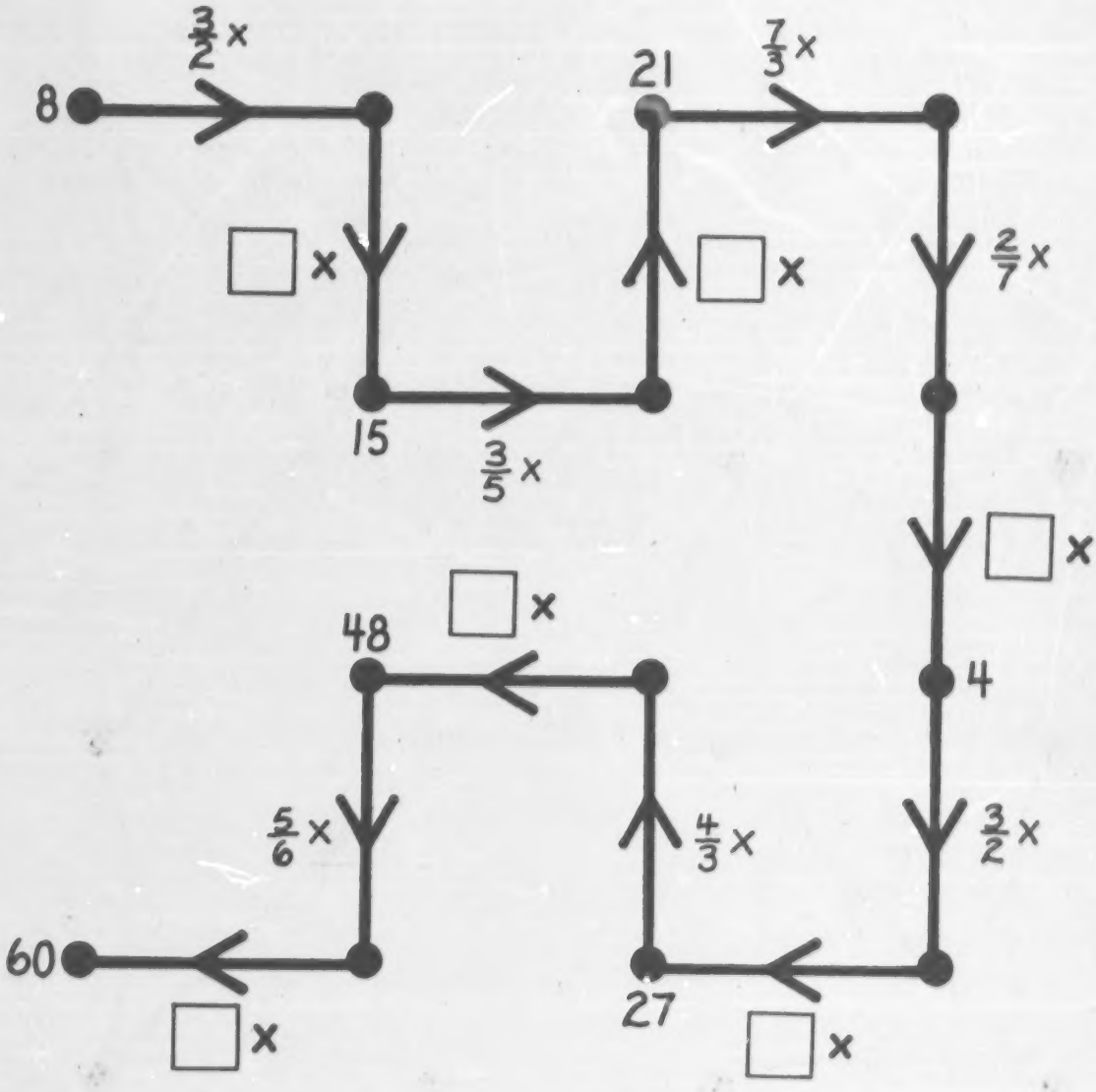






Name _____ **N 64** ***

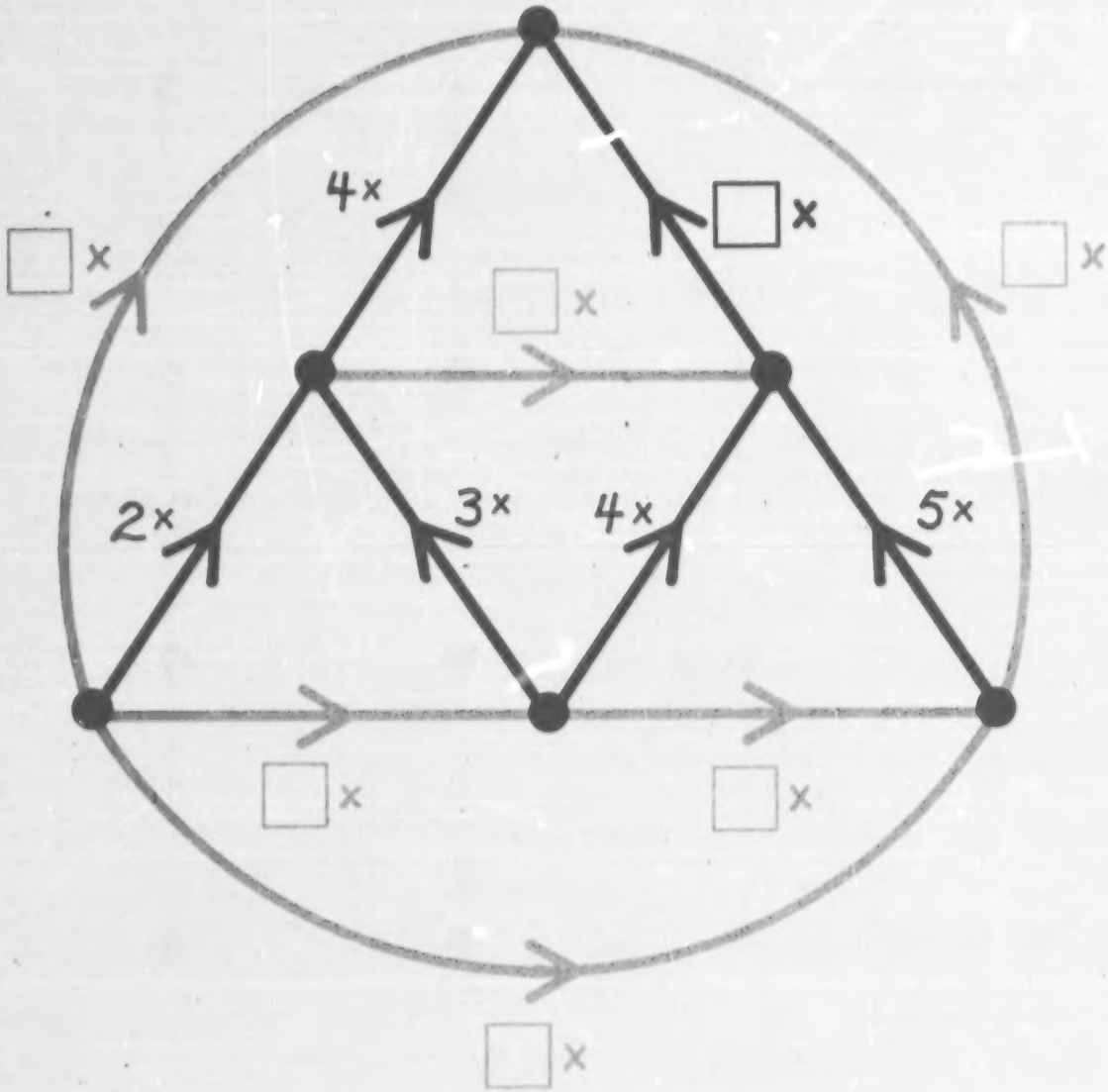
Label the dots and fill in the boxes for the arrows.



Name _____

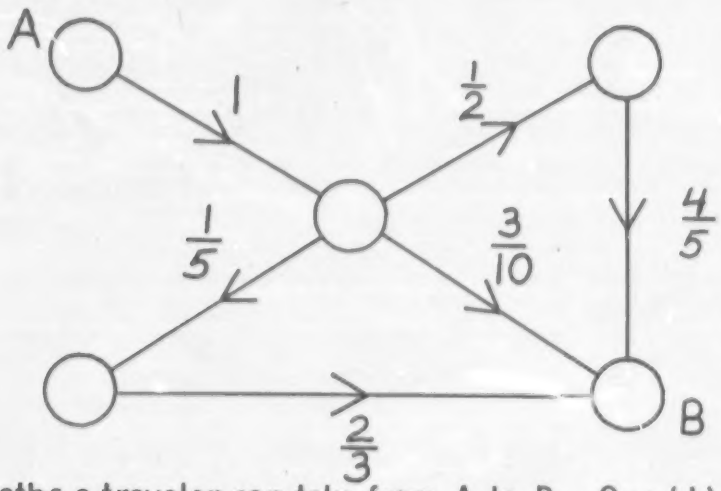
N 65 ****

Fill in the boxes for the arrows.



Name _____ **N66** *

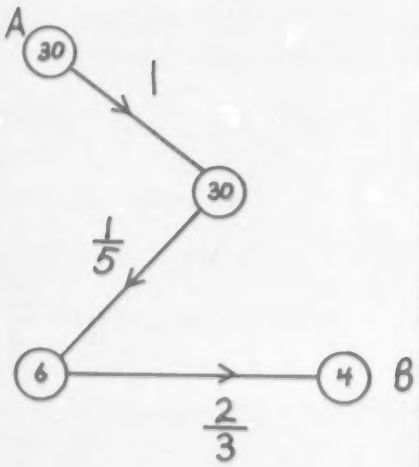
This is the map of the roads from town A to town B.



There are three paths a traveler can take from A to B. One (I) is shown below; draw the other paths (II and III).

Suppose 30 people start from town A. For each path, calculate the number of people we would expect to survive. The calculation for path I is done for you.

I



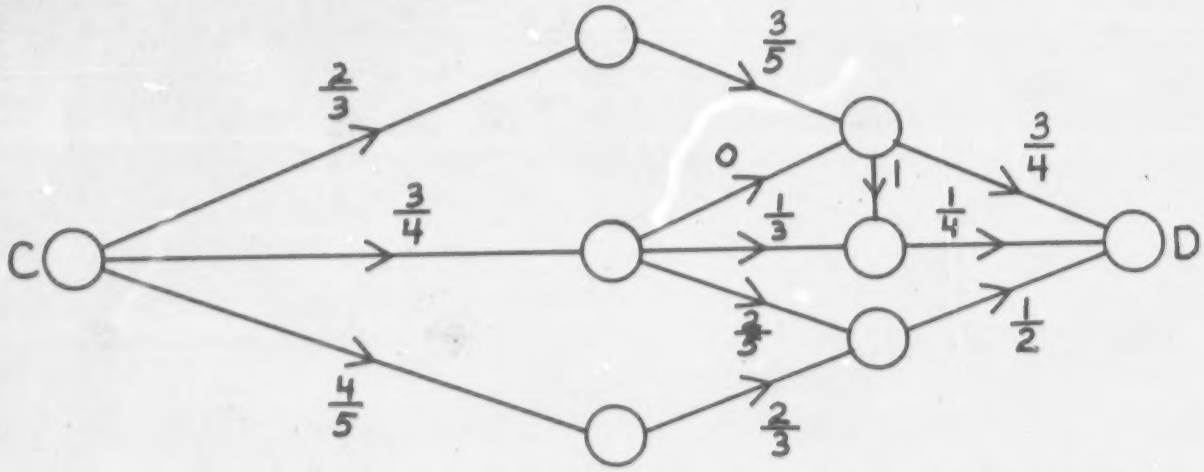
II

III

Which path is the safest? _____

Name _____ **N 67** **

This is the map of the roads from town C to town D.



Draw each path a traveler might take from C to D. If there is a road that should definitely be avoided, do not include it in any of the paths you draw.

Suppose 60 people start from C. For each path, calculate the number of people we would expect to survive.

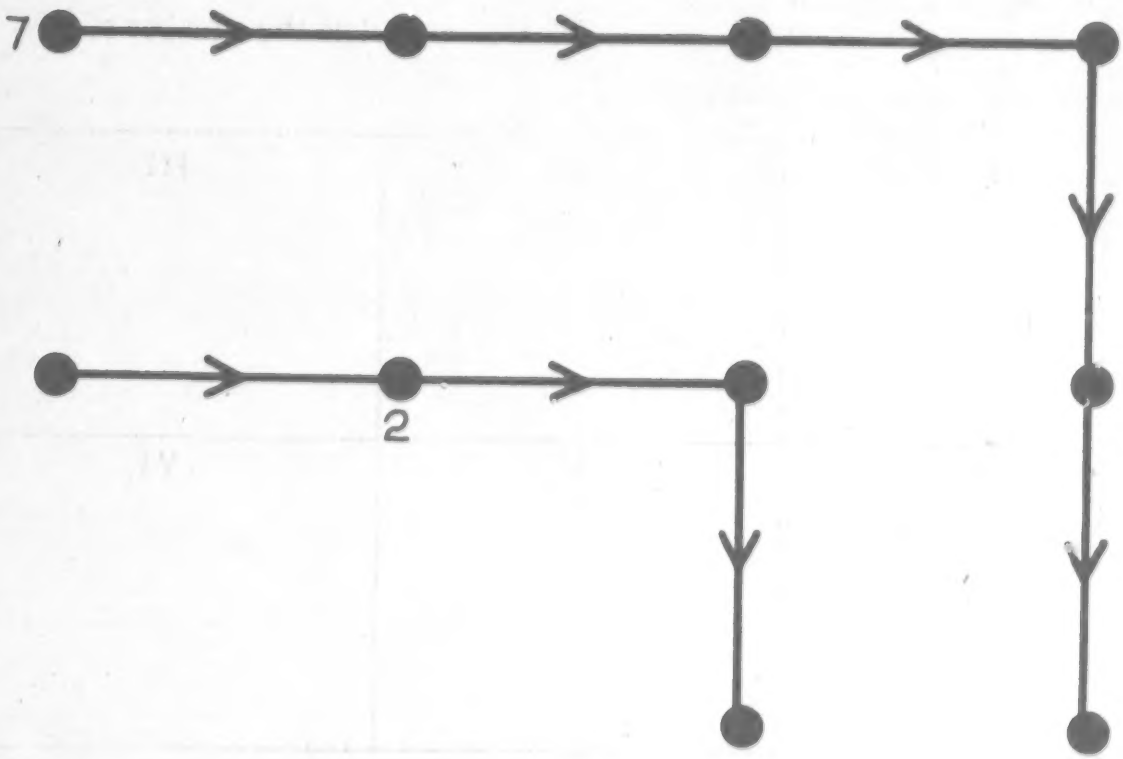
I	II	III
IV	V	VI

Which path is the safest? _____

Name _____ **N 68** *

$(2 \times s) \div e = 20$

Label the dots.

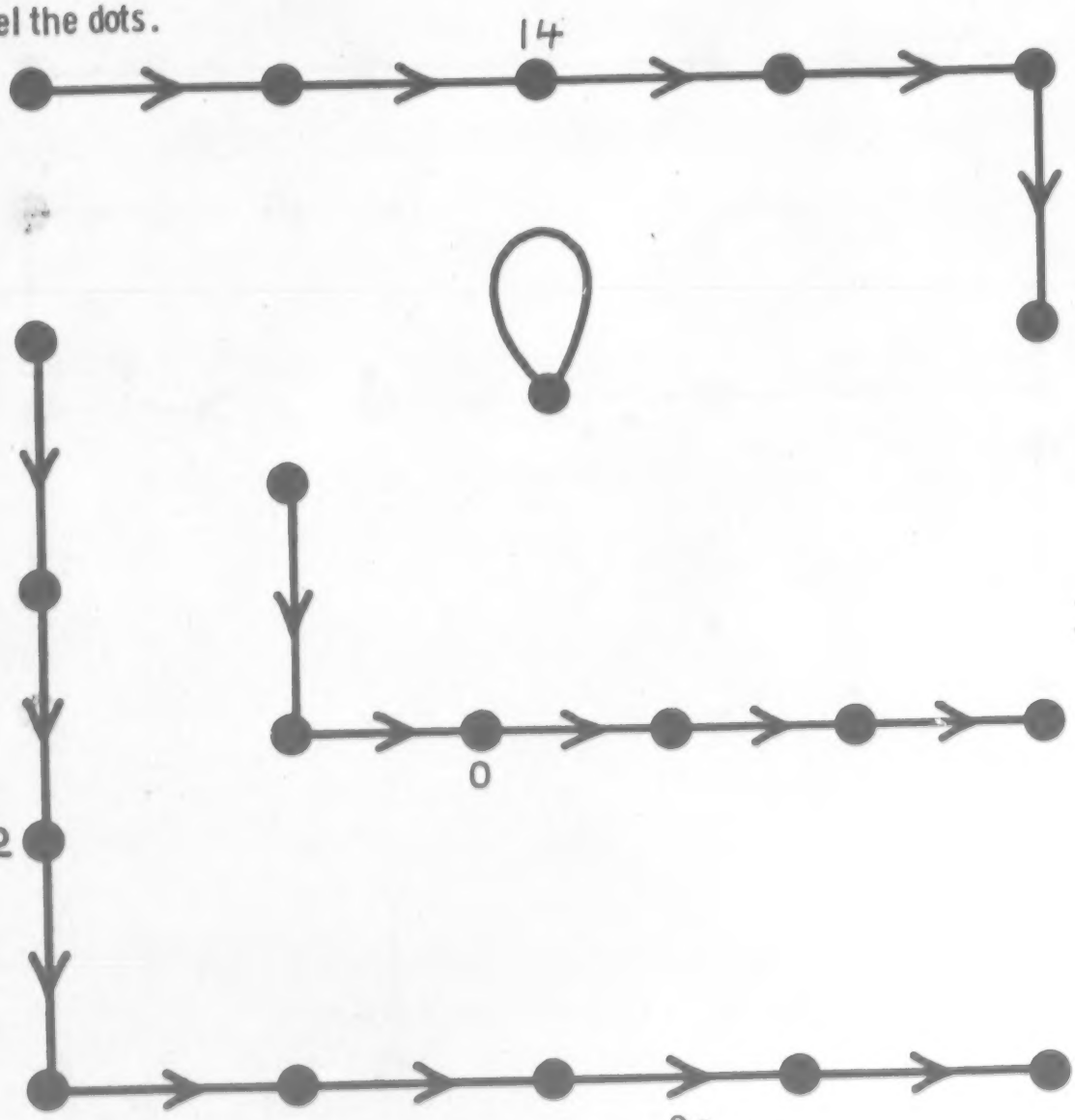


Name _____

N 69 **

$(2 \times s) + e = 30$

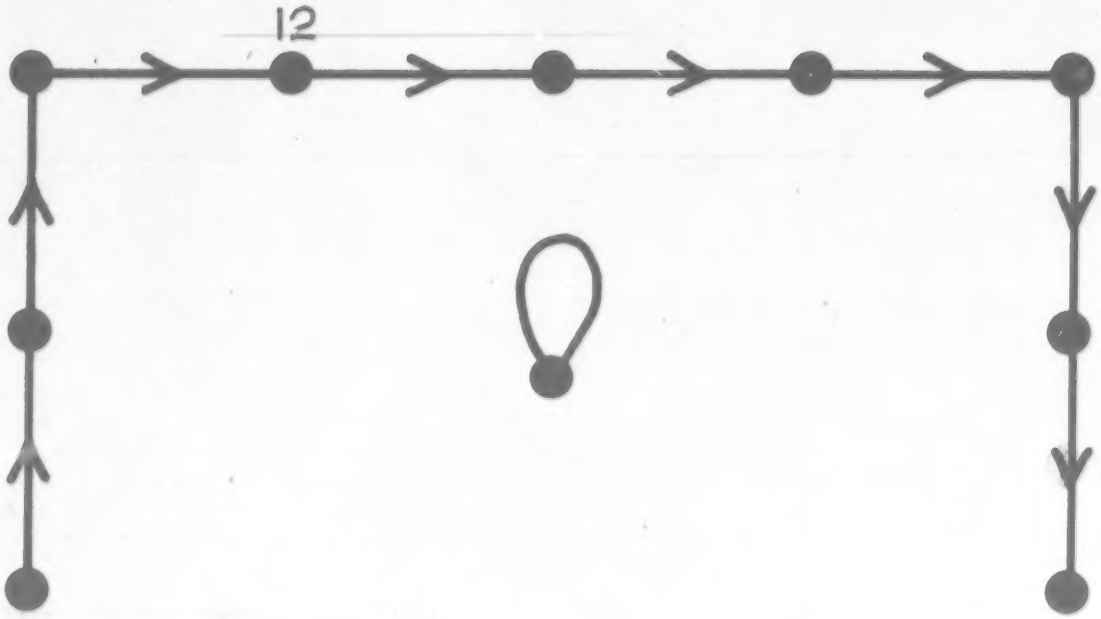
Label the dots.



Name _____ **N 70** ***

$(2 \times \Delta) + e = 10$

Label the dots.



Flop is in this arrow picture. Flop is between 3 and 4.

Flop is _____.

Name _____ **N 71** *

Put these numbers on the Base Three abacus.

2,187	729	243	81	27	9	3	1			

= 25

2,187	729	243	81	27	9	3	1			

= 100

2,187	729	243	81	27	9	3	1			

= 300

2,187	729	243	81	27	9	3	1			

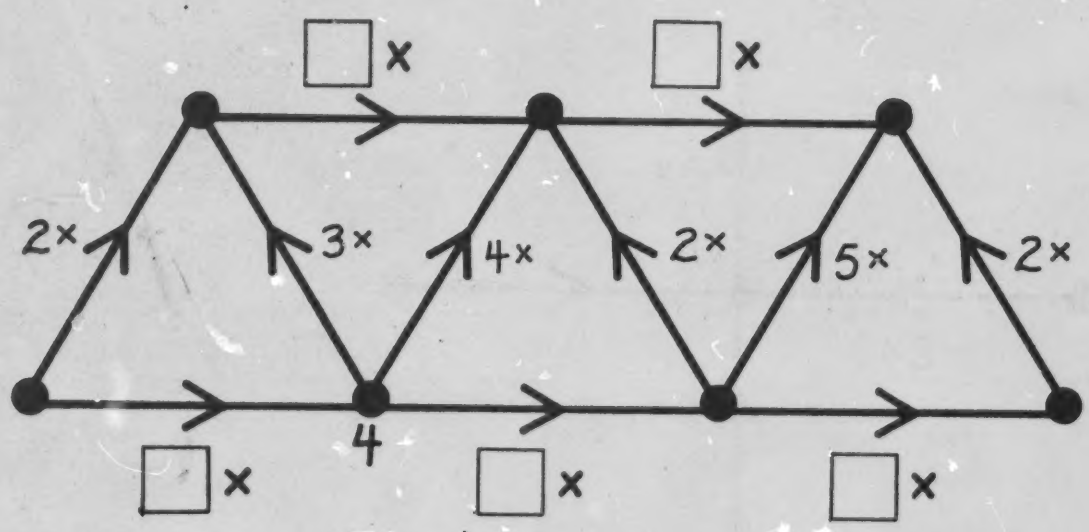
= 900

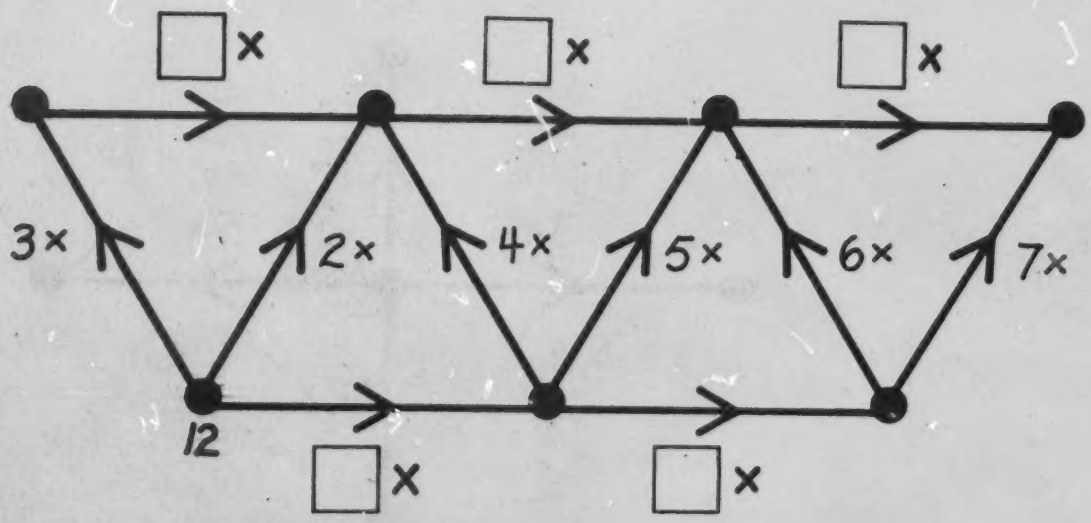
2,187	729	243	81	27	9	3	1			

= 2,700

Name _____ **N 72** *

Label each dot. Fill in the boxes for the blue arrows.

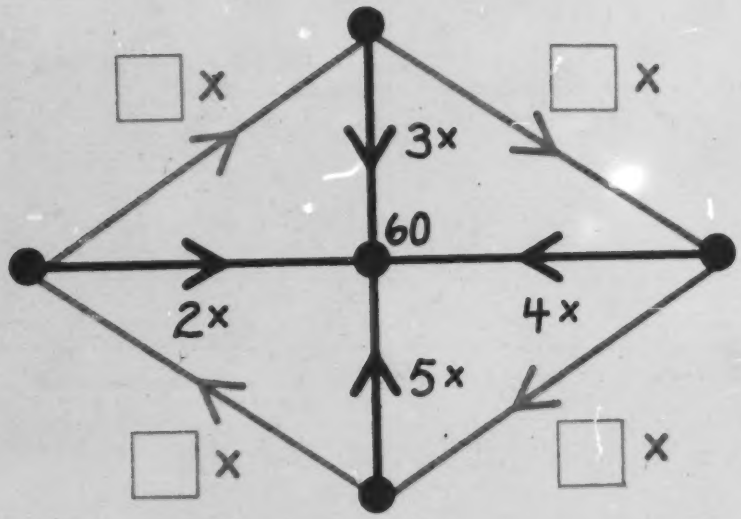


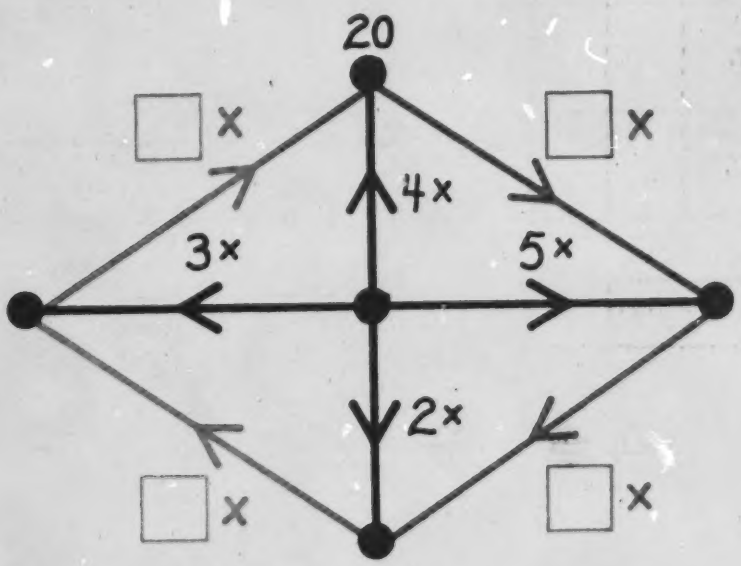


Name _____

N 73 **

Label each dot. Fill in the boxes for the blue arrows.

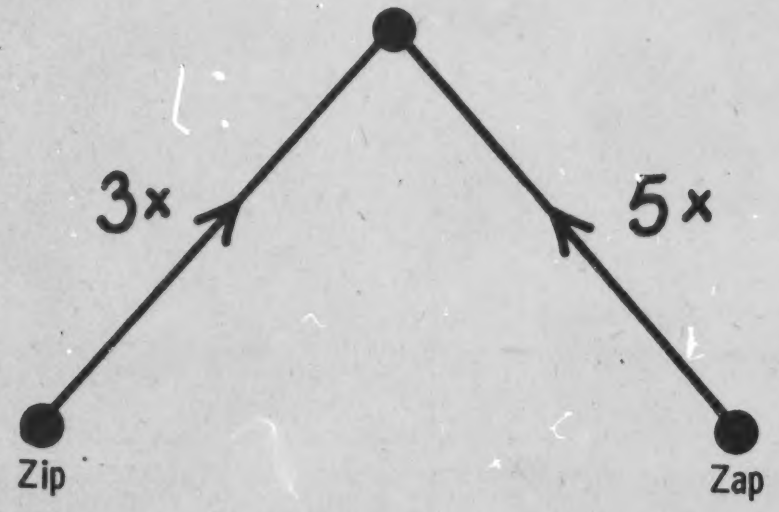




Name _____

N 74 ***

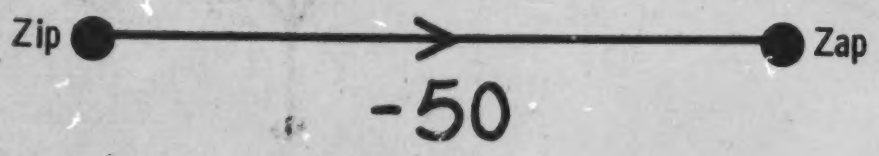
First clue



Complete this chart with pairs of numbers that Zip and Zap could be.
Many solutions are possible.

Zip			10									200		
Zap						30								150

Second clue



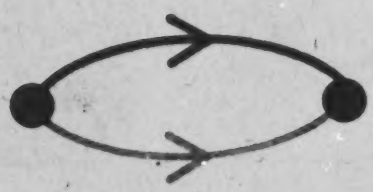
Zip is _____. Zap is _____.

Name _____

N 75 ****

Label the dots.

$\frac{3}{2}\times$



+50

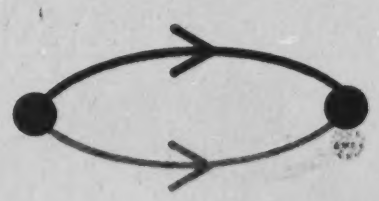
$\frac{2}{3}\times$

−40

$\frac{4}{5}\times$

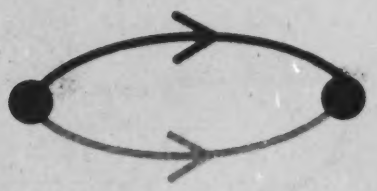
−20

$\frac{3}{4}\times$



-30

$\frac{6}{5}\times$



+15

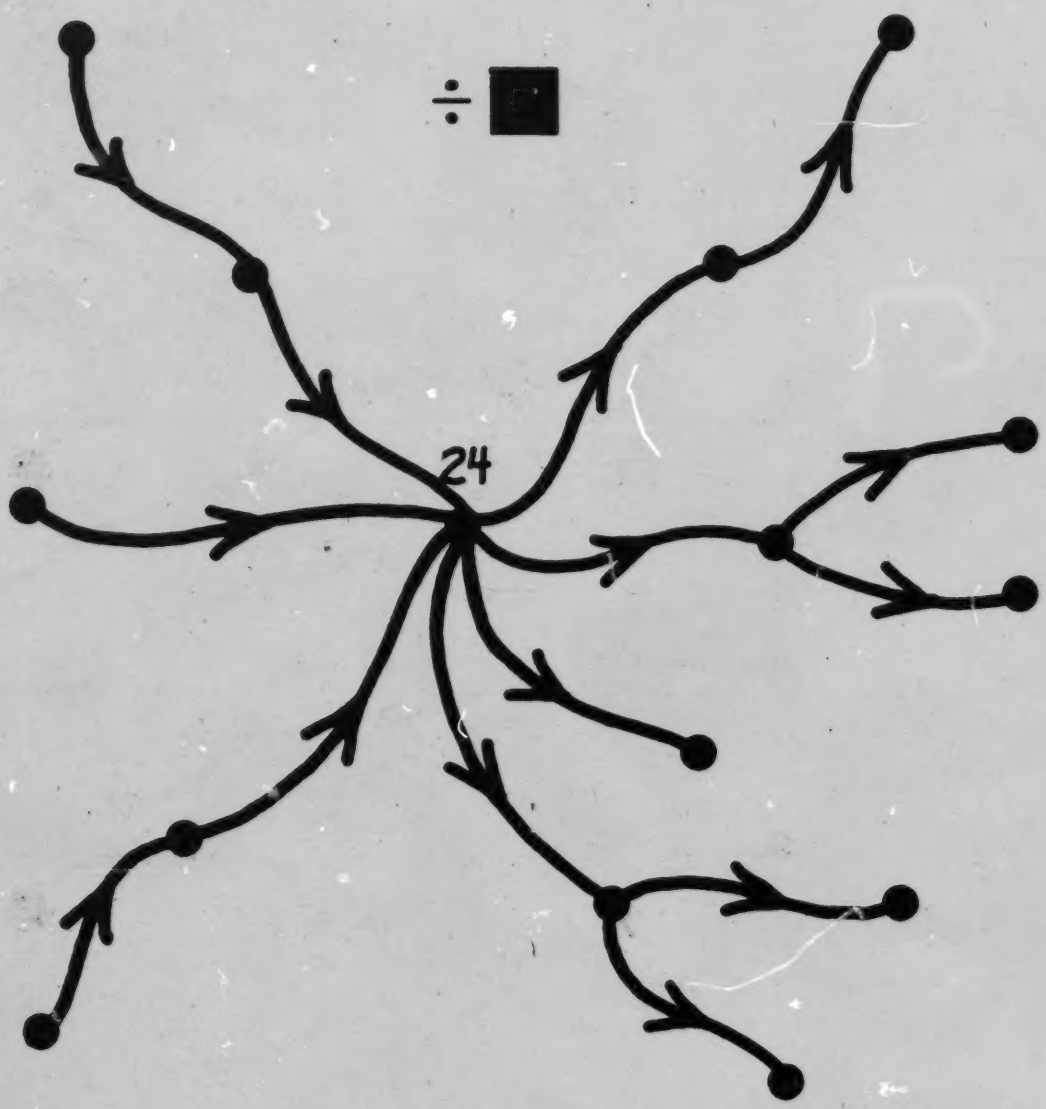
$\frac{5}{6}\times$

−25

Name _____ **N 76** *

" ÷ ■ " means each red arrow is for divided by some WHOLE number.

Label the dots. Label each arrow " ÷ □ ". Many solutions are possible.

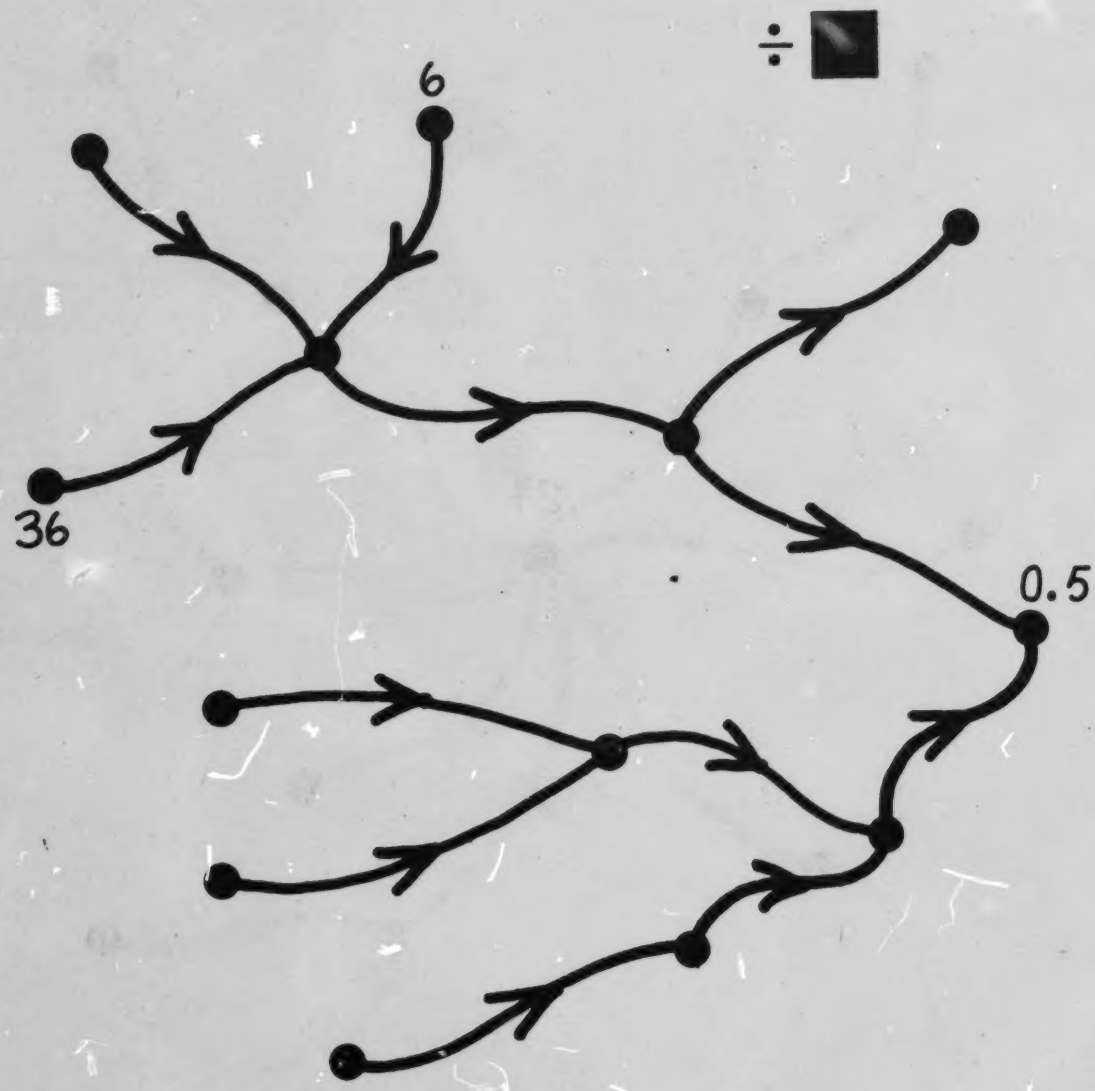


Name _____

N 77 **

" ÷ ■ " means each red arrow is for divided by some WHOLE number.

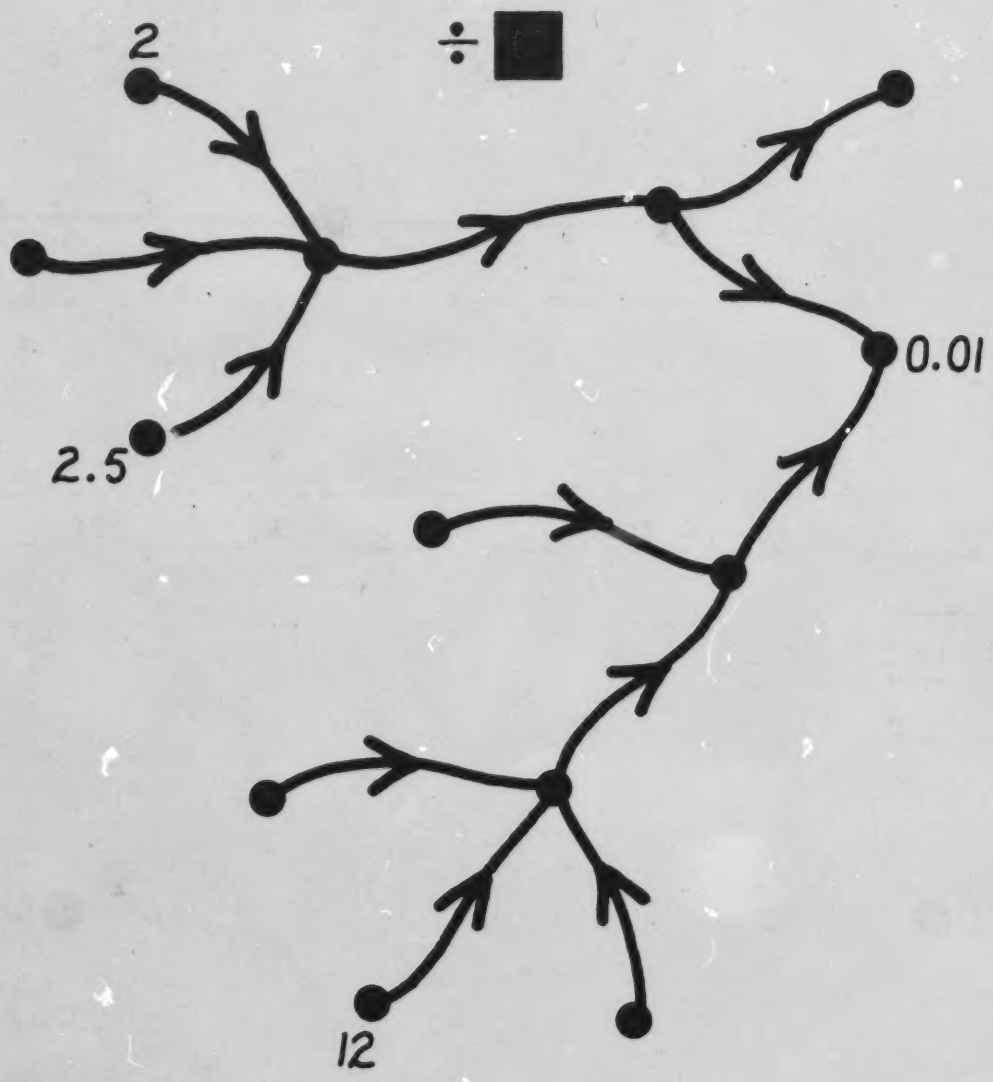
Label the dots. Label each arrow " ÷ □ ". Many solutions are possible.



Name _____ **N 78** ***

" ÷ ■ " means each red arrow is for divided by some WHOLE number.

Label the dots. Label each arrow " ÷ □ ". Many solutions are possible.

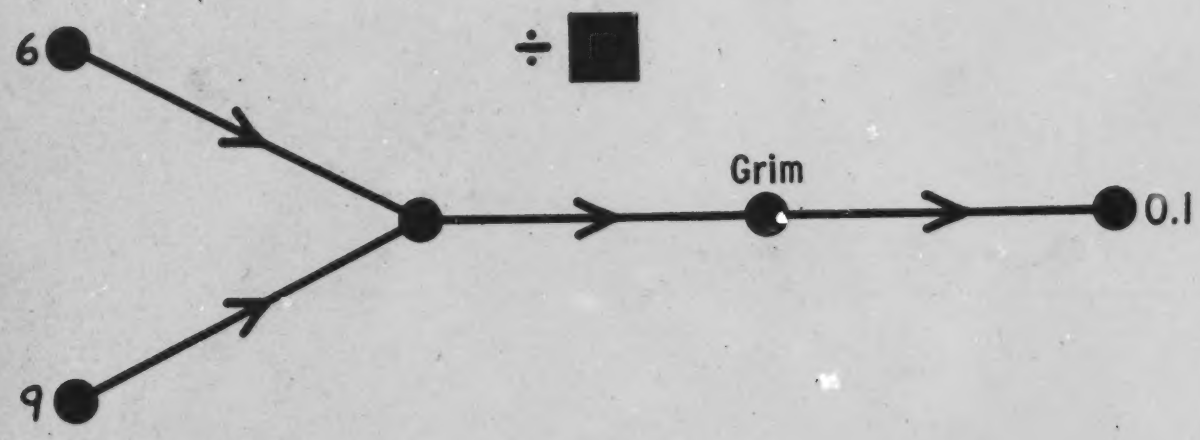


Name _____

N 79 ****

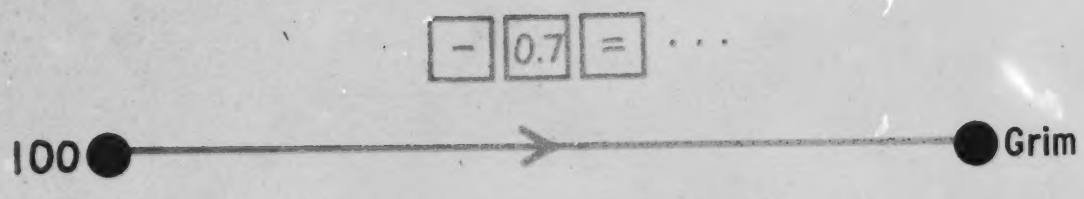
First clue

" ÷ ■ " means each red arrow is for divided by some WHOLE number.



Grim is _____ or _____ or _____ or _____ or _____ or _____.

Second clue



Grim is _____.

Name _____ **N 80** *

Could Zig be 82? _____

Could Zig be 75? _____

Could Zig be 0? _____

Could Zig be -5? _____

Could Zig be 80? _____

Could Zag be 115? _____

Could Zag be 120? _____

Could Zag be 1,000,000? _____

What is the largest number Zig could be? _____

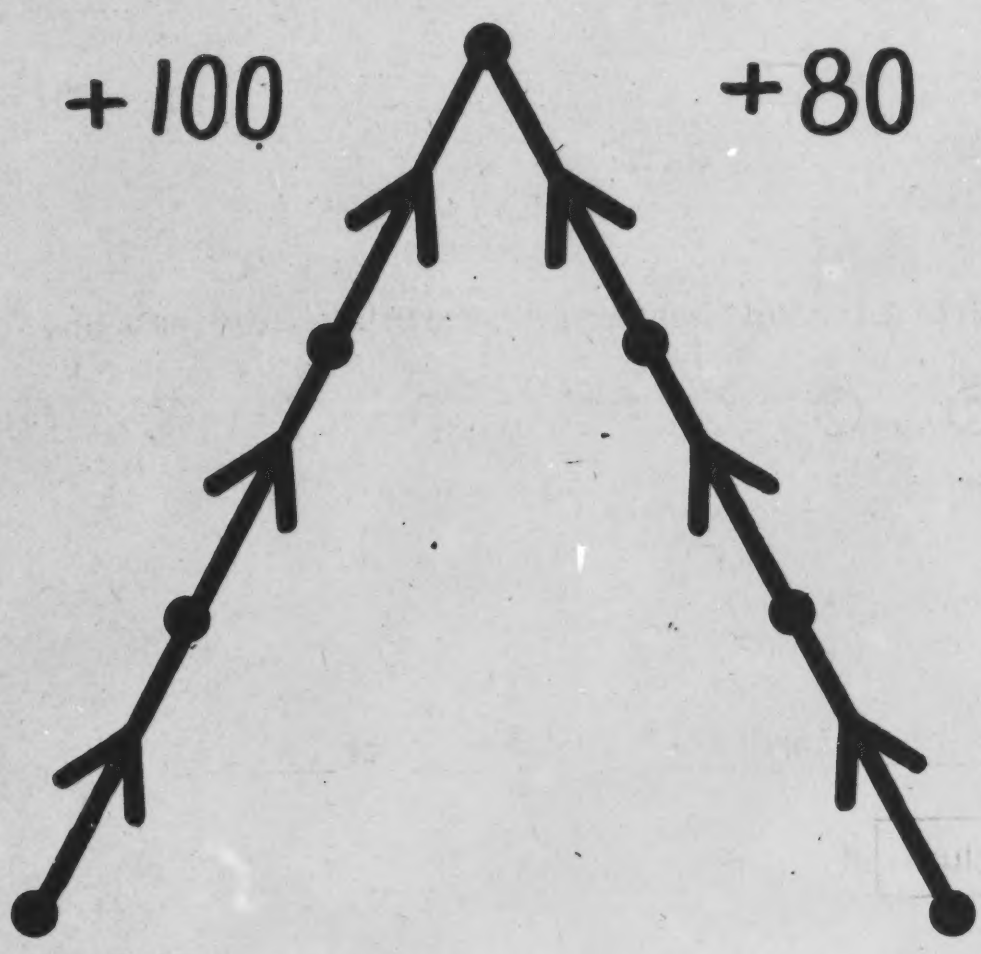
What is the smallest number Zag could be? _____

Name _____

N 81 *

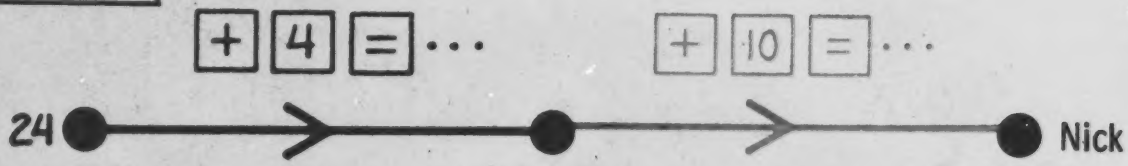
Second clue

Zig and Zag are the two smallest numbers in this picture.



Name _____ **N 82** *

First clue



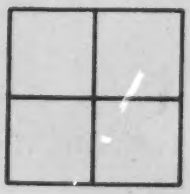
Nick could be _____; _____; _____; _____; _____; _____; _____; _____;

_____; _____; _____; _____; _____; _____; _____; _____; and so on.

Second clue

Nick can be put on this Minicomputer board using exactly one of these checkers:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨



Nick is _____ or _____ or _____ or _____.

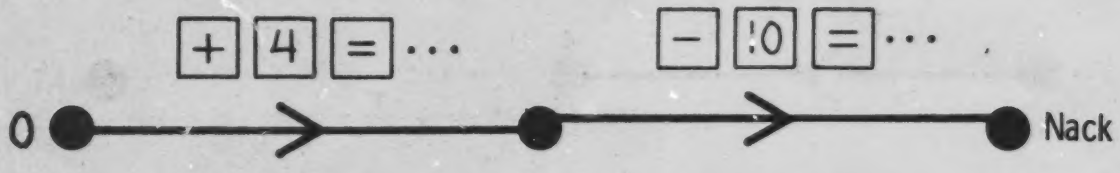
Third clue

Nick is a square number. Nick is _____.

Name _____

N 83 **

First clue



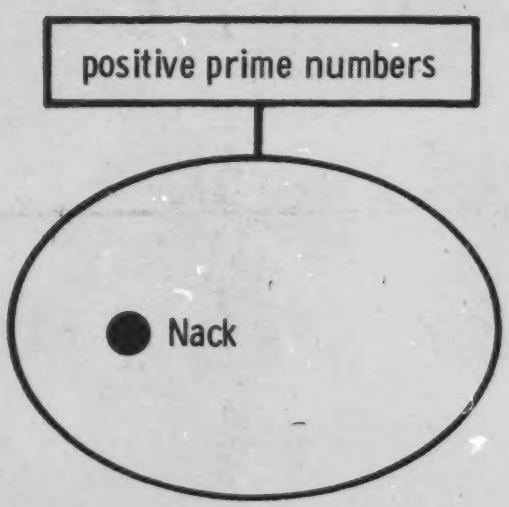
Nack could be _____; _____; _____; _____; _____; _____; _____; _____;

_____; _____; _____; _____; _____; _____; _____; _____; and so on.

What do you notice about the numbers that Nack could be?

_____

Second clue

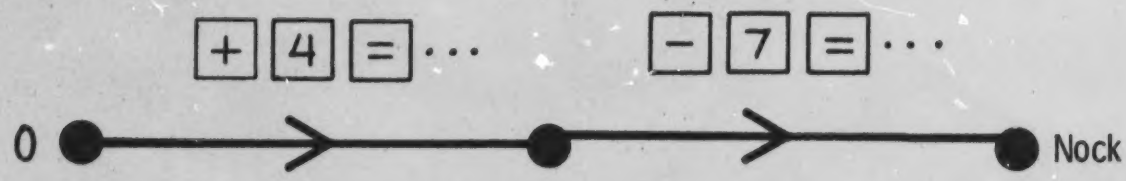


Nack is _____.

Name _____

N 84 ***

First clue



Nock could be _____; _____; _____; _____; _____; _____; _____; _____;

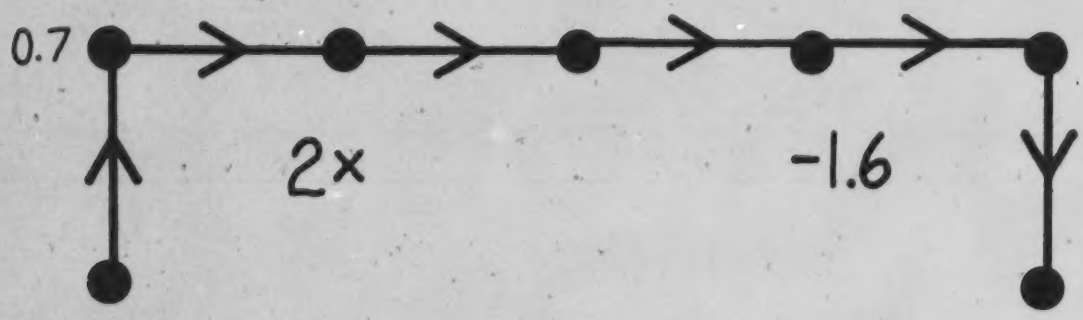
_____; _____; _____; _____; _____; _____; _____; _____; and so on.

What do you notice about the numbers that Nock could be?

_____

Second clue

Nock is in this arrow picture.



Nock is _____.

N 85

is larger than

is smaller than

is at least 20 smaller than

is at least 20 larger than

is a multiple of

is a positive divisor of

is the square of

$10 \times$

$\div 10$

$2 \times$

$\div 2$

$\frac{2}{3} \times$

$\frac{3}{2} \times$

$\boxed{+}\ \boxed{=} \cdots$

$\boxed{+}\ \boxed{2}\ \boxed{=} \cdots$

$\boxed{-}\ \boxed{2}\ \boxed{=} \cdots$

$\boxed{+}\ \boxed{3}\ \boxed{=} \cdots$

$\boxed{-}\ \boxed{3}\ \boxed{=} \cdots$

$\boxed{+}\ \boxed{4}\ \boxed{=} \cdots$

$\boxed{-}\ \boxed{4}\ \boxed{=} \cdots$

$\boxed{+}\ \boxed{5}\ \boxed{=} \cdots$

$\boxed{-}\ \boxed{5}\ \boxed{=} \cdots$

is larger than

is smaller than

is at least 20 smaller than

is at least 20 larger than

is a multiple of

is a positive divisor of

is the square of

$10\times$

$\div\ 10$

$2\times$

$\div\ 2$

$\frac{2}{3}\times$

$\frac{3}{2}\times$

$\boxed{+}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{2}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{2}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{3}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{3}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{4}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{4}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{5}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{5}\ \boxed{=}\ \cdots$

N 86

is larger than

is smaller than

is at least 20 smaller than

is at least 20 larger than

is a multiple of

is a positive divisor of

is the square of

$10 \times$

$\div 10$

$2 \times$

$\div 2$

$\frac{2}{3} \times$

$\frac{3}{2} \times$

$\boxed{+}\ \boxed{=} \cdots$

$\boxed{+}\ \boxed{2}\ \boxed{=} \cdots$

$\boxed{-}\ \boxed{2}\ \boxed{=} \cdots$

$\boxed{+}\ \boxed{3}\ \boxed{=} \cdots$

$\boxed{-}\ \boxed{3}\ \boxed{=} \cdots$

$\boxed{+}\ \boxed{4}\ \boxed{=} \cdots$

$\boxed{-}\ \boxed{4}\ \boxed{=} \cdots$

$\boxed{+}\ \boxed{5}\ \boxed{=} \cdots$

$\boxed{-}\ \boxed{5}\ \boxed{=} \cdots$

is larger than

is smaller than

is at least 20 smaller than

is at least 20 larger than

is a multiple of

is a positive divisor of

is the square of

$10\times$

$\div 10$

$2\times$

$\div 2$

$\frac{2}{3}\times$

$\frac{3}{2}\times$

$\boxed{+}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{2}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{2}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{3}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{3}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{4}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{4}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{5}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{5}\ \boxed{=}\ \cdots$

N 87

is larger than

is smaller than

is at least 20 smaller than

is at least 20 larger than

is a multiple of

is a positive divisor of

is the square of

$10 \times$

$\div\ 10$

$2 \times$

$\div\ 2$

$\frac{2}{3} \times$

$\frac{3}{2} \times$

$\boxed{+}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{2}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{2}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{3}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{3}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{4}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{4}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{5}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{5}\ \boxed{=}\ \cdots$

is larger than

is smaller than

is at least 20 smaller than

is at least 20 larger than

is a multiple of

is a positive divisor of

is the square of

$10\times$

$\div\ 10$

$2\times$

$\div\ 2$

$\frac{2}{3}\times$

$\frac{3}{2}\times$

$\boxed{+}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{2}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{2}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{3}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{3}\ \boxed{=}\ \cdots$

$\boxed{+}\ \boxed{4}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{4}\ \boxed{=}\ \cdots$

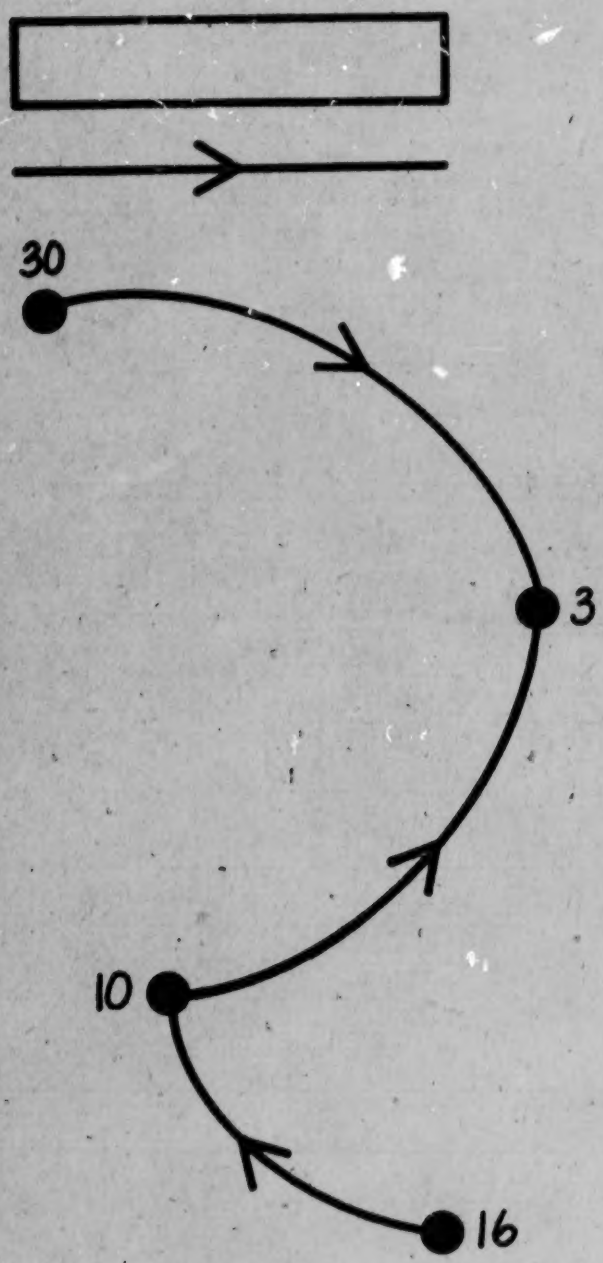
$\boxed{+}\ \boxed{5}\ \boxed{=}\ \cdots$

$\boxed{-}\ \boxed{5}\ \boxed{=}\ \cdots$

Name _____

N 88 *

Complete the arrow label.



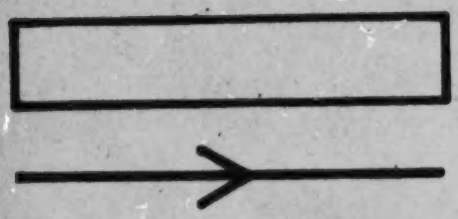
The red arrow could be for:

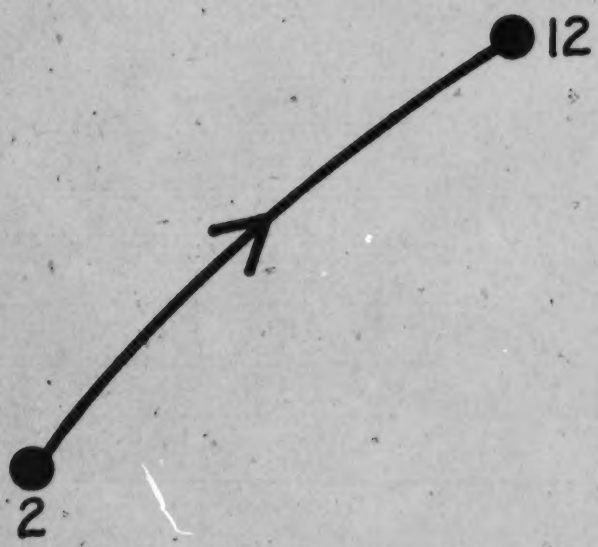
- is larger than
- is smaller than
- is at least 20 smaller than
- is at least 20 larger than
- is a multiple of
- is a positive divisor of
- is the square of
- 10 ×
- 10
- 2 ×
- ÷ 2
- $\frac{3}{5}$ ×
- $\frac{3}{2}$ ×
- [+] [=] ...
- [+] [2] [=] ...
- [−] [2] [=] ...
- [+] [3] [=] ...
- [−] [3] [=] ...
- [+] [4] [=] ...
- [−] [4] [=] ...
- [+] [5] [=] ...
- [−] [5] [=] ...

Name _____

N 89 **

Complete the arrow label.





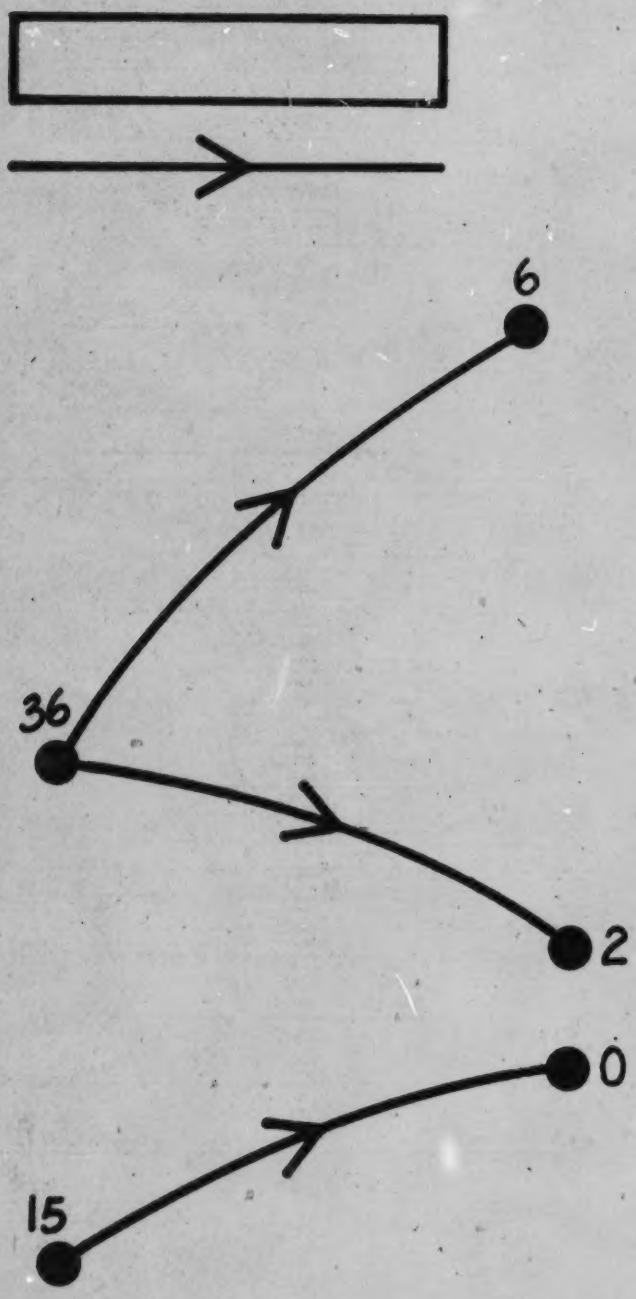
The red arrow could be for:

- is larger than
- is smaller than
- is at least 20 smaller than
- is at least 20 larger than
- is a multiple of
- is a positive divisor of
- is the square of
- 10 ×
- ÷ 10
- 2 ×
- ÷ 2
- $\frac{1}{5}$ ×
- $\frac{1}{2}$ ×
- [+] [=] ...
- [+] [2] [=] ...
- [−] [2] [=] ...
- [+] [3] [=] ...
- [−] [3] [=] ...
- [+] [4] [=] ...
- [−] [4] [=] ...
- [+] [5] [=] ...
- [−] [5] [=] ...

Name _____

N 90 ***

Complete the arrow label.



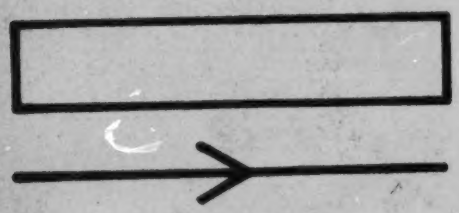
The red arrow could be for:

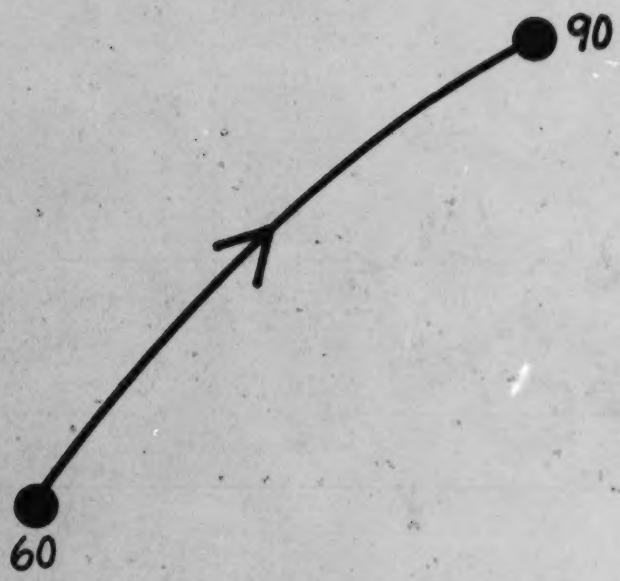
- is larger than
- is smaller than
- is at least 20 smaller than
- is at least 20 larger than
- is a multiple of
- is a positive divisor of
- is the square of
- 10 ×
- ÷ 10
- 2 ×
- ÷ 2
- $\frac{3}{5}$ ×
- $\frac{3}{2}$ ×
- [+] [=] ...
- [+] [2] [=] ...
- [−] [2] [=] ...
- [+] [3] [=] ...
- [−] [3] [=] ...
- [+] [4] [=] ...
- [−] [4] [=] ...
- [+] [5] [=] ...
- [−] [5] [=] ...

Name _____

N 91 ****

Complete the arrow label.





The red arrow could be for:

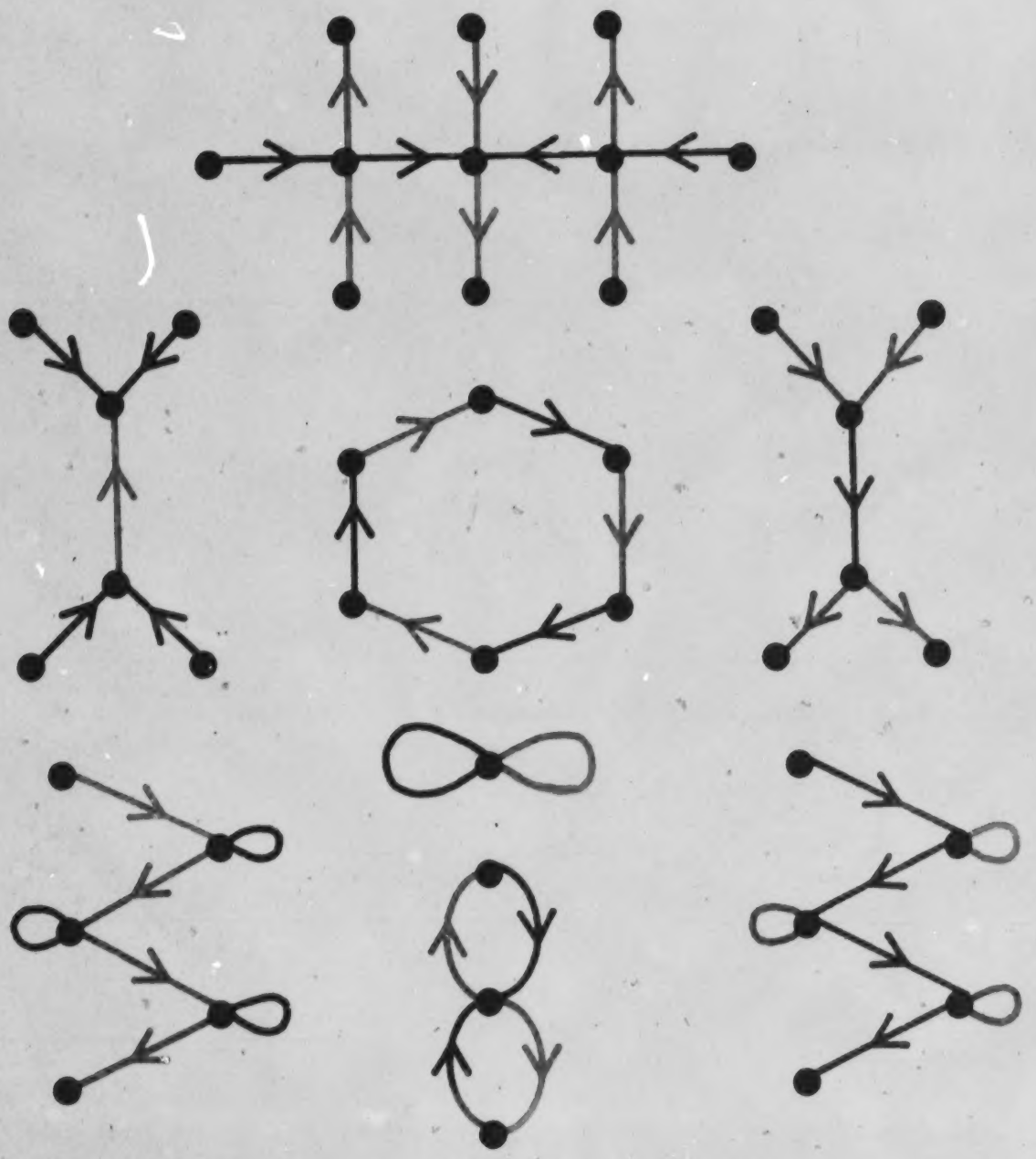
- is larger than
- is smaller than
- is at least 20 smaller than
- is at least 20 larger than
- is a multiple of
- is a positive divisor of
- is the square of
- 10 ×
- ÷ 10
- 2 ×
- ÷ 2
- $\frac{3}{5}$ ×
- $\frac{3}{2}$ ×
- [+] [=] ...
- [+] [2] [=] ...
- [−] [2] [=] ...
- [+] [3] [=] ...
- [−] [3] [=] ...
- [+] [4] [=] ...
- [−] [4] [=] ...
- [+] [5] [=] ...
- [−] [5] [=] ...

Name _____

L 01 *

Draw the missing green arrows and loops.

R S R∘S

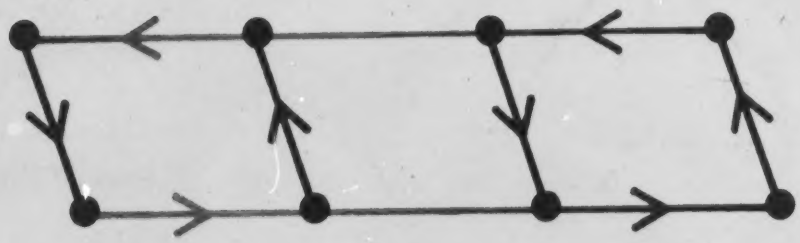


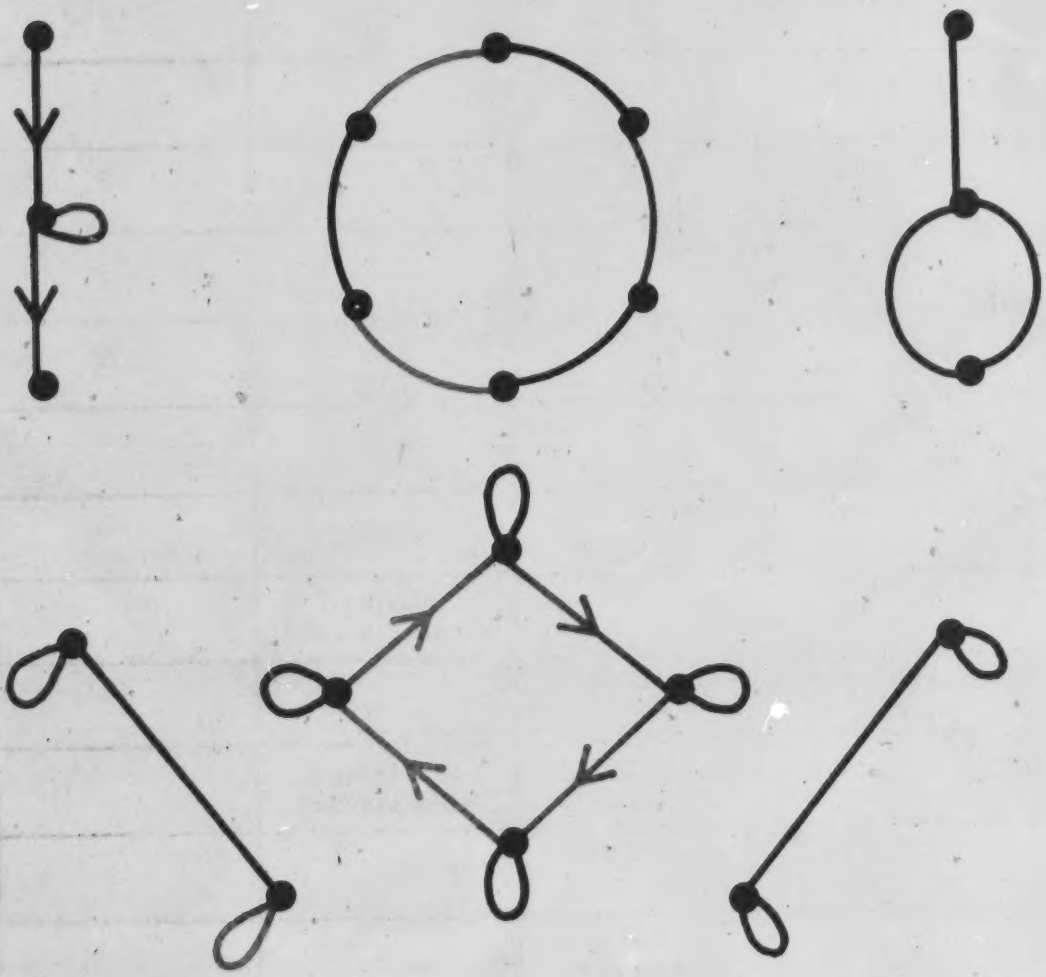
Name _____

L 02 **

Draw the missing green and black arrows, loops, and cords.

R **S** **R∘S** **S∘S**





Name _____ **L03** ***

Complete the table.

R	S	RoS	SoR
-6	-13		
5×	4×		
3×		6×	
	-7		-4
8×		4×	
you are my son	you are my sister		
		you are my maternal grandfather	
you are my daughter	you are my husband		
		you are my friend's brother	
you are 5 years older than I	you are 3 years younger than I		
you are older than I	you are the same age as I		

Name _____ **L 04 ***

Complete:

2 ⊕ 4 ⊕ 6 ⊕ 8 = _____

2 ⊗ 4 ⊗ 6 ⊗ 8 = _____

1 ⊕ 3 ⊕ 5 ⊕ 7 ⊕ 9 = _____

1 ⊗ 3 ⊗ 5 ⊗ 7 ⊗ 9 = _____

3 ⊕ 4 ⊕ 5 ⊕ 6 = _____

3 ⊗ 4 ⊗ 5 ⊗ 6 = _____

The number in the box could be:

□ ⊗ 7 = 3 _____.

4 ⊗ □ = 2 _____ or _____.

5 ⊗ □ = 5 ___ or ___ or ___ or ___ or ___.

Name _____

L 05 **

$3^1 = 3$

$3^2 = 3 \times 3 =$ _____

$3^3 = 3 \times 3 \times 3 =$ _____

$3^4 = 3 \times 3 \times 3 \times 3 =$ _____

$3^5 =$ _____ $3^9 =$ _____

$3^6 =$ _____ $3^{10} =$ _____

$3^7 =$ _____ $3^{11} =$ _____

$3^8 =$ _____ $3^{12} =$ _____

$3^{25} =$ _____

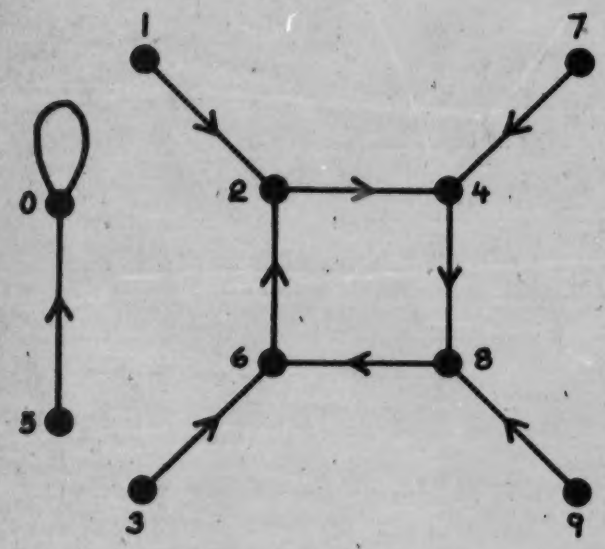
$3^{47} =$ _____

$3^{100} =$ _____

Name _____ **L06** *

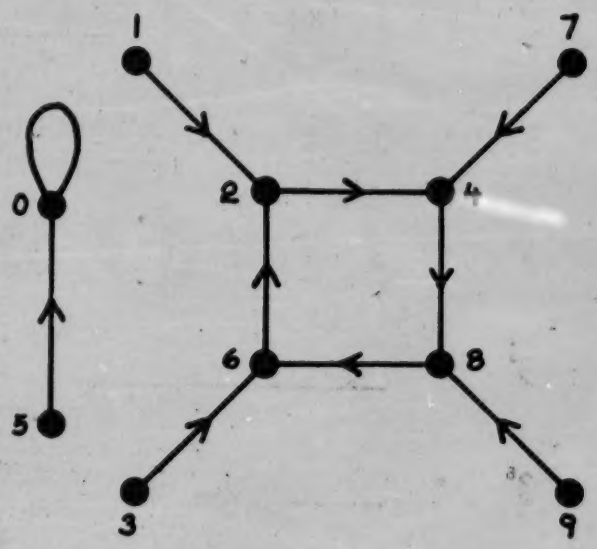
$R = \otimes 2$

$\mathbb{R}^2 = \otimes \square$



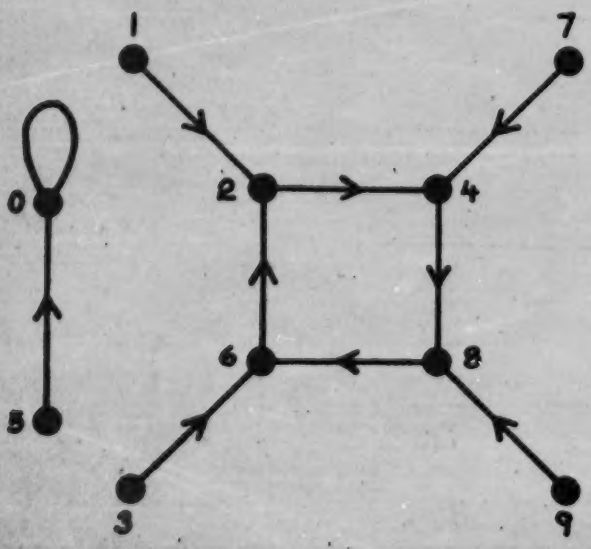
$R = \otimes 2$

$\mathbb{R}^3 = \otimes \square$



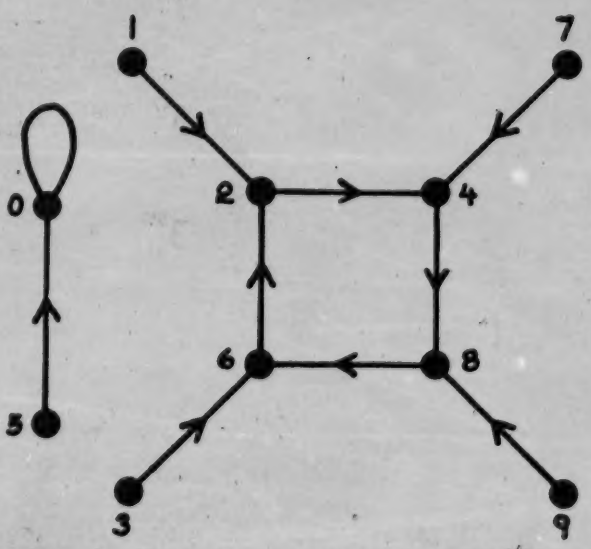
$R = \otimes 2$

$\mathbb{R}^4 = \otimes \square$



$R = \otimes 2$

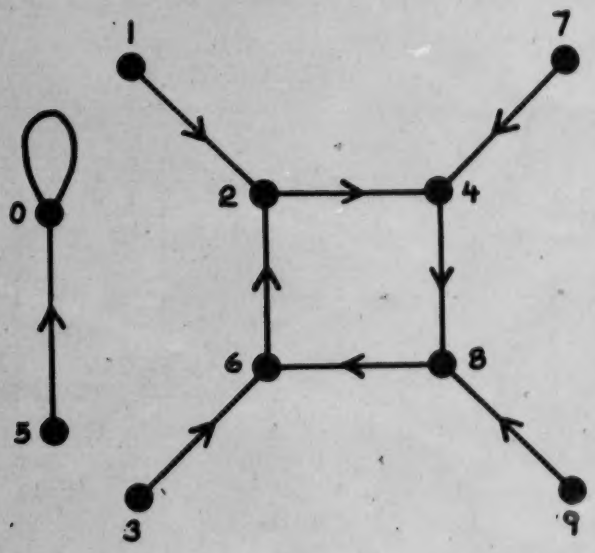
$\mathbb{R}^5 = \otimes \square$



Name _____ **L07** **

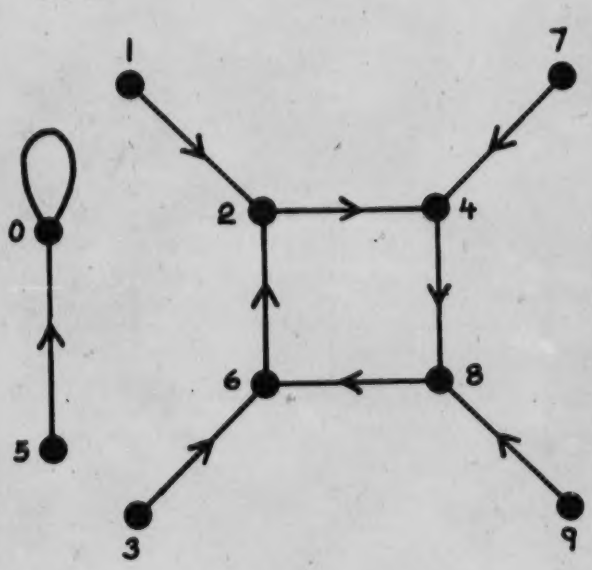
$R = \otimes 2$

$\mathbb{R}^6 = \otimes \square$



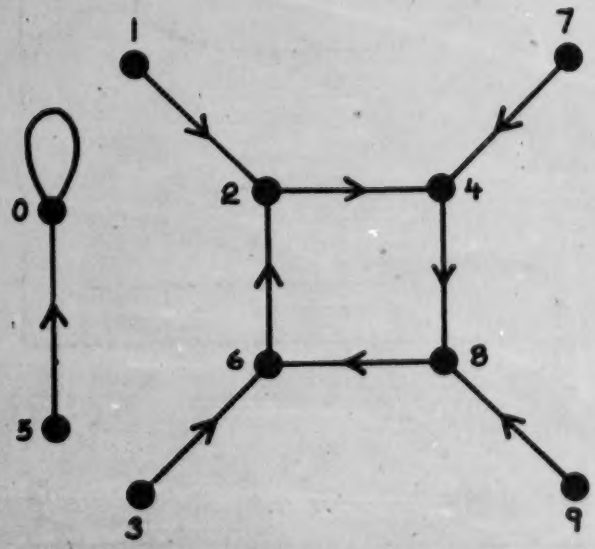
$R = \otimes 2$

$\mathbb{R}^7 = \otimes \square$



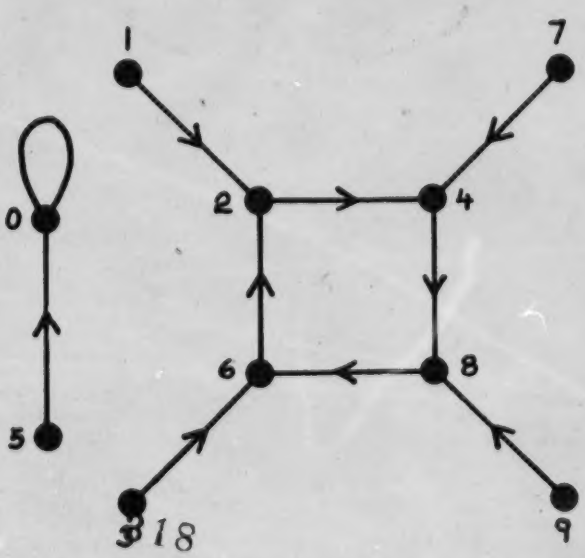
$R = \otimes 2$

$\mathbb{R}^8 = \otimes \square$



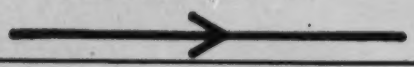
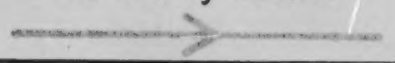
$R = \otimes 2$

$\mathbb{R}^9 = \otimes \square$



Name _____

L 08 *

You are my mother	You are my teacher
	

Decode:

- - - - - - - - - - - - - -

- - - - - - - - - - - - - -

- - - - - - - - - - - - - -

- - - - - - - - - - - - - -

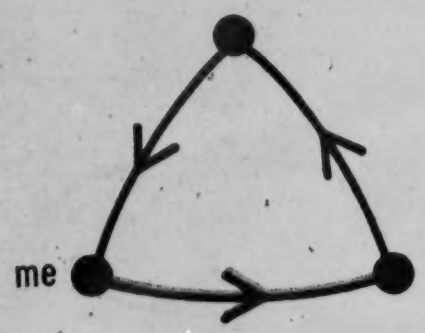
Name _____

L 09 **

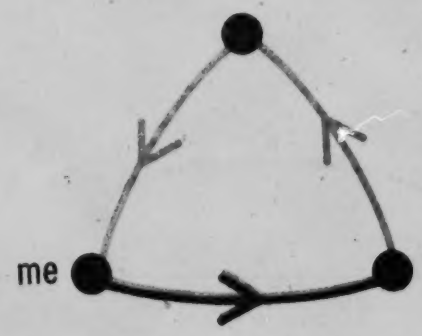
You are my mother

You are my teacher

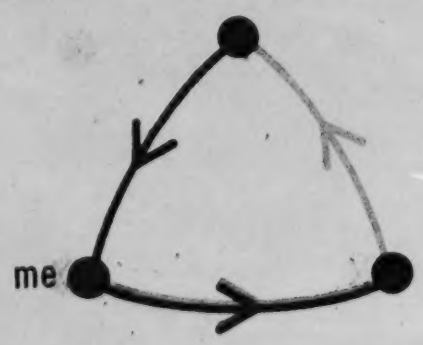
What could the purple arrow be for ?



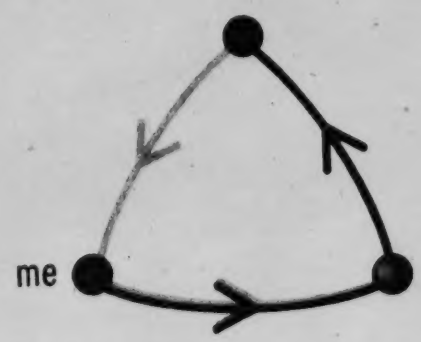
_____



_____



_____



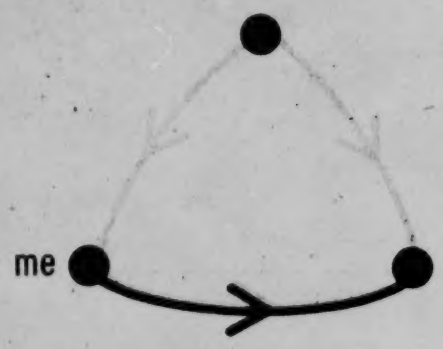
_____

Name _____ **L 10** ***

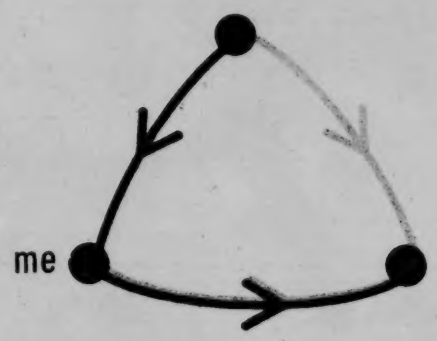
You are my mother

You are my teacher

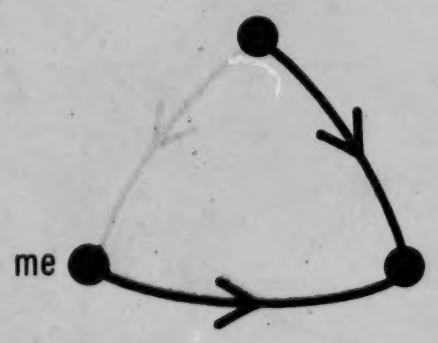
What could the purple arrow be for ?



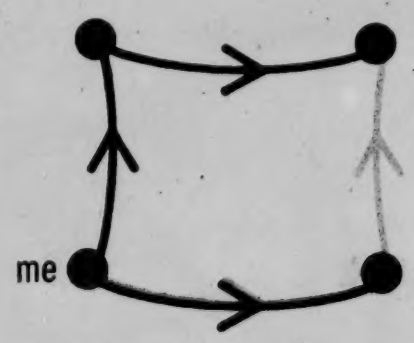
- - - - - - - - - - - - -



- - - - - - - - - - - -



- - - - - - - - - - - - -



- - - - - - - - - - - -

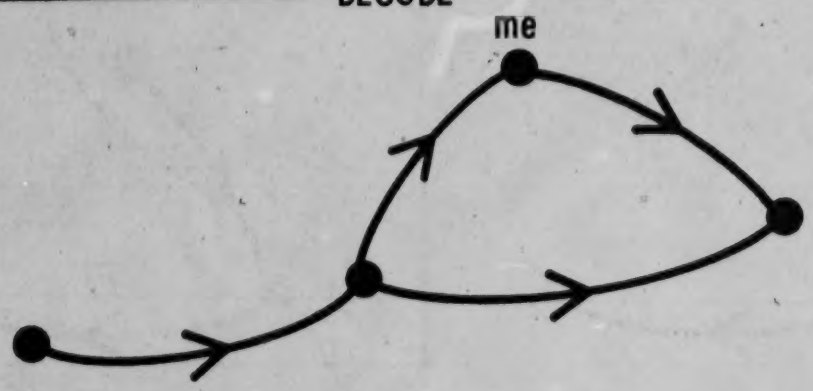
Name _____

L 11 **

You are my mother →

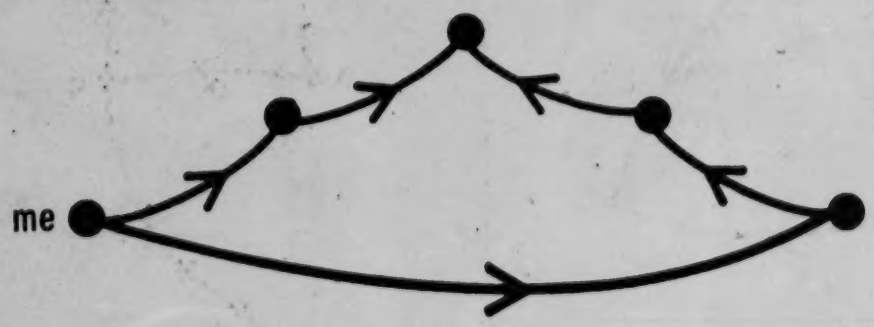
You are my teacher →

DECODE



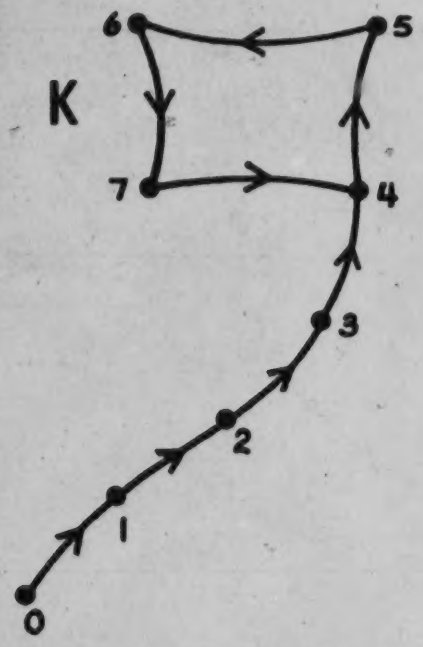
_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

What could the purple arrow be for ?



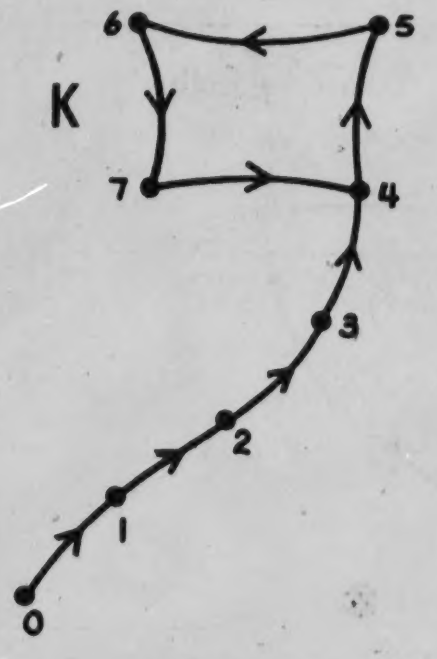
_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Name _____ **L12** *



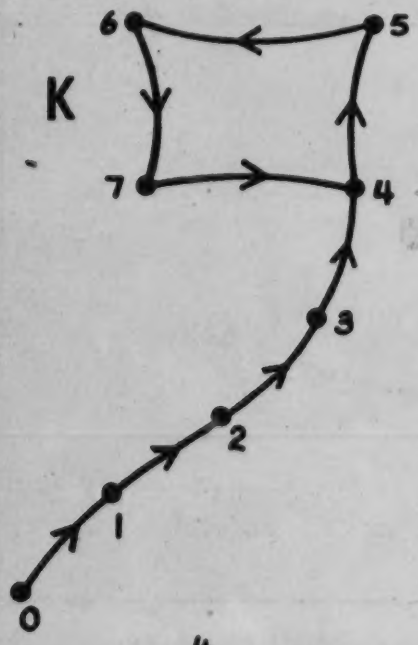
K^2 ⟨

0	1	2	3	4	5	6	7



K^3 ⟨

0	1	2	3	4	5	6	7



K^4 ⟨

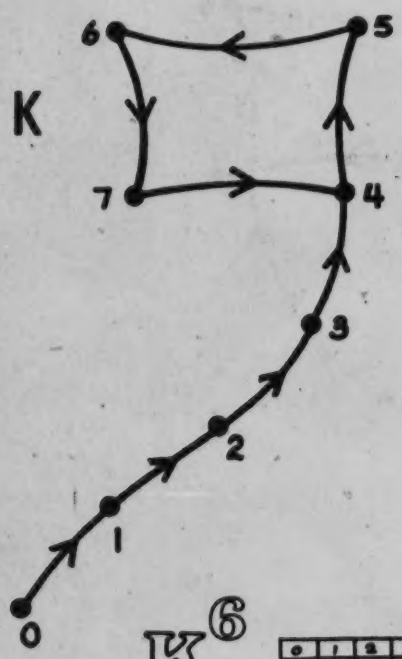
0	1	2	3	4	5	6	7

K^5 ⟨

0	1	2	3	4	5	6	7

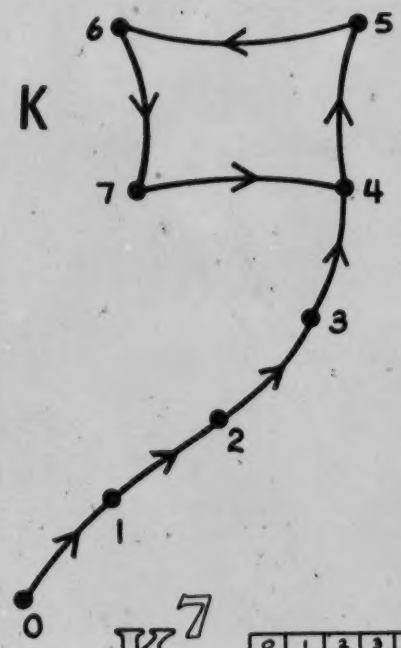
Name _____

L 13 **



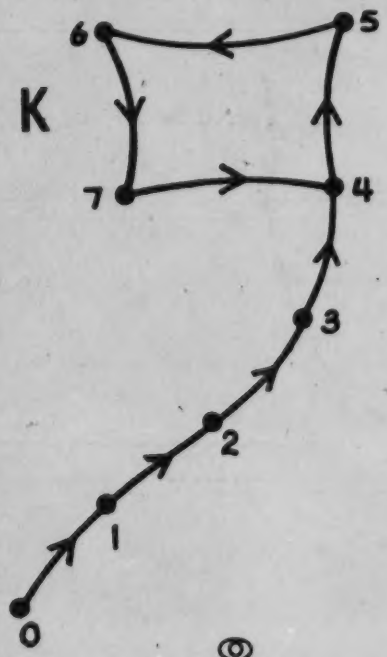
K^6

0	1	2	3	4	5	6	7



K^7

0	1	2	3	4	5	6	7



K^8

0	1	2	3	4	5	6	7

K^9

0	1	2	3	4	5	6	7

Name _____ **L 14 ***

Complete:

$59 +_{80} 36 =$ _____

$34 +_{80} 46 +_{80} 62 =$ _____

$47 +_{80} 26 +_{80} 72 =$ _____

$10 \times_{80} 42 =$ _____

$3 \times_{80} 40 \times_{80} 5 =$ _____

$4 \times_{80} 40 \times_{80} 78 =$ _____

The number in the box could be:

$\square +_{80} 52 = 15$ _____.

$4 \times_{80} \square = 40$ _____ or _____ or _____ or _____.

$\square \times_{80} 5 = 60$ _____ or _____ or _____ or _____ or _____.

Name _____ **L 15** **

Complete:

$3^1 = 3$

$3^2 = 3 \times_{80} 3 =$ _____

$3^3 = 3 \times_{80} 3 \times_{80} 3 =$ _____

$3^4 = 3 \times_{80} 3 \times_{80} 3 \times_{80} 3 =$ _____

$3^5 =$ _____

$3^6 =$ _____

$3^7 =$ _____

$3^8 =$ _____

$3^9 =$ _____

If you see a pattern, find

$3^{43} =$ _____.

(Continue on the back.)

Complete:

$5^1 = 5$

$5^2 = 5 \times_{80} 5 =$ _____

$5^3 = 5 \times_{80} 5 \times_{80} 5 =$ _____

$5^4 = 5 \times_{80} 5 \times_{80} 5 \times_{80} 5 =$ _____

$5^5 =$ _____

$5^6 =$ _____

$5^7 =$ _____

$5^8 =$ _____

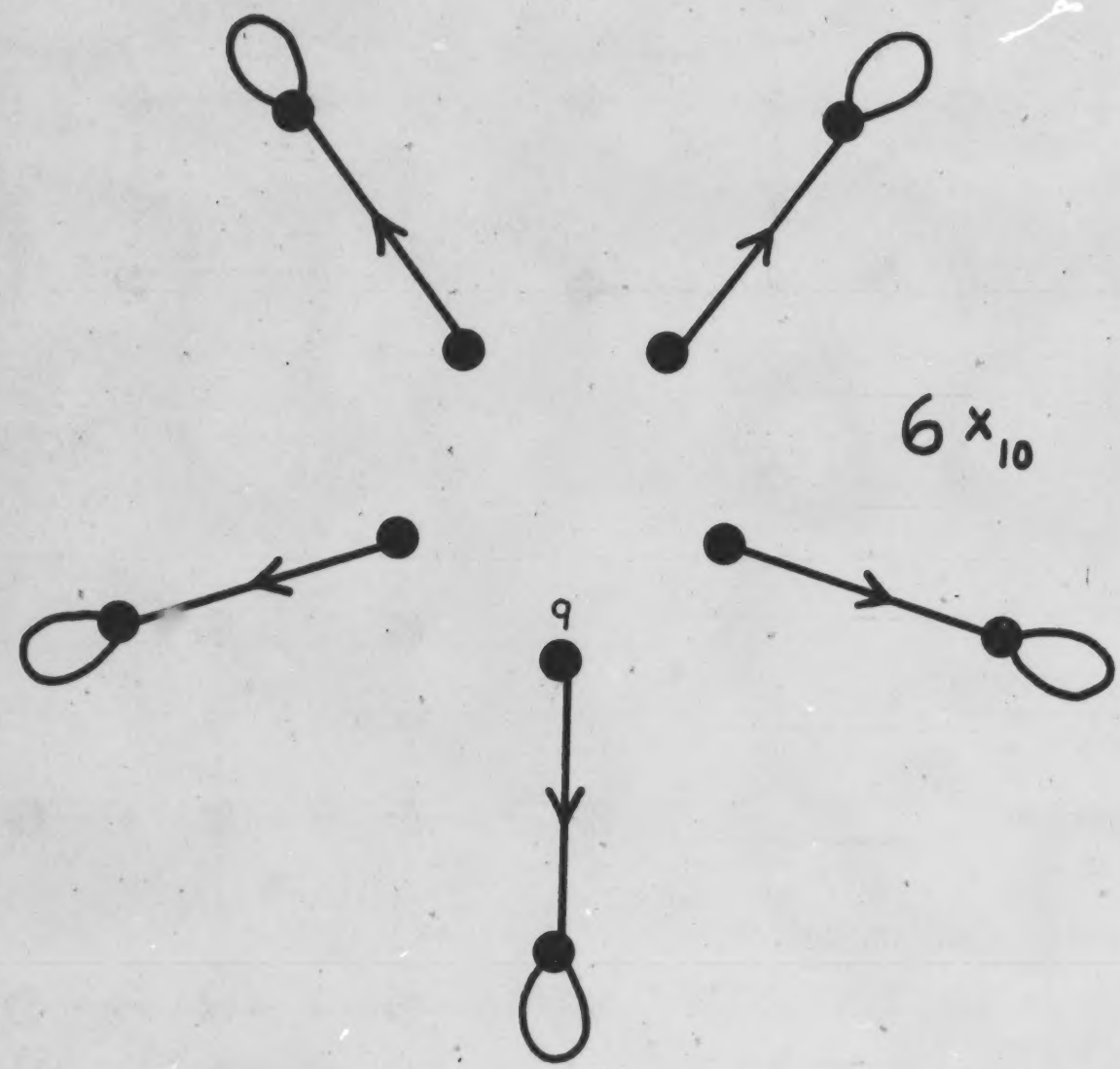
$5^9 =$ _____

If you see a pattern, find

$5^{57} =$ _____.

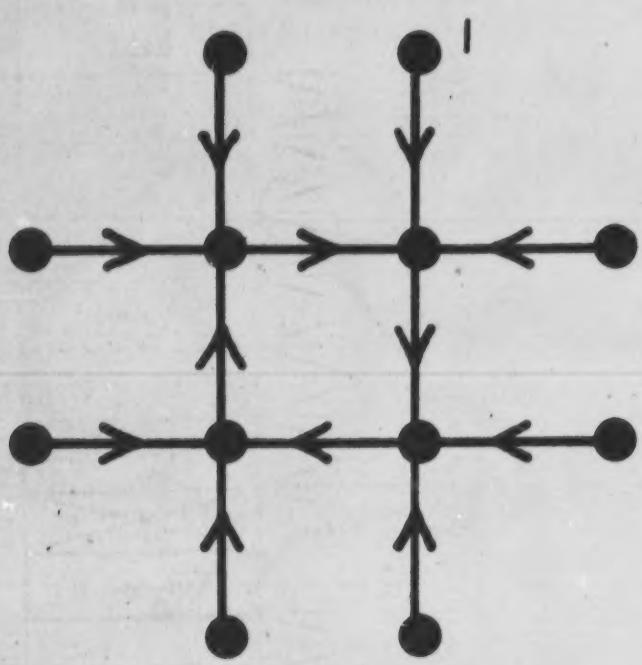
Name _____ **L 16** *

Put the numbers 0, 1, 2, 3, 4, 5, 6, 7, and 8 on this picture.

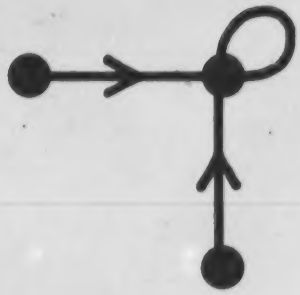


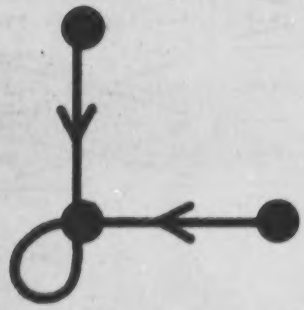
Name _____ **L17** **

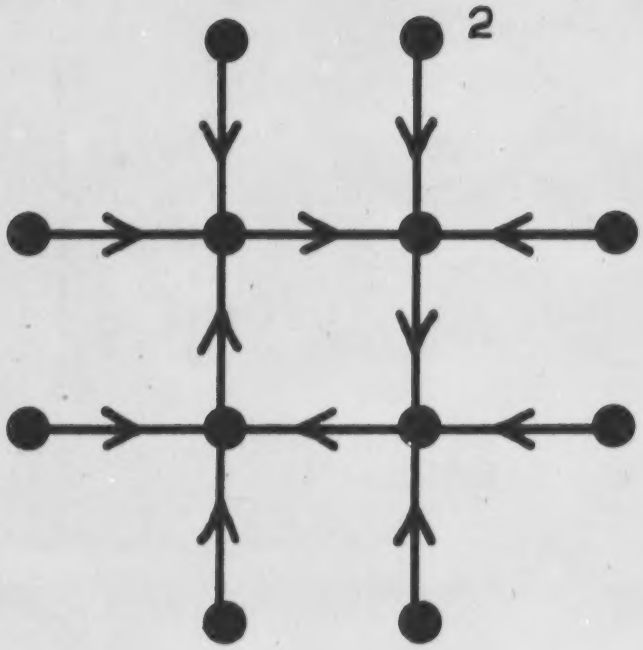
Put the numbers from 0 to 29 on this picture.



$3 \times_{30}$

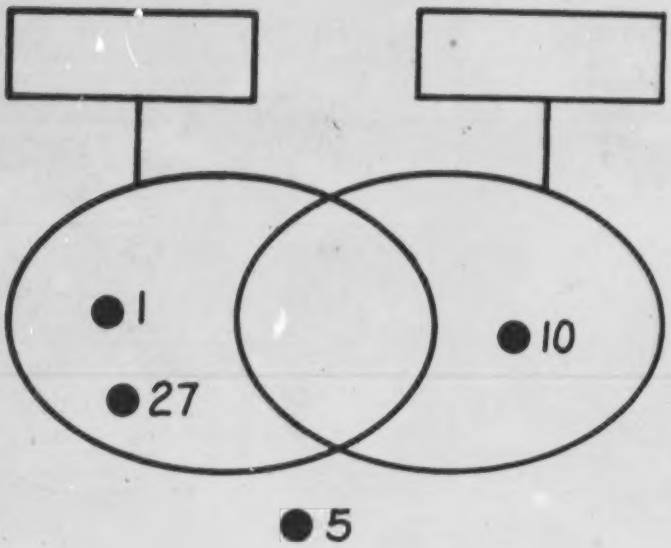






Name _____ **L 18** *

Cross out the labels that the strings <u>cannot</u> have. Some are done for you.



RED	BLUE
~~Multiples of 2~~	Multiples of 2
~~Multiples of 3~~	Multiples of 3
~~Multiples of 4~~	~~Multiples of 4~~
~~Multiples of 5~~	Multiples of 5
~~Multiples of 10~~	Multiples of 10
~~Odd Numbers~~	Odd Numbers
~~Positive Prime Numbers~~	~~Positive Prime Numbers~~
Larger than 50	Larger than 50
~~Smaller than 50~~	Smaller than 50
Larger than ~10	Larger than ~10
Smaller than ~10	Smaller than ~10
~~Positive Divisors of 12~~	~~Positive Divisors of 12~~
Positive Divisors of 18	~~Positive Divisors of 18~~
~~Positive Divisors of 20~~	Positive Divisors of 20
Positive Divisors of 24	~~Positive Divisors of 24~~
Positive Divisors of 27	~~Positive Divisors of 27~~

The label for the red string is [ ].

The label for the blue string could be

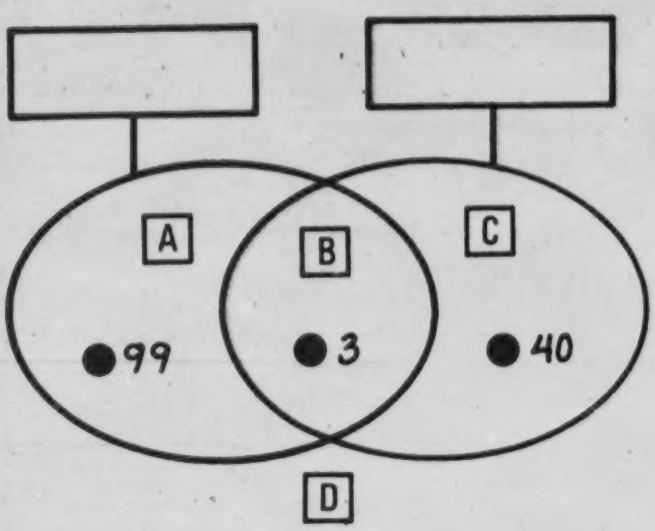
[ ] or [ ].

Circle one of the following: T (True)
F (False)
CT (Can't Tell)

1) 9 is in the red string. T F CT
2) 7 is in the blue string. T F CT
3) 99 is outside both strings. T F CT
4) 2 is in the blue string. T F CT
5) 3 is in both strings. T F CT
6) 4 is outside both strings. T F CT

Name _____ **L 19** **

Cross out the labels the strings cannot have. Some are done for you.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	~~Multiples of 3~~
Multiples of 4	~~Multiples of 4~~
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
~~Positive Prime Numbers~~	~~Positive Prime Numbers~~
Larger than 50	Larger than 50
~~Smaller than 50~~	Smaller than 50
~~Larger than ¯10~~	~~Larger than ¯10~~
Smaller than ¯10	Smaller than ¯10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

The label for the blue string is _____.

The label for the red string could be

_____ or _____.

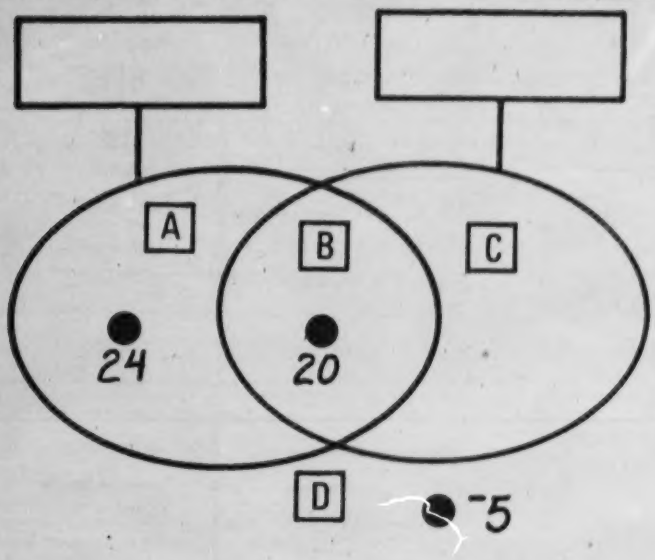
In the string picture, four regions are labeled: A, B, C, D.

Circle one of the following: T (True)
F (False)
CT (Can't Tell)

1) 20 is in region C. T F CT
2) 2 is in region B. T F CT
3) 105 is in region B. T F CT
4) 105 is in region A. T F CT
5) 7 is in region B. T F CT
6) 7 is in region D. T F CT
7) 100 is in region D. T F CT
8) 60 is in region A. T F CT

Name _____ **L 20** ***

Cross out the labels that the strings cannot have.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50
Smaller than 50	Smaller than 50
Larger than ¯10	Larger than ¯10
Smaller than ¯10	Smaller than ¯10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

The label for the red string could be

_____ or _____.

The label for the blue string could be

_____ or _____.

In the string picture, four regions are labeled: A, B, C, D.

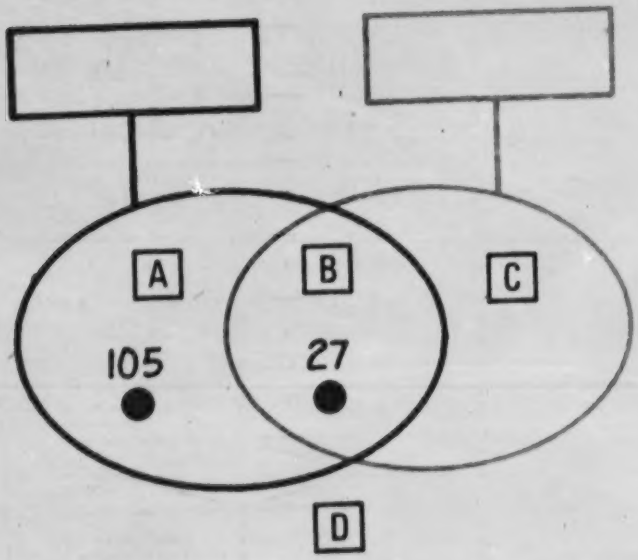
Circle one of the following: T (True)
F (False)
CT (Can't Tell)

1) 4 is in region C. T F CT
2) 2 is in region D. T F CT
3) 10 is in region A. T F CT
4) 8 is in region A. T F CT
5) 10 is in region C. T F CT
6) 100 is in region B. T F CT
7) 0 is in region D. T F CT
8) 1 is in region C. T F CT
9) ¯1 is in region D. T F CT
10) 50 is in region C. T F CT

Name _____

L 21 ****

Cross out the labels that the strings cannot have.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50
Smaller than 50	Smaller than 50
Larger than ¯10	Larger than ¯10
Smaller than ¯10	Smaller than ¯10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

In this string picture, four regions are labeled:

A, B, C, D.

Circle one of the following: T (True)
F (False)
CT (Can't Tell)

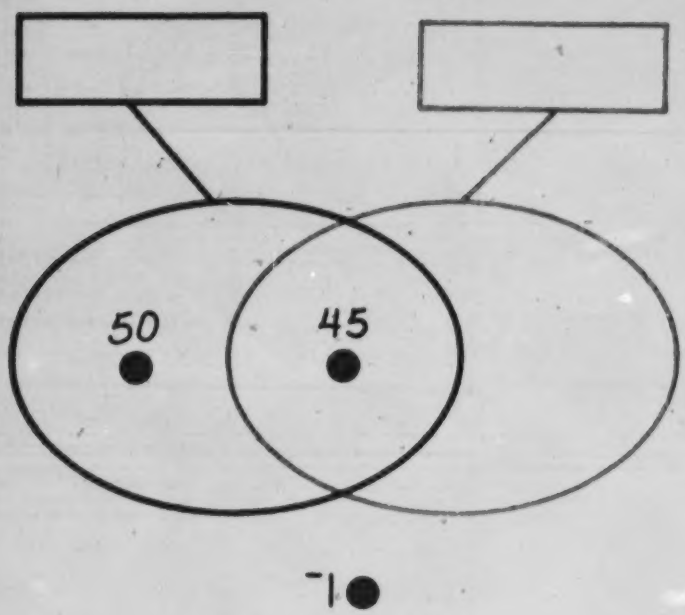
1) 45 is in region C. T F CT
2) 45 is in region A. T F CT
3) ¯15 is in region B. T F CT
4) 9 is in region B. T F CT
5) 3 is in region C. T F CT
6) 1 is in region A. T F CT
7) ¯80 is in region D. T F CT
8) ¯100 is in region A. T F CT
9) 99 is in region A. T F CT
10) 60 is in region D. T F CT

Name _____

L 22 *

Cross out the labels that the strings <u>cannot</u> have. Some are done for you.

Complete the tags.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
~~Odd Numbers~~	~~Odd Numbers~~
Positive Prime Numbers	Positive Prime Numbers
~~Larger than 50~~	~~Larger than 50~~
~~Smaller than 50~~	~~Smaller than 50~~
~~Larger than ⁻10~~	~~Larger than ⁻10~~
Smaller than ⁻10	Smaller than ⁻10
Positive Divisors of 12	Positive Divisors of 12
~~Positive Divisors of 18~~	~~Positive Divisors of 18~~
~~Positive Divisors of 20~~	~~Positive Divisors of 20~~
~~Positive Divisors of 24~~	~~Positive Divisors of 24~~
~~Positive Divisors of 27~~	~~Positive Divisors of 27~~

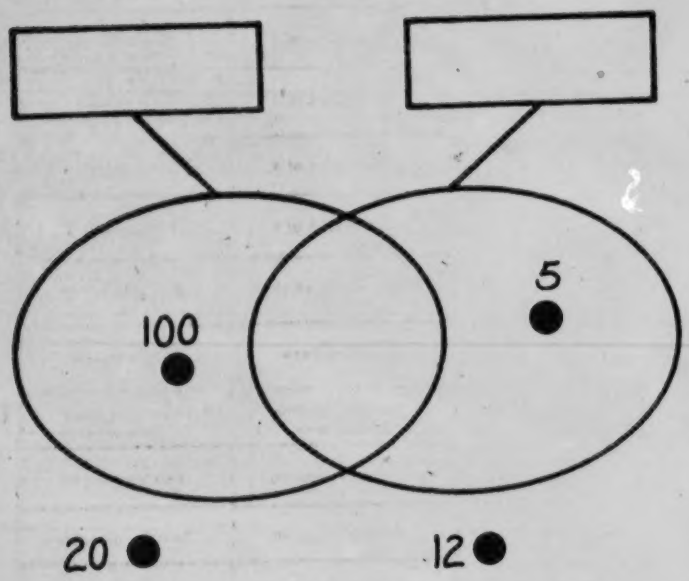
Put these numbers in the string picture.

⁻55; ⁻15; 0; 6; 7; 8; 20; 27; 99; 105

Name _____

L 23 **

Cross out the labels that the strings cannot have. Some are done for you.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50
Smaller than 50	Smaller than 50
Larger than ⁻10	Larger than ⁻10
Smaller than ⁻10	Smaller than ⁻10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

The label for the red string is _____.

The label for the blue string could be

_____ or _____.

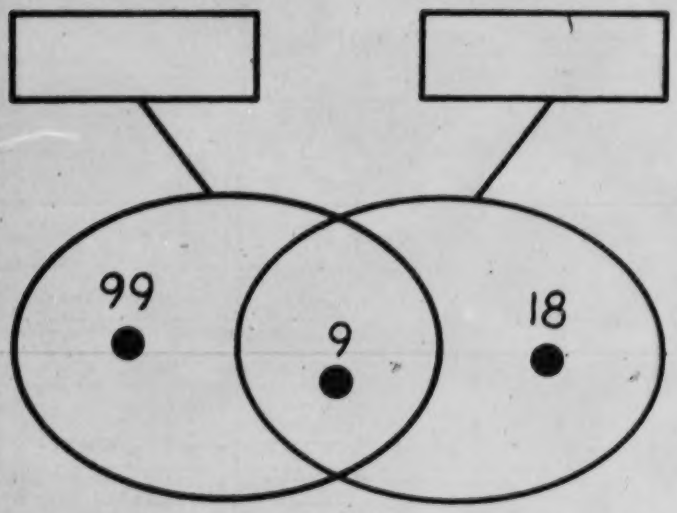
Exactly three of the numbers below cannot be put in the string picture because the blue string is not known. Circle these four numbers. Put all the other numbers in the string picture.

⁻80; ⁻15; 2; 3; 7; 24; 50; 60; 99; 105

Name _____

L 24 ***

Cross out the labels that the strings cannot have.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50
Smaller than 50	Smaller than 50
Larger than ⁻10	Larger than ⁻10
Smaller than ⁻10	Smaller than ⁻10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

The label for the red string is _____.

The label for the blue string could be

_____ or _____.

It is your turn in the String Game.
You want to find the blue string.

1) You can find the label for the blue string by playing exactly one of these numbers, even if you get a NO answer. Circle the number you should play.

3; 105; 60; 7; 2.

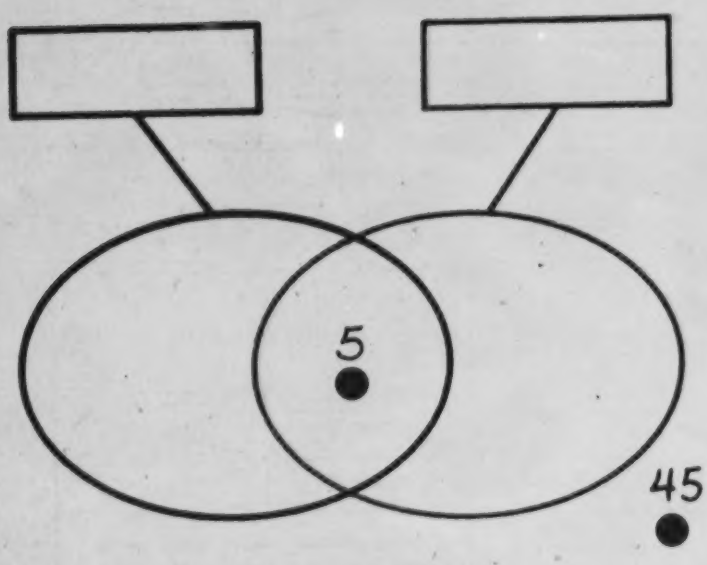
2) Repeat problem (1) but with these numbers.

20; 100; 6; 55; 1.

Name _____

L 25 ****

Cross out the labels that the strings cannot have.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50
Smaller than 50	Smaller than 50
Larger than ⁻10	Larger than ⁻10
Smaller than ⁻10	Smaller than ⁻10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

The label for the red string could be

_____ or _____.

The label for the blue string could be

_____ or _____.

Some of these numbers cannot be put in the string picture because the string labels are not known. Circle them. Put the others in the string picture.

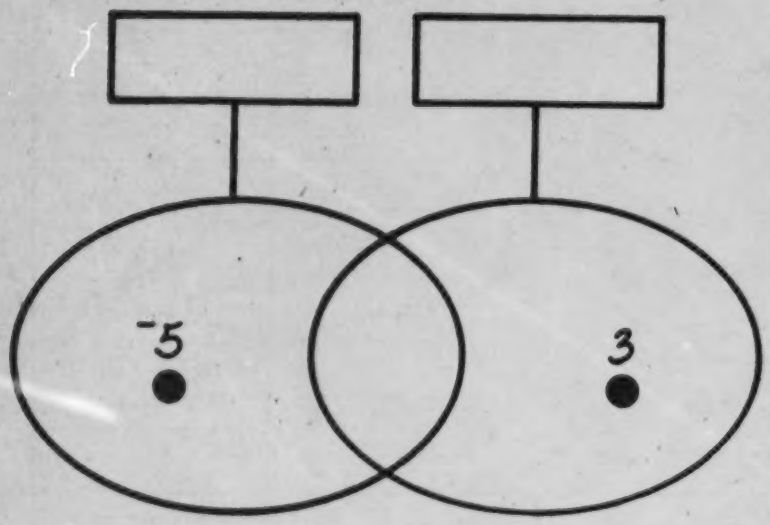
⁻55; ⁻15; 2; 3; 4; 8; 9; 10; 24; 105.

It is your turn in the String Game. By playing exactly one of these numbers you can find both of the string labels, even if you get a NO answer. Circle the number you should play.

20; 100; 6; 55; ⁻1.

Name _____ **L 26** *

Cross out the labels that the strings cannot have. Some are done for you.



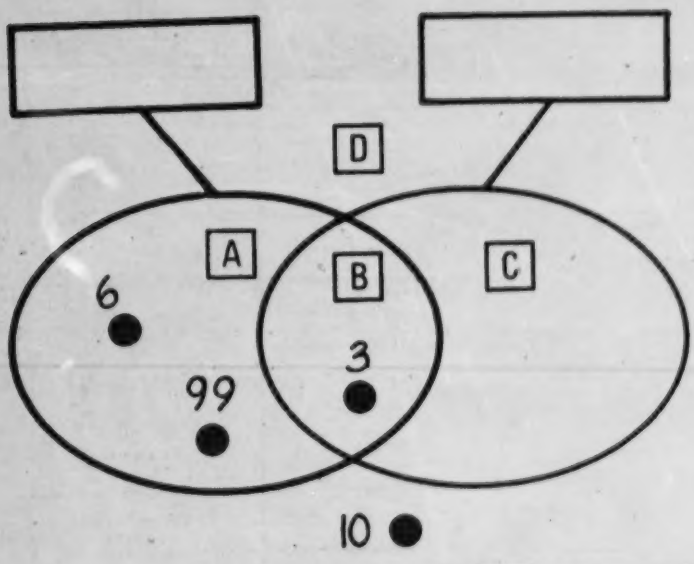
RED	BLUE
~~Multiples of 2~~	~~Multiples of 2~~
Multiples of 3	Multiples of 3
~~Multiples of 4~~	~~Multiples of 4~~
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers
~~Larger than 50~~	~~Larger than 50~~
~~Smaller than 50~~	~~Smaller than 50~~
Larger than ⁻10	Larger than ⁻10
Smaller than ⁻10	Smaller than ⁻10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
~~Positive Divisors of 20~~	~~Positive Divisors of 20~~
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

Circle one of the following: T (True)
F (False)
CT (Can't Tell)

1. The red string is for the numbers smaller than ⁻10. T F CT
2. The blue string is for the positive prime numbers. T F CT
3. The red string is for the odd numbers. T F CT
4. The blue string is for the odd numbers. T F CT
5. The red string is for the multiples of 5. T F CT
6. The blue string is for the positive divisors of 27. T F CT

Name _____ **L 27** **

Cross out the labels that the strings cannot have. Some are done for you.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	~~Multiples of 3~~
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
~~Odd Numbers~~	~~Odd Numbers~~
Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50
Smaller than 50	Smaller than 50
~~Larger than ⁻10~~	~~Larger than ⁻10~~
Smaller than ⁻10	Smaller than ⁻10
Positive Divisors of 12	Positive Divisors of 12
~~Positive Divisors of 18~~	~~Positive Divisors of 18~~
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
~~Positive Divisors of 27~~	Positive Divisors of 27

The label for the red string is _____.

The label for the blue string could be

_____ or _____.

In the string picture, four regions are labeled:

A, B, C, D.

Circle one of the following: T (True)
F (False)
CT (Can't Tell)

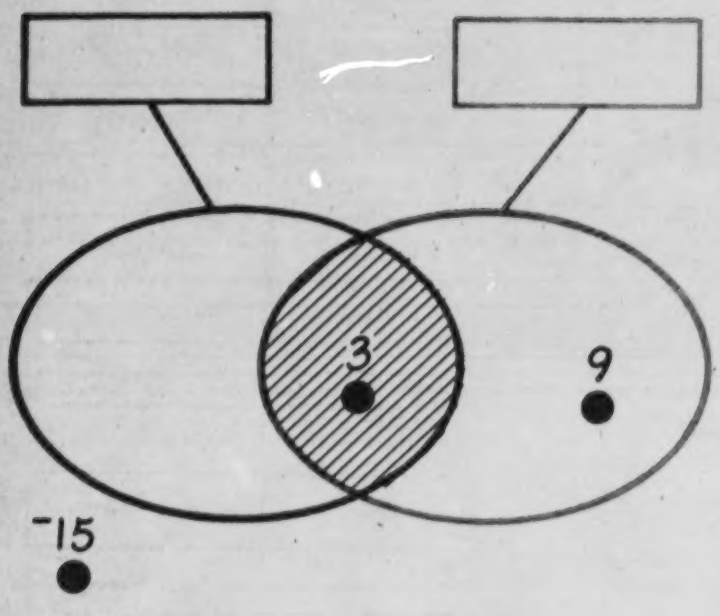
1) ⁻15 is in region A. T F CT
2) 1 is in region C. T F CT
3) 0 is in region B. T F CT
4) 20 is in region A. T F CT
5) 20 is in region D. T F CT
6) 9 is in region B. T F CT
7) 105 is in region D. T F CT
8) 27 is in region A. T F CT

Name _____

L 28 ***

Cross out the labels that the strings cannot have.
The hatching is also a clue.

Complete the tags.



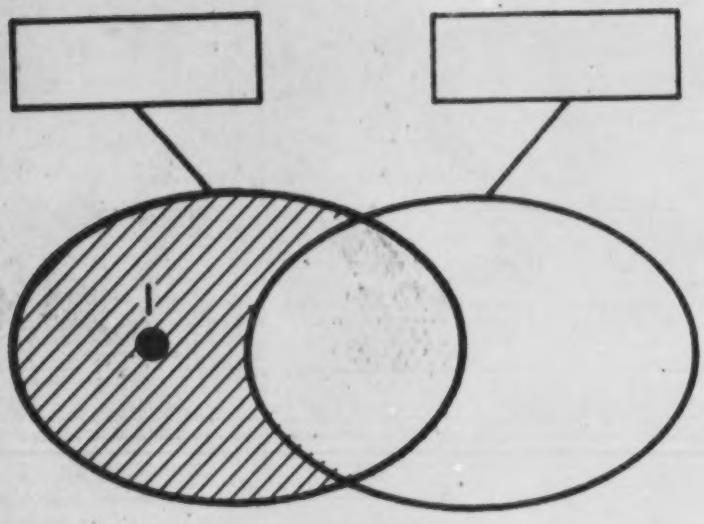
RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50
Smaller than 50	Smaller than 50
Larger than -10	Larger than -10
Smaller than -10	Smaller than -10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

Name _____

L 29 **

Cross out the labels that the strings cannot have.
The hatching is also a clue.

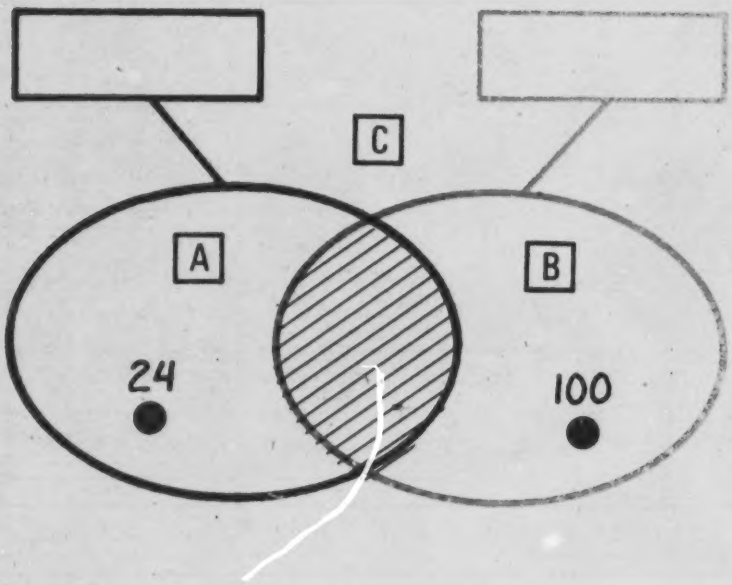
Complete the tags.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50
Smaller than 50	Smaller than 50
Larger than ¯10	Larger than ¯10
Smaller than ¯10	Smaller than ¯10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

Name _____ **L 30 *******

Cross out the labels that the strings <u>cannot</u> have.
The hatching is also a clue.



RED	BLUE
Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50
Smaller than 50	Smaller than 50
Larger than ¯10	Larger than ¯10
Smaller than ¯10	Smaller than ¯10
Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of 18	Positive Divisors of 18
Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27

In this string picture, three regions are labeled:

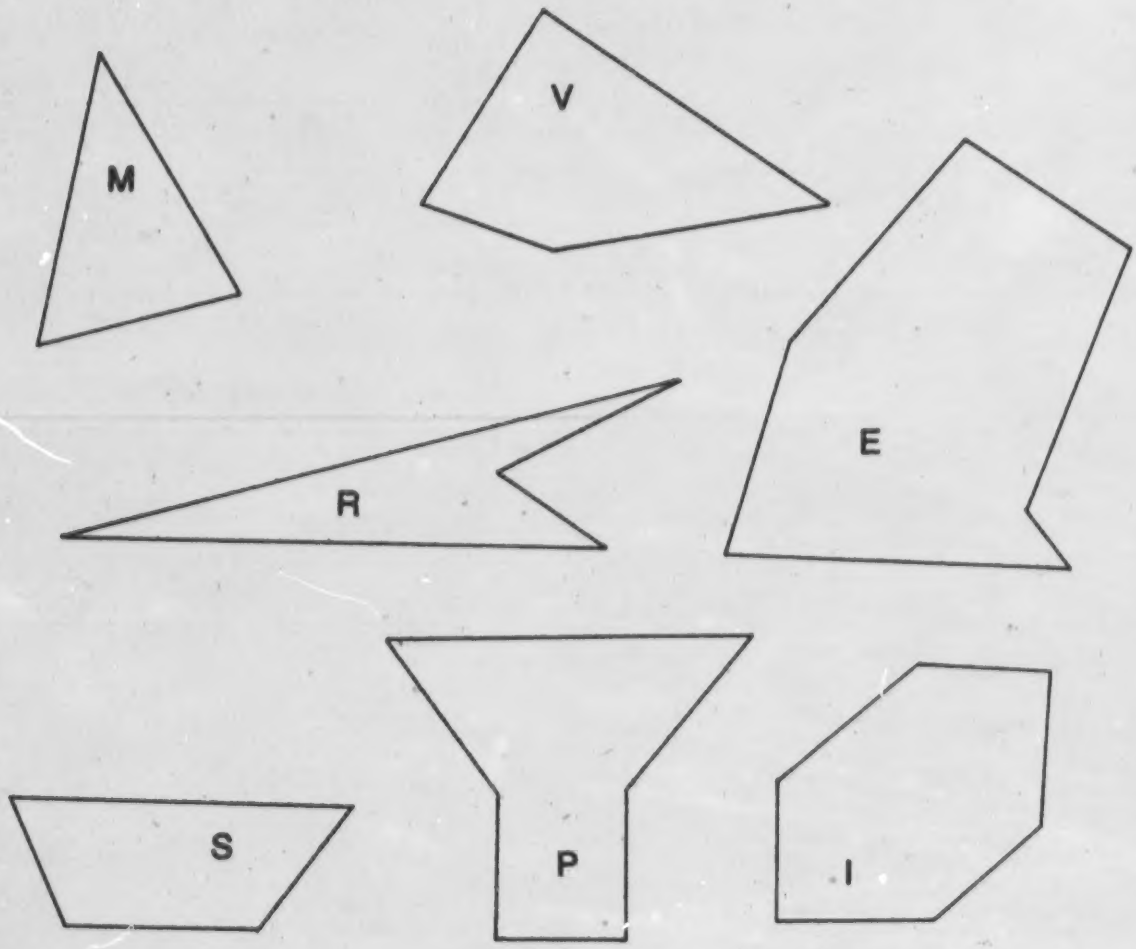
A, B, C.

Circle one of the following: T (True)
F (False)
CT (Can't Tell)

1) 60 is in region A. T F CT
2) 6 is in region A. T F CT
3) 60 is in region B. T F CT
4) ¯100 is in region B. T F CT
5) 12 is in region C. T F CT
6) 0 is in region C. T F CT
7) 20 is in region A. T F CT
8) 20 is in region B. T F CT
9) 1 is in region A. T F CT
10) 27 is in region B. T F CT

Name _____ **G01** *

Measure the sides of each shape to the nearest centimeter.
Write the length of each side next to it.
Inside each shape, write the total of the lengths of its sides.



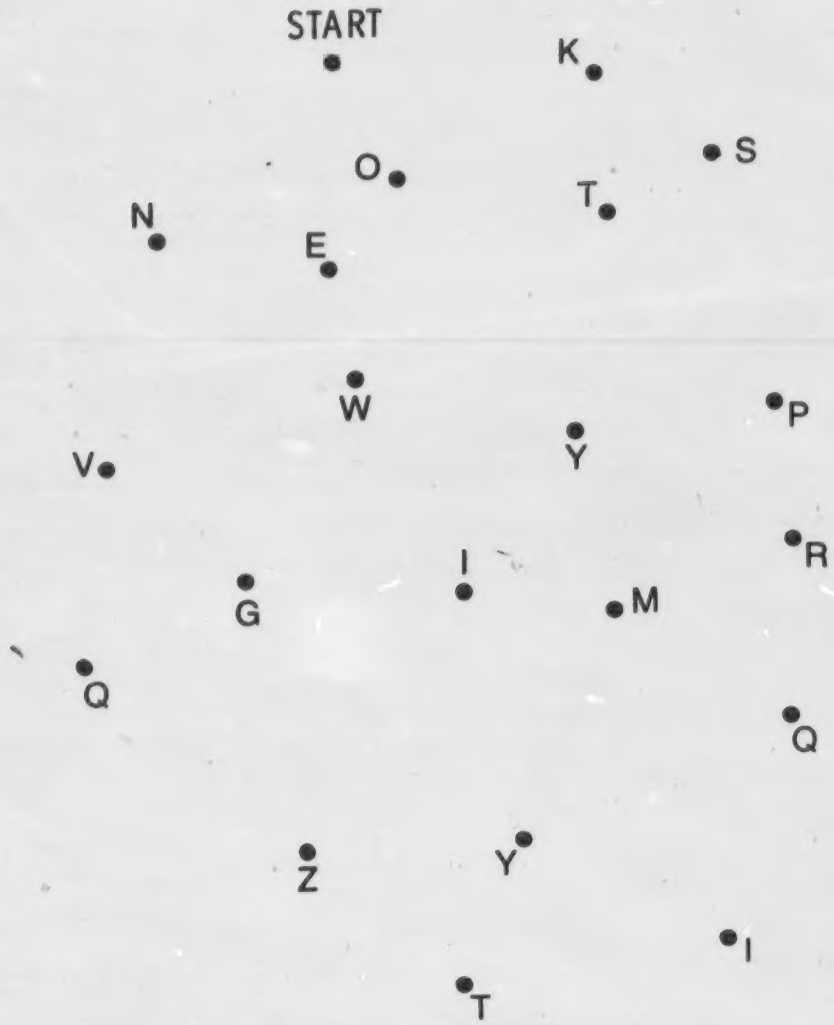
The letter of a shape is matched to the number inside the shape.
Use the matching to decode this message.

$\overline{11}$ $\overline{13}$ $\overline{12}$ $\overline{12}$ $\overline{13}$ $\overline{12}$ $\overline{12}$ $\overline{13}$ $\overline{17}$ $\overline{17}$ $\overline{13}$ $\overline{22}$ $\overline{13}$ $\overline{14}$ $\overline{20}$ $\overline{22}$

Name _____ **G 02** **

This is a treasure map. Start at the dot marked START and draw a zigzag. Use the list of lengths for the segments of the zigzag. The second dot is 5.7 cm from the dot marked START. Write the letters from the dots in order below to find the name of the treasure.

5.7 cm
3.8 cm
2.0 cm
4.9 cm
6.8 cm
5.1 cm
6.7 cm
3.9 cm
5.1 cm
6.5 cm



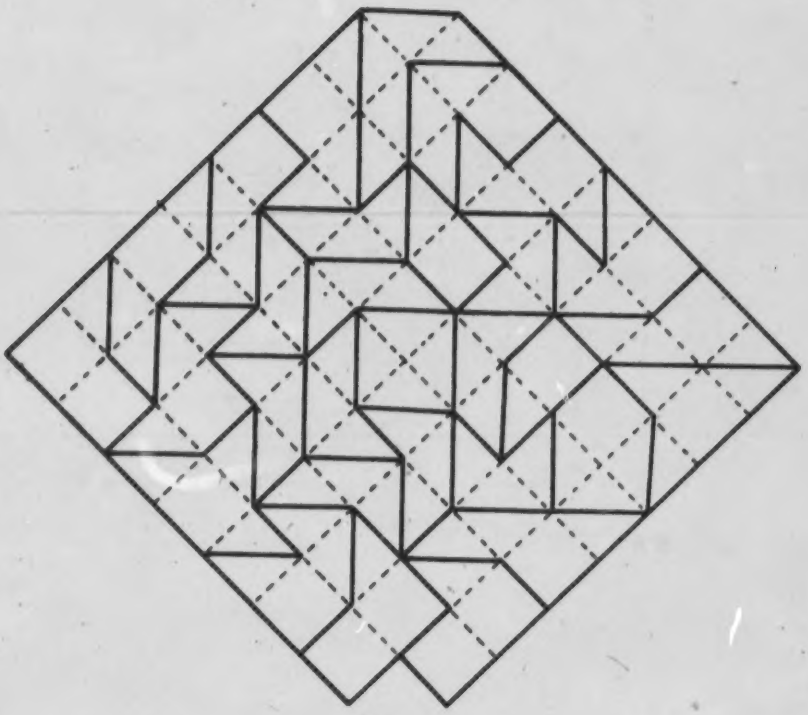
_____ _____ _____ _____ _____ _____ _____ _____ _____ _____

Name _____

G 03 *

Color all shapes of area $1\frac{1}{2}$ cm^2 with the same color.

Use another color to color all shapes of area $2\frac{1}{2}$ cm^2

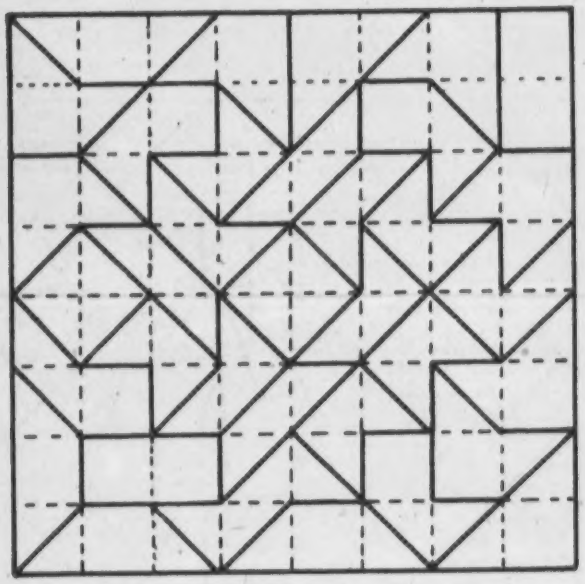


Name _____

G 04 **

Color all shapes of area 2 cm^2 the same color.

Use another color to color all shapes of area $1\frac{1}{2}$ cm^2.



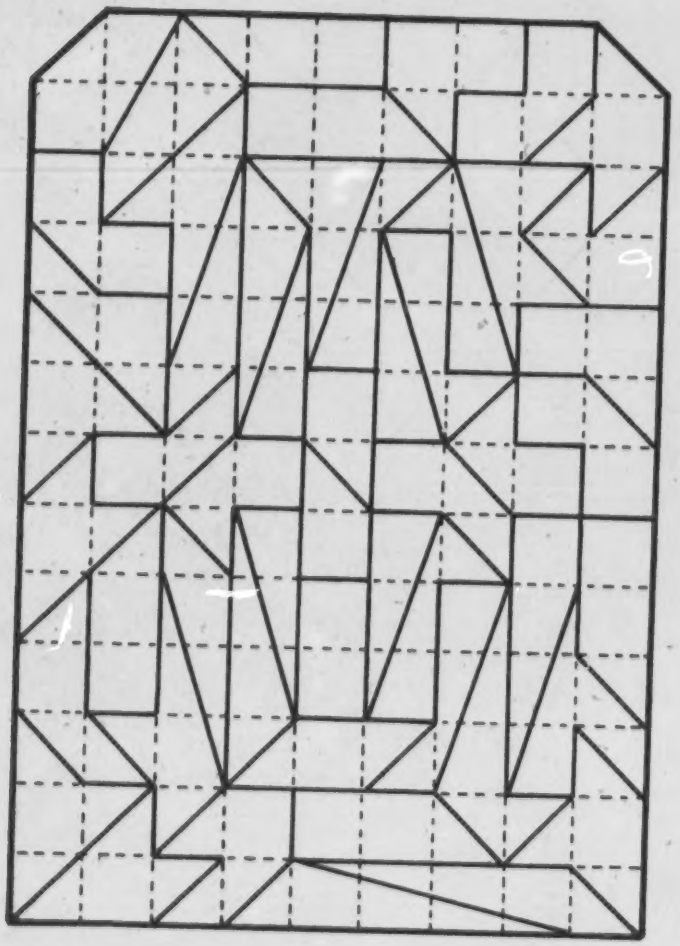
Name _____

G 05 ***

Use one color to color all shapes of area $1\frac{1}{2}$ cm^2.

Use another color to color all shapes of area 2 cm^2.

Use a third color to color all shapes of area $2\frac{1}{2}$ cm^2.



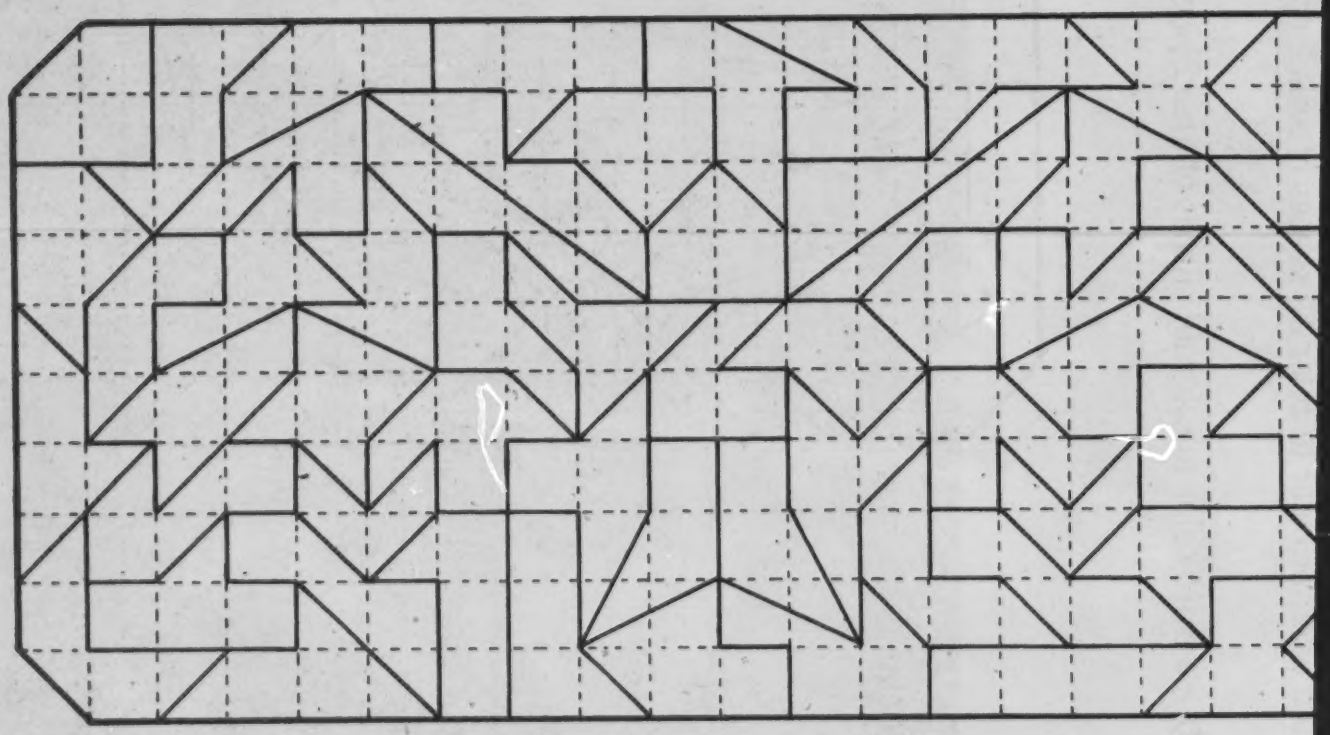
Name _____

G 06 ****

Use one color to color all shapes of area 2 cm^2.

Use another color to color all shapes of area 3 cm^2.

Use a third color to color all shapes of area $3\frac{1}{2}$ cm^2.



Name _____ **G07** *

Draw a two piece zigzag starting in one circle and ending in the other. Make one piece 5 cm long and the other 7 cm long.

Draw a three piece zigzag starting in one circle and ending in the other. The lengths of the pieces must be 2 cm, 4 cm, and 8 cm.

Draw a three piece zigzag starting in one circle and ending in the other. The lengths of the pieces must be 4.8 cm, 5.9 cm, and 6.2 cm.

Name _____

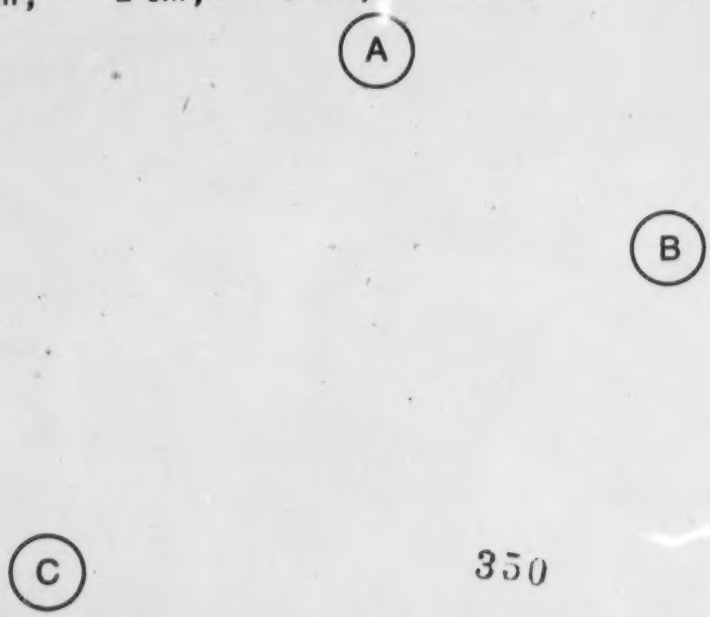
G 08 **

Draw a zigzag from circle A to circle B to circle C. Use each of these lengths exactly once:

5 cm, 6 cm, 7 cm, 8 cm.

Draw a zigzag from circle A to circle B to circle C. Use each of these lengths exactly once:

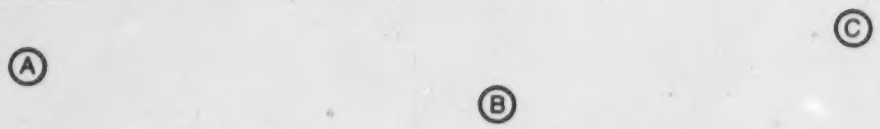
1 cm, 2 cm, 3 cm, 6 cm, 9 cm, 11 cm.



Name _____ **G09** ***

Draw a zigzag from circle A to circle B to circle C. Use each of these lengths exactly once:

4.3 cm, 5.5 cm, 7.2 cm, 8.4 cm.



Draw a zigzag from circle A to circle B to circle C. Use each of these lengths exactly once:

2.5 cm, 3.6 cm, 4.9 cm, 10.4 cm, 11.2 cm.

Name _____

G 10 *

What is the largest possible area ?

perimeter
100 m

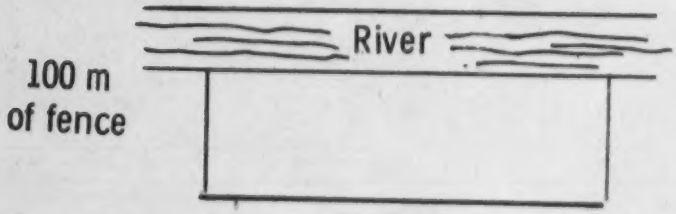
● ●	● ●	Area
20 m	30 m	600 m²

Use the other side to record other solutions.

Name _____

G 11 *

What is the largest possible area ?



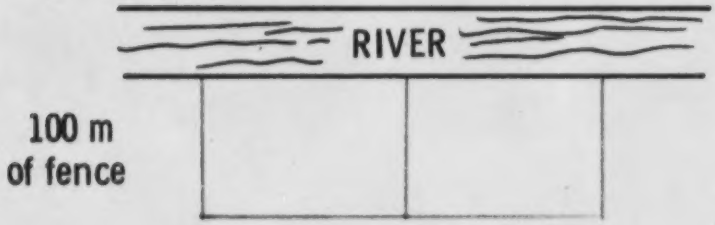
● ●	●	Area
20 m	60 m	1200 m²

Use the other side to record other solutions.

Name _____

G12 *

What is the largest possible area ?



● ● ●	●	TOTAL AREA
15 m	55 m	825 m²

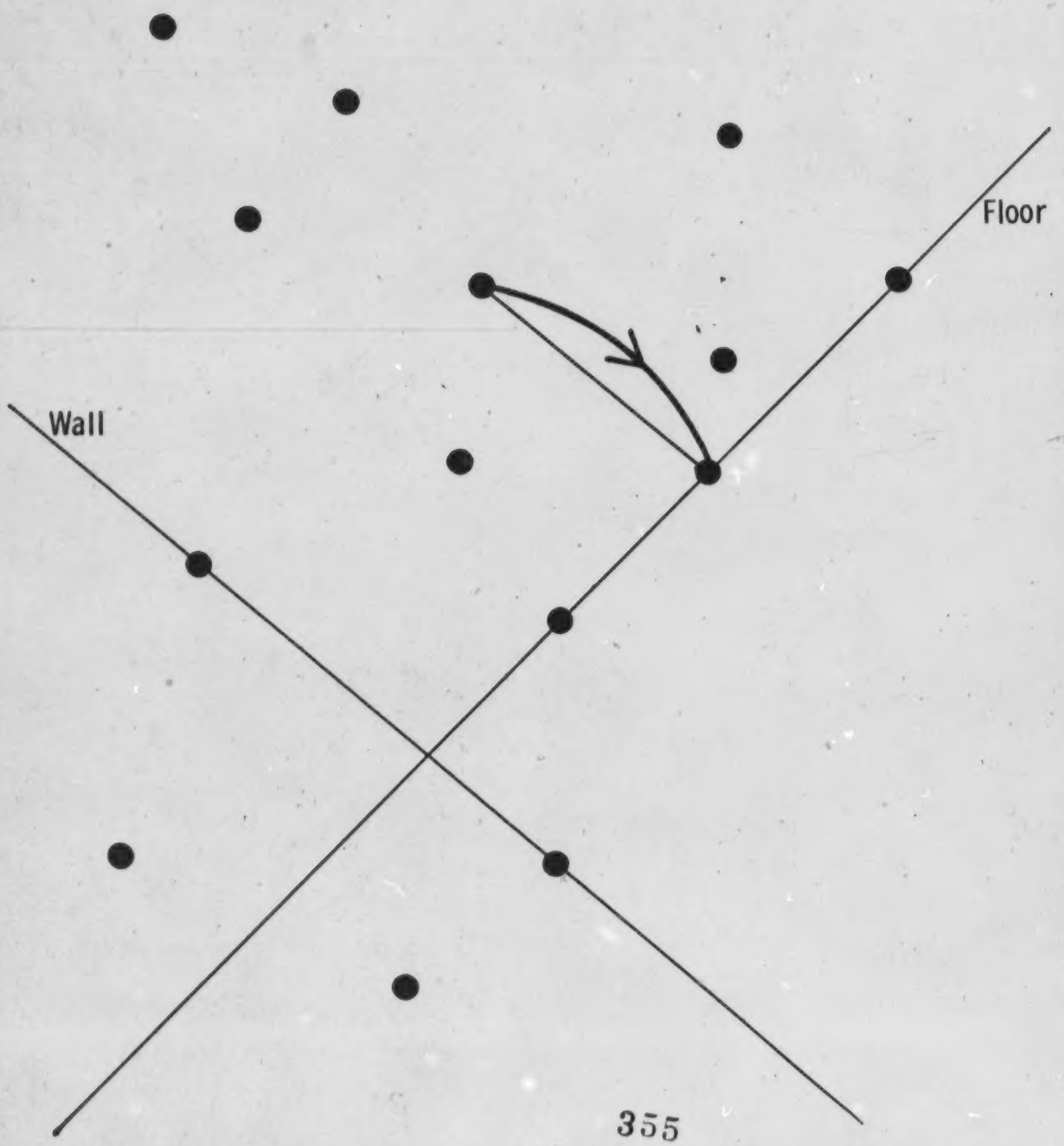
Use the other side to record other solutions.

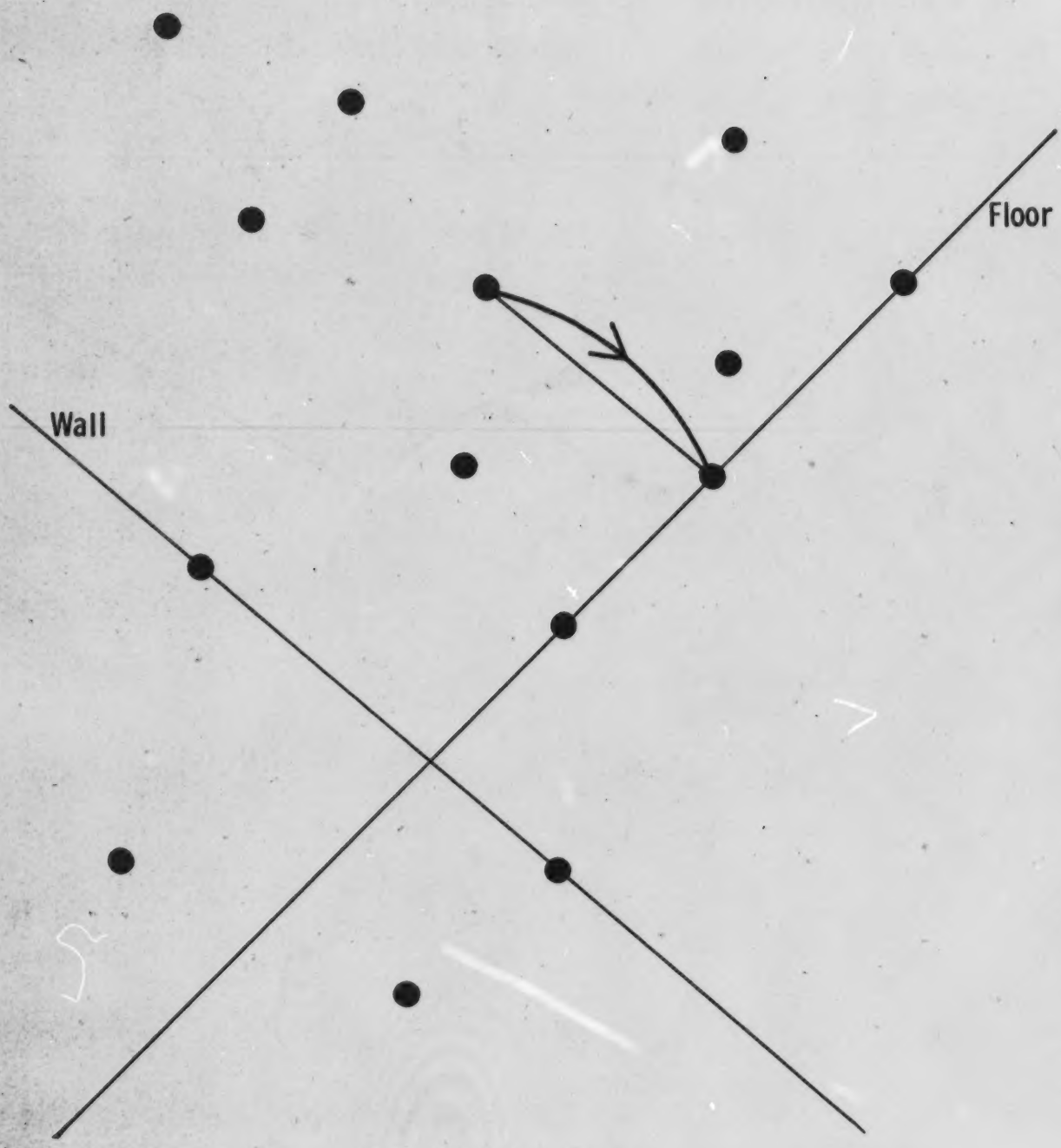
Name _____

G13 *

Project each point onto the floor parallel to the wall.

Draw a red arrow from each point to its image.

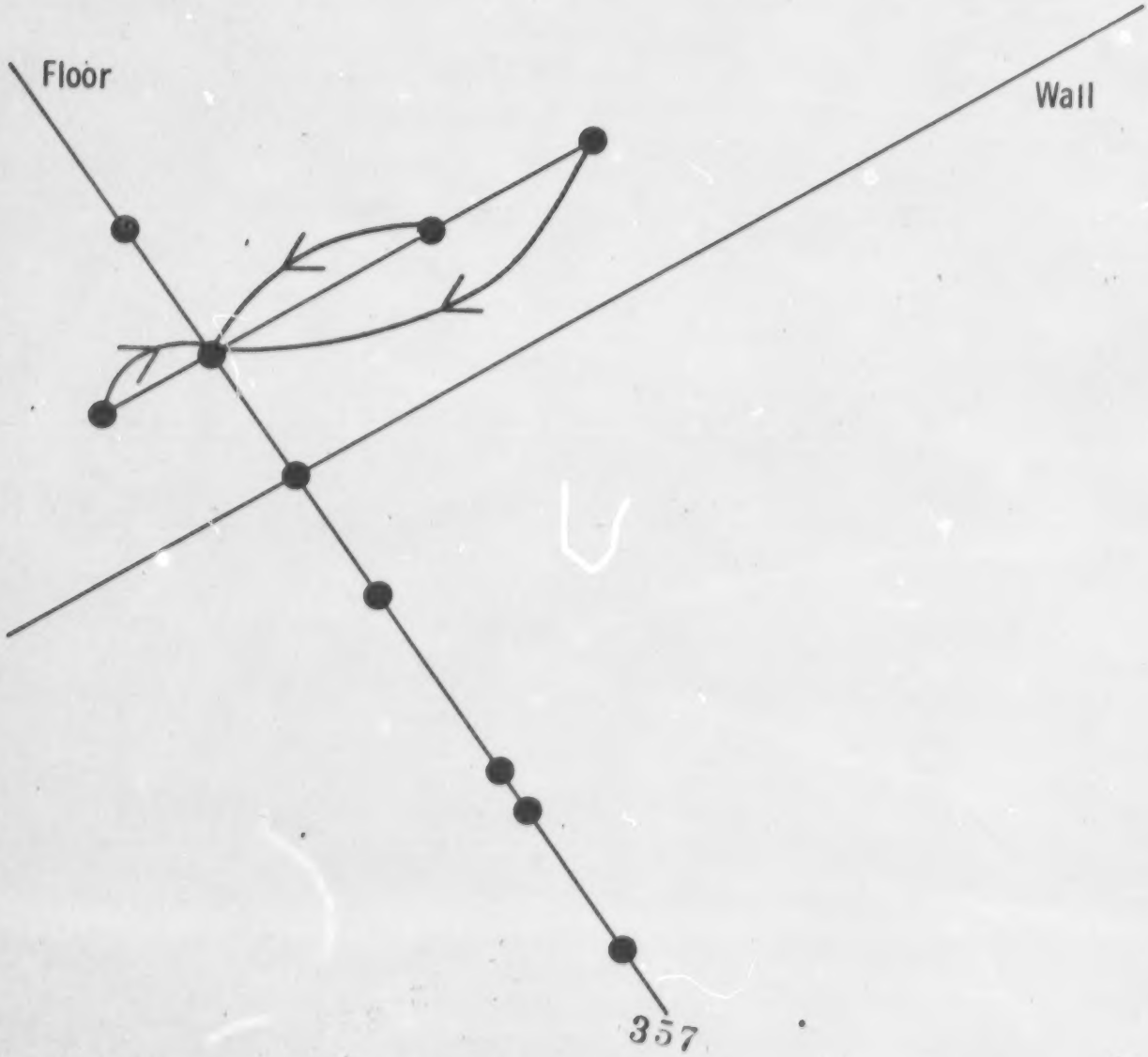


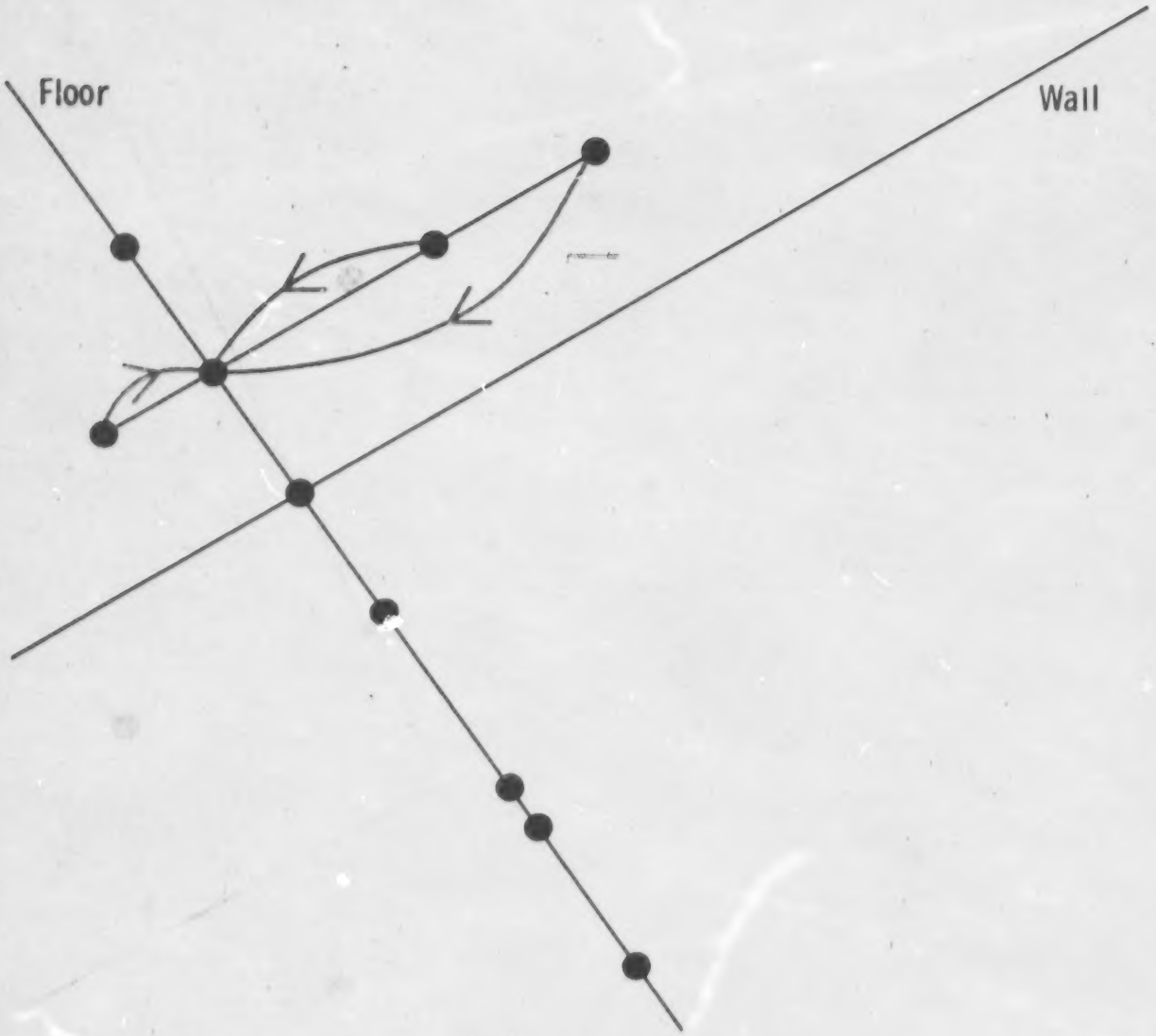

Floor
Wall

Name _____ **G14** **

Each point on the floor is the image of many points when they are projected onto the floor parallel to the wall. An example of three points with the same image point on the floor is shown for you.

For each point on the floor, find many points all having this point as image. Draw a blue arrow from a point to its image.

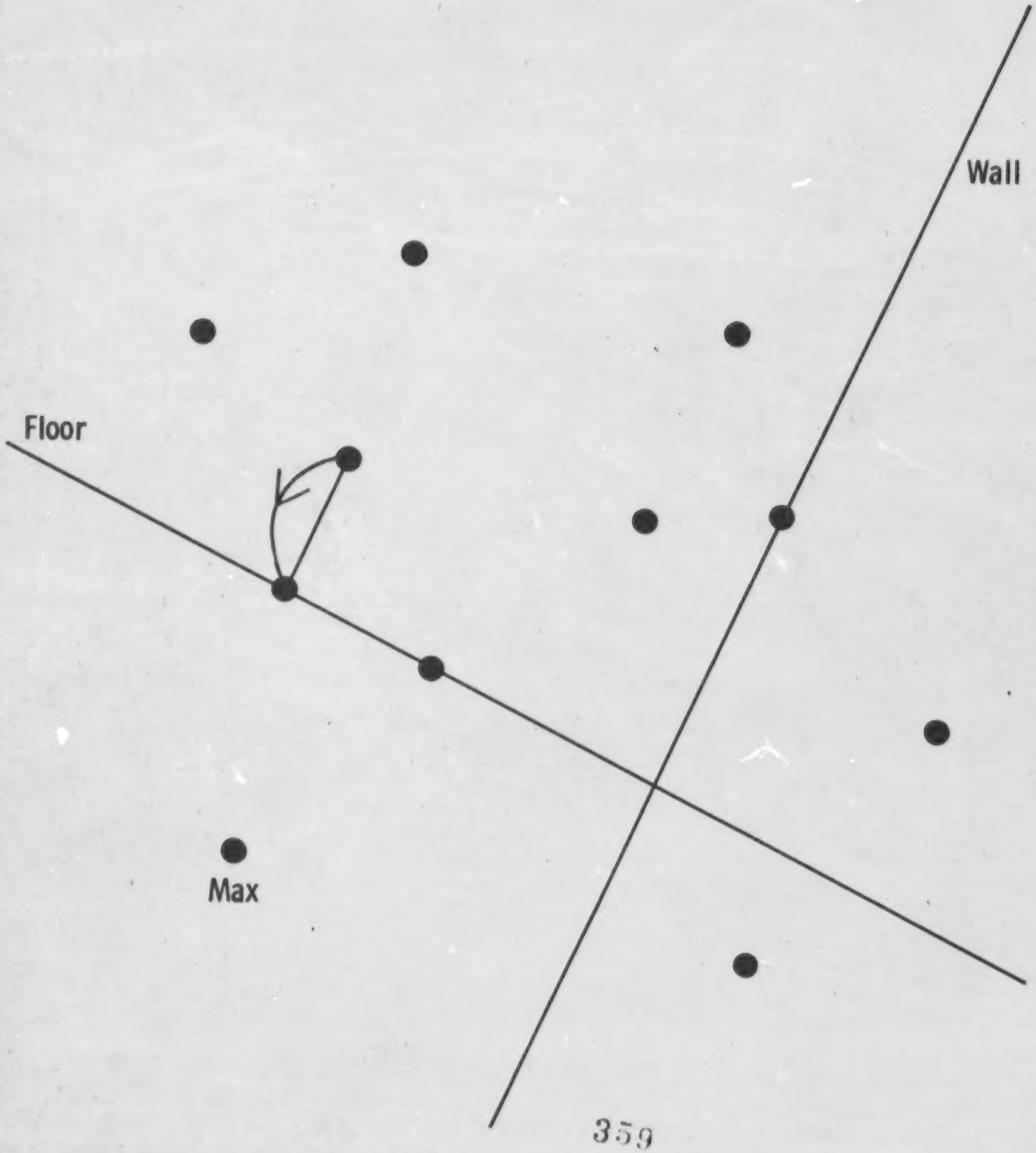


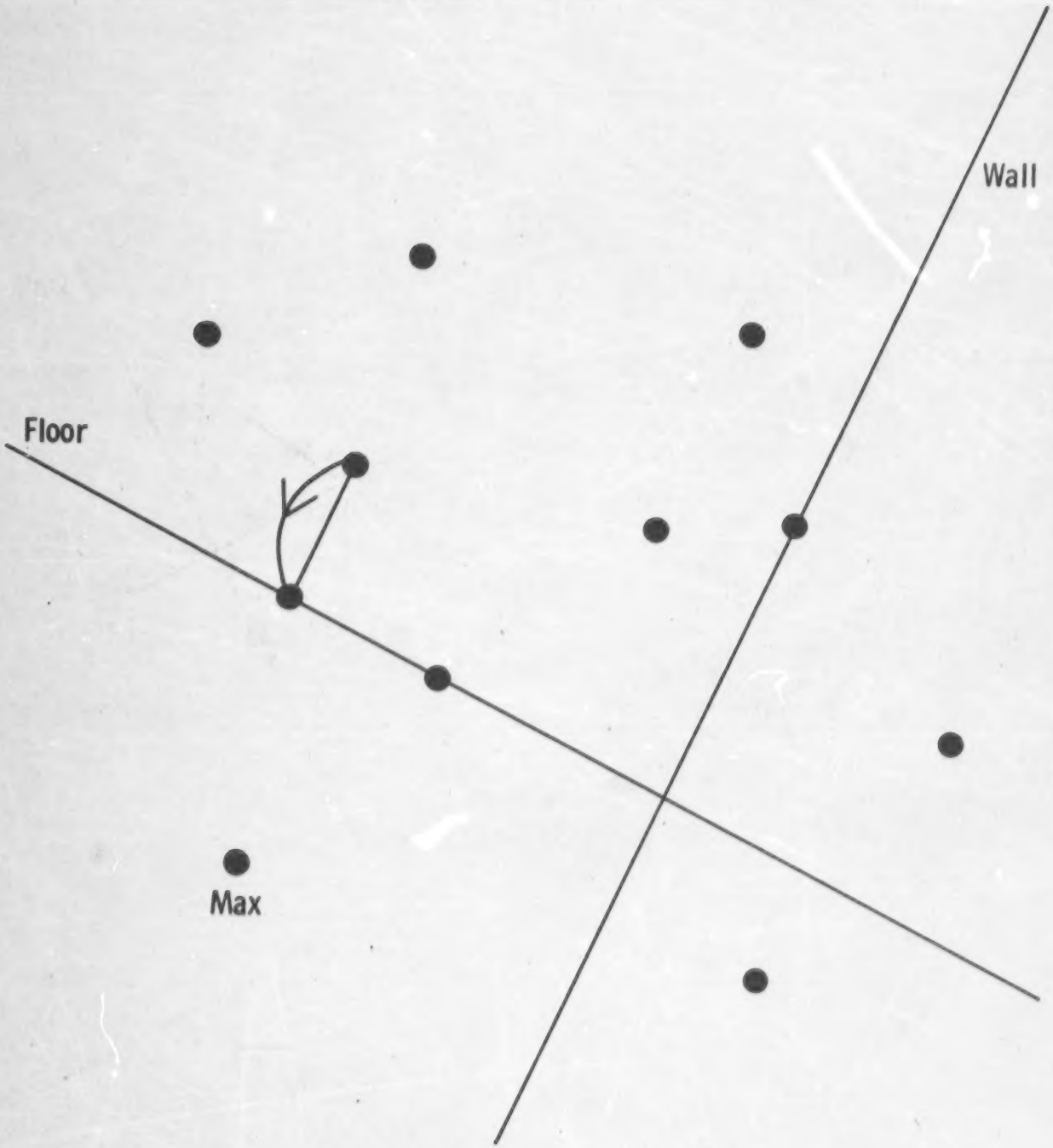

Floor
Wall

Name _____ **G15 ***

Project each point onto the floor parallel to the wall.

Then find some other points that have the same image as Max.

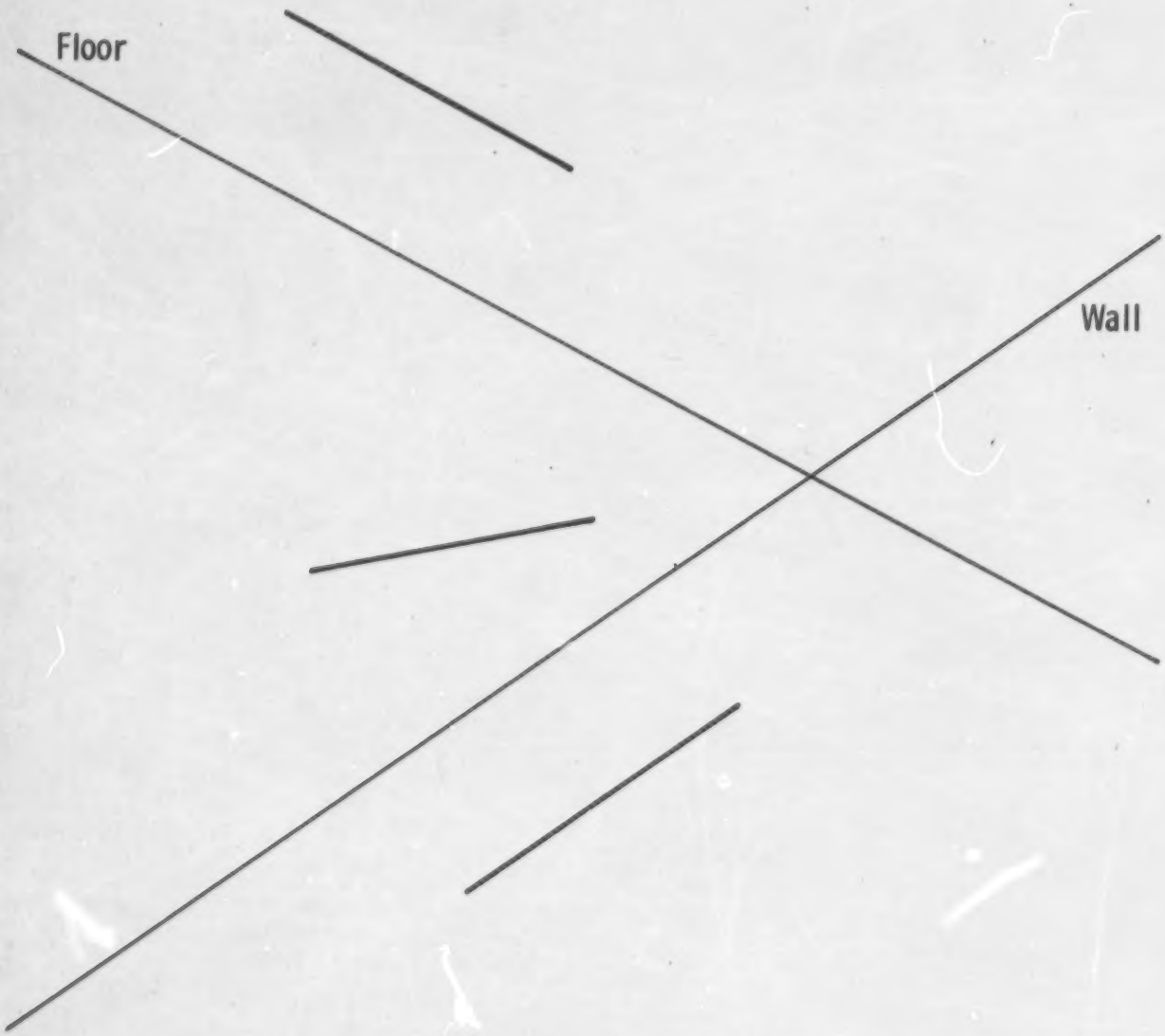


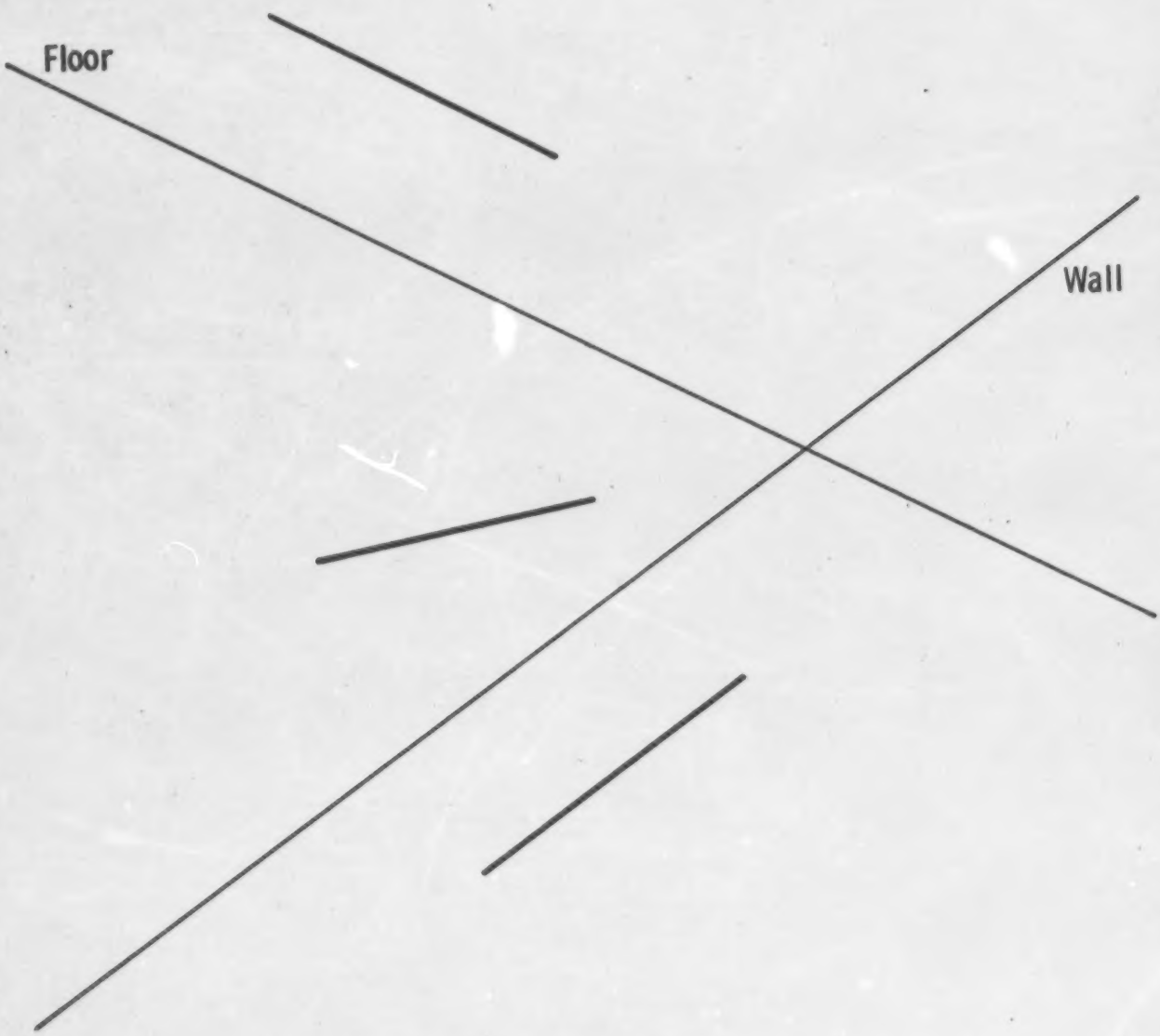

Wall
Floor
Max

Name _____ **G16** **

Project the three line segments onto the floor parallel to the wall.

Use a different color for the image of each line segment.



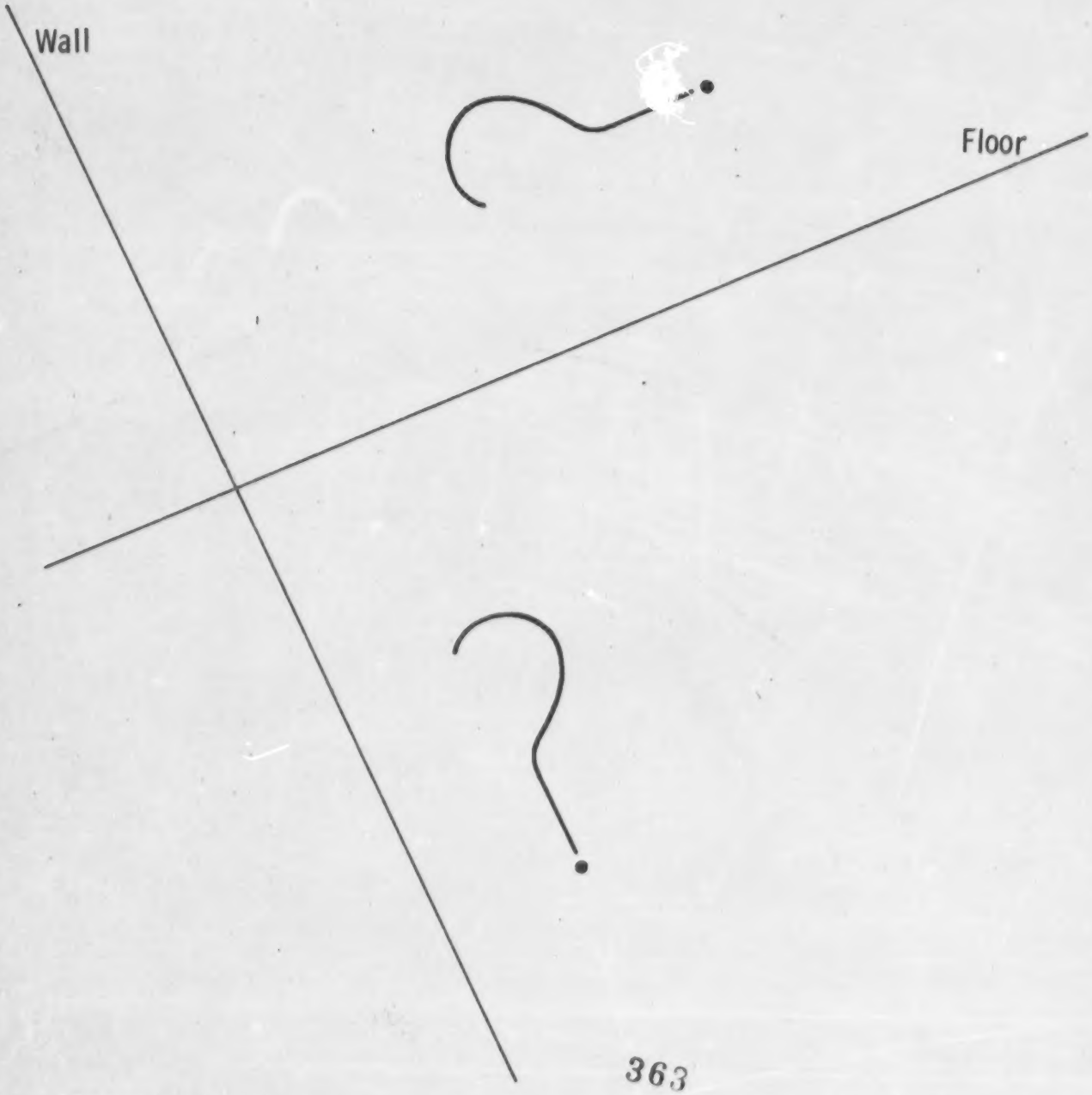

Floor
Wall

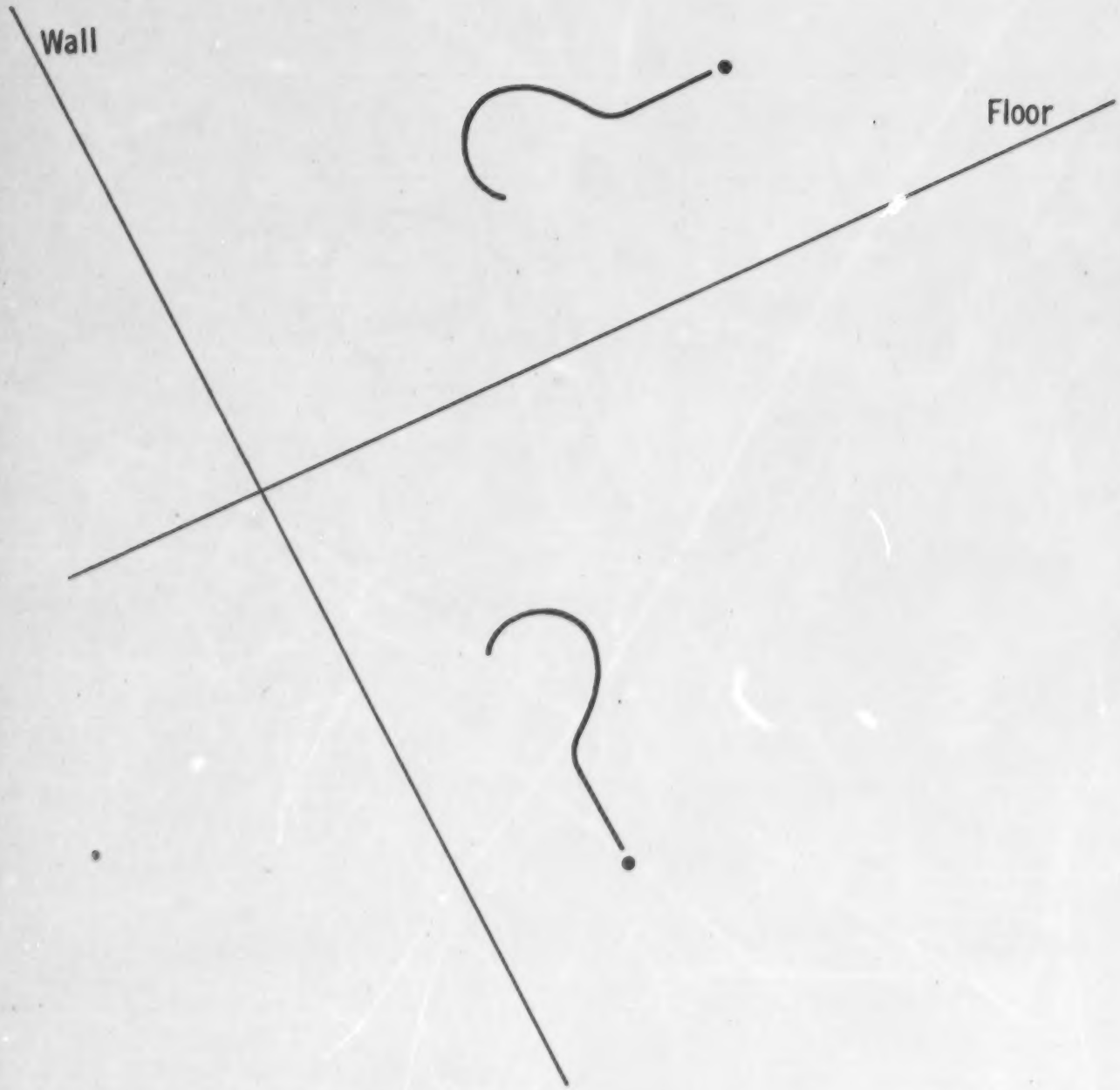
Name _____ **G17** ***

Project each question mark onto the floor parallel to the wall.

Use different colors for the two images.

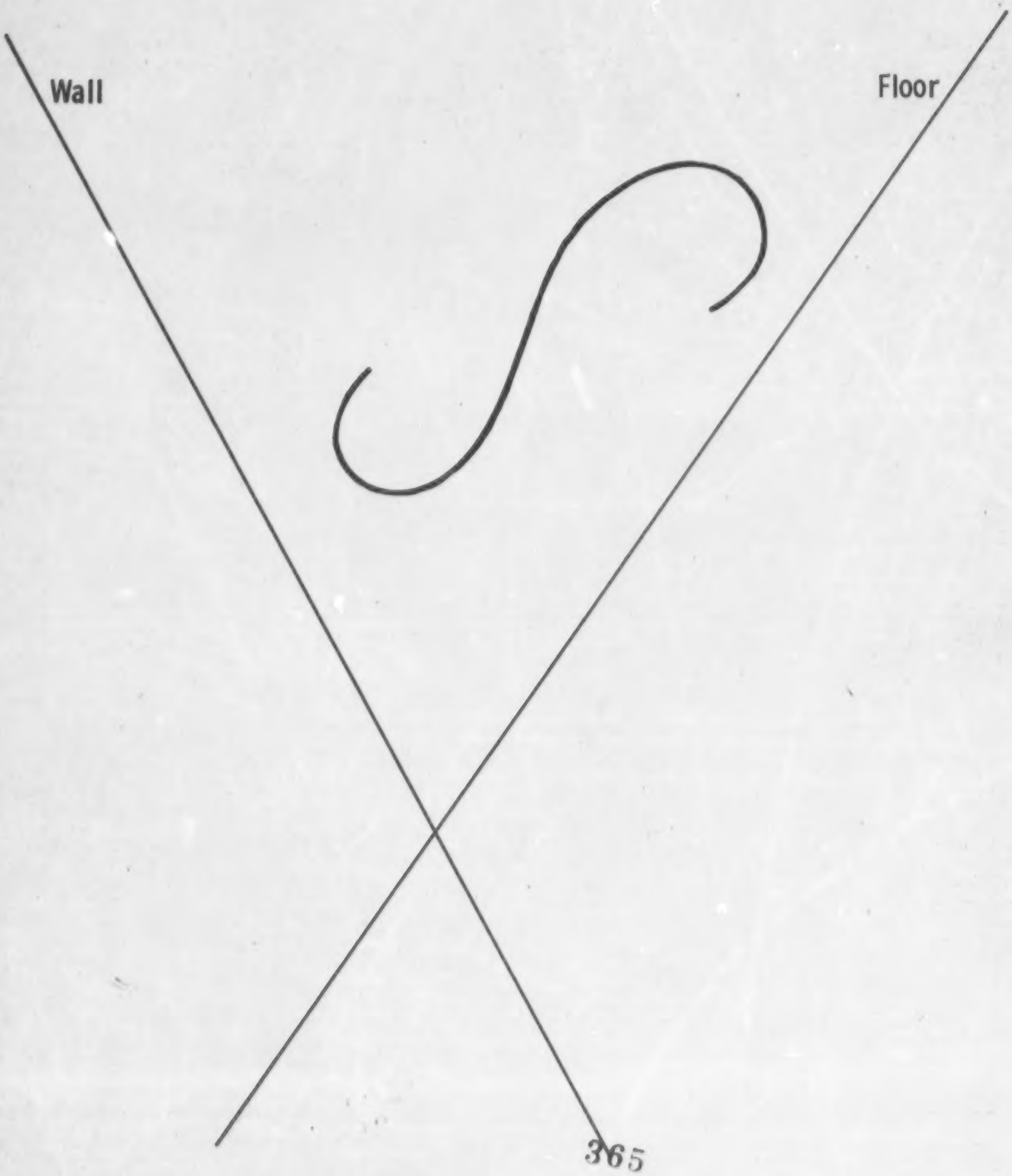
Be careful, each question mark has two pieces.

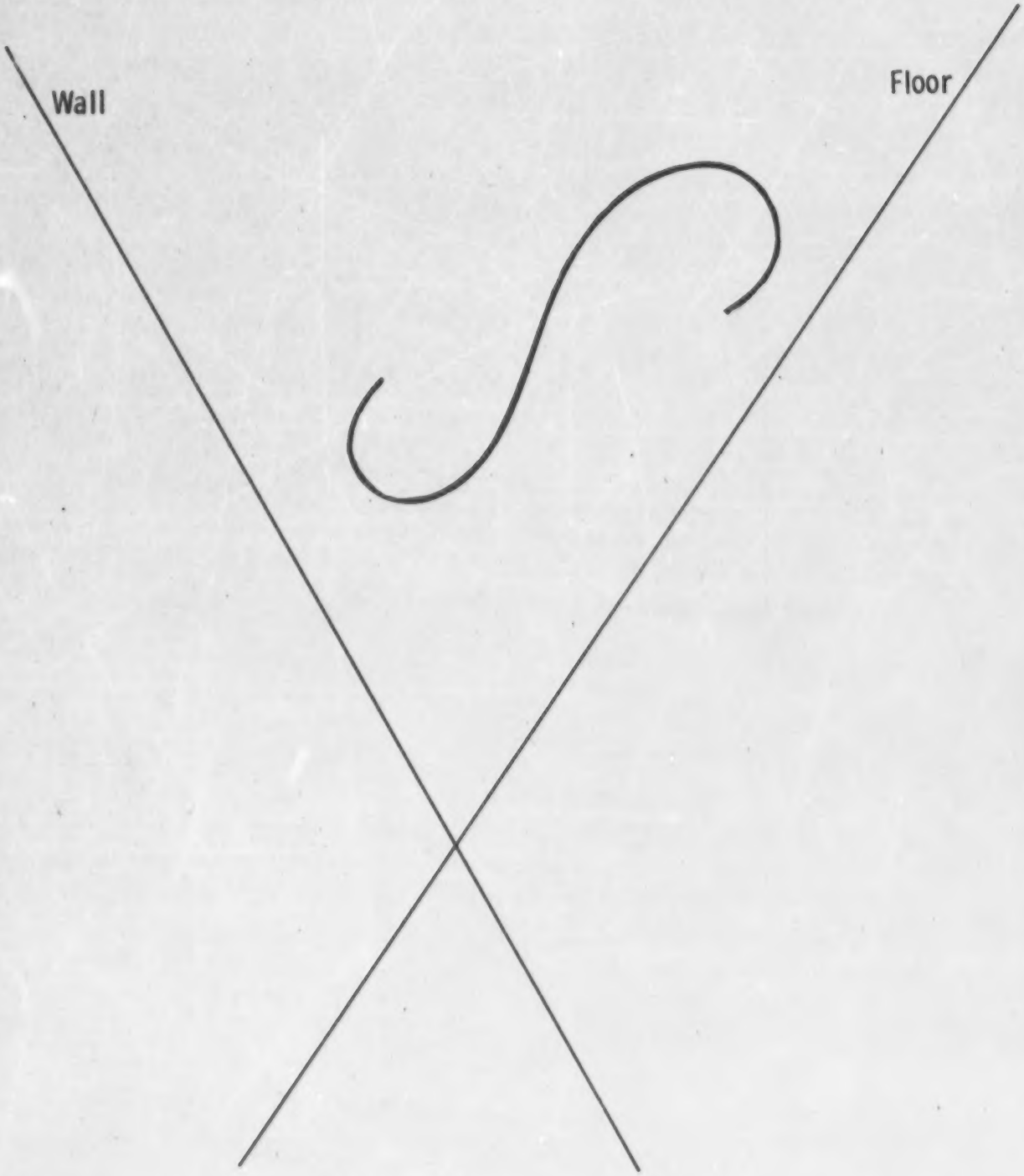



Wall
Floor

Name _____ **G18** ****

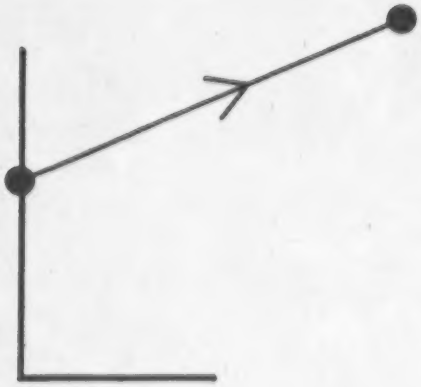
Project this graceful curve onto the floor parallel to the wall.
Show the image in red.

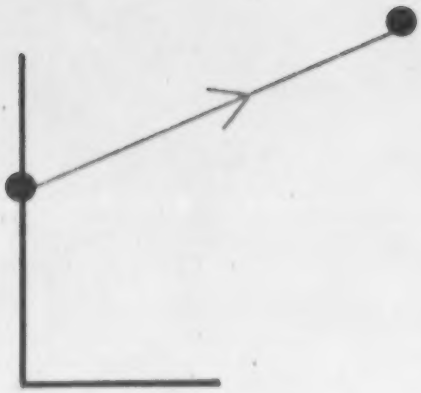



Wall
Floor

Name _____ **G19** *

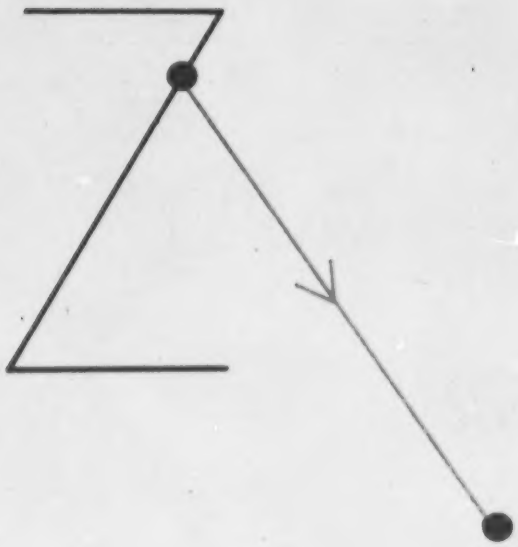
Find the image of " L " by the translation determined by the red couple .

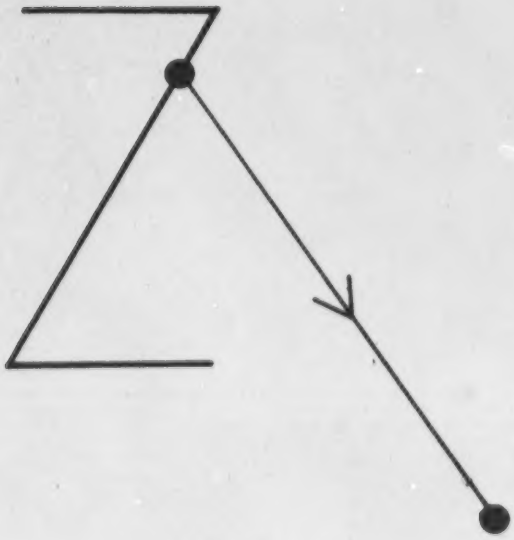




Name _____ **G20** **

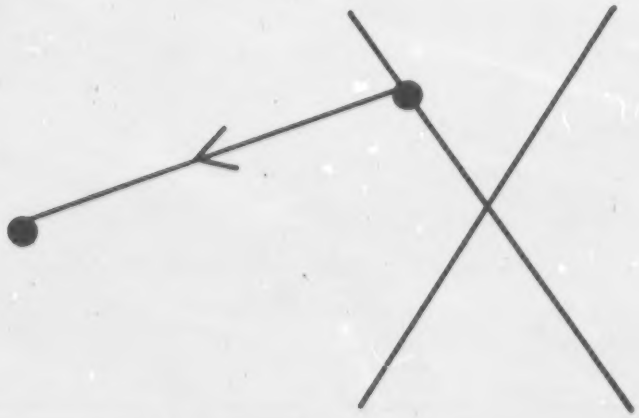
Find the image of " Z " by the translation determined by the blue couple .

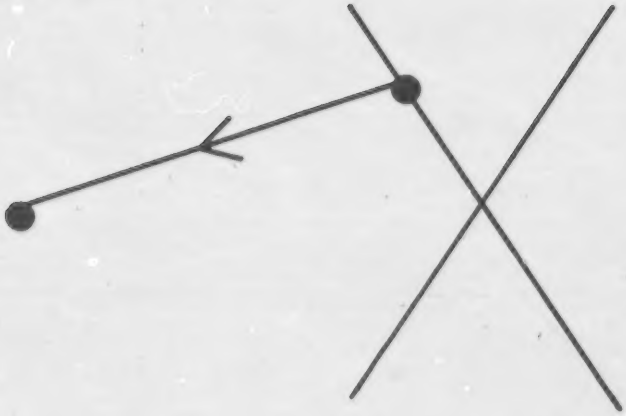




Name _____ **G21** ***

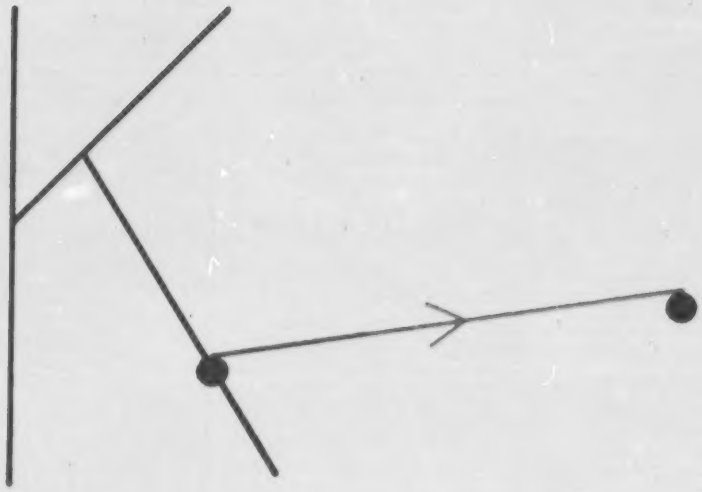
Find the image of " X " by the translation determined by the red couple.

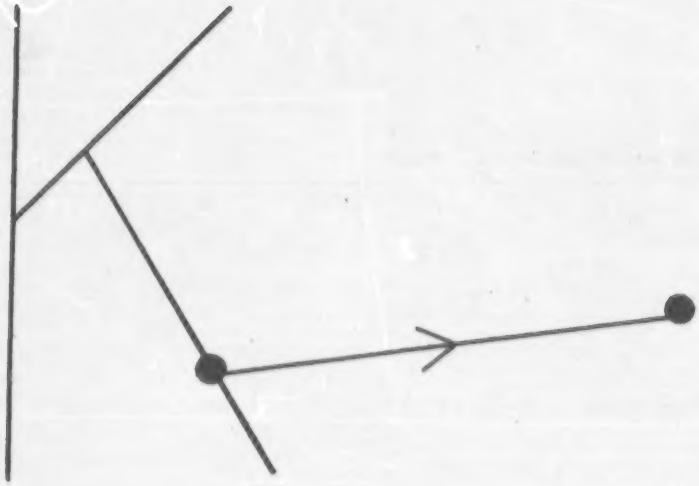




Name _____ **G22 ****

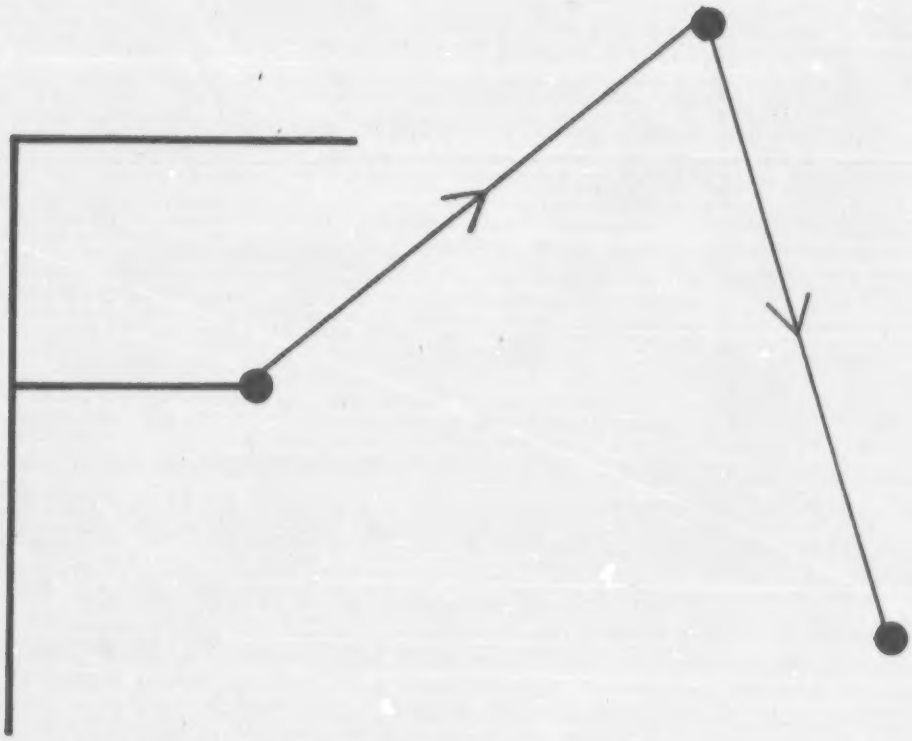
Find the image of " K " by the translation determined by the blue couple.





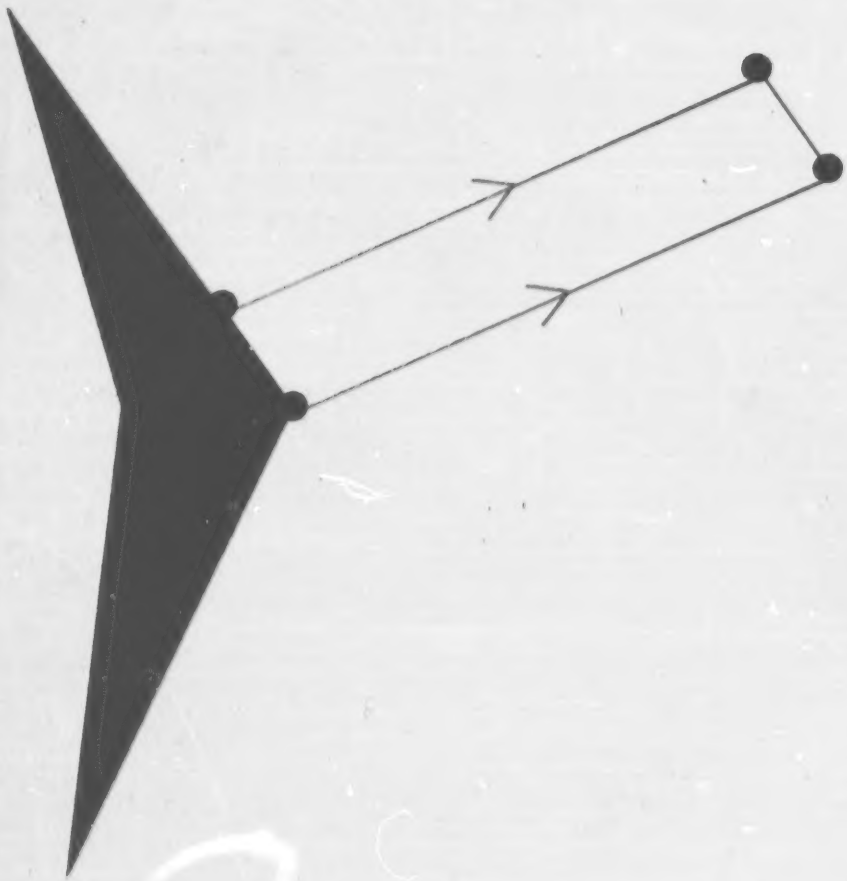
Name _____ **G 23 *******

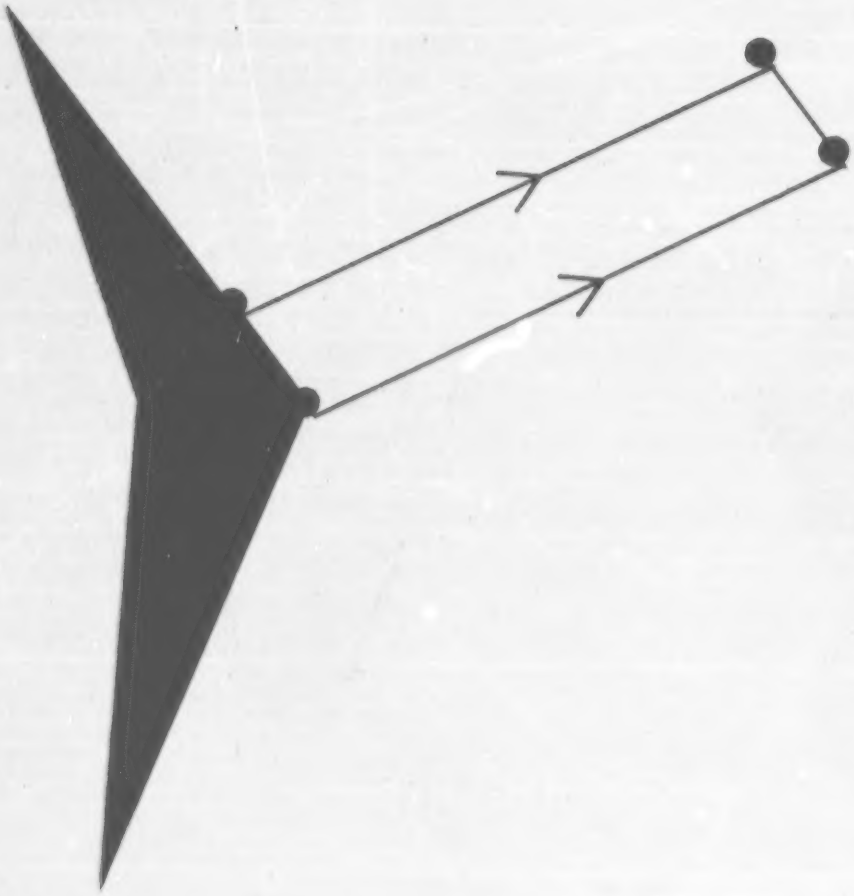
First find the image of "F" by the translation determined by the blue couple. Then find the image of the image of the original "F" by the translation determined by the red couple.



Name _____ **G 24** *

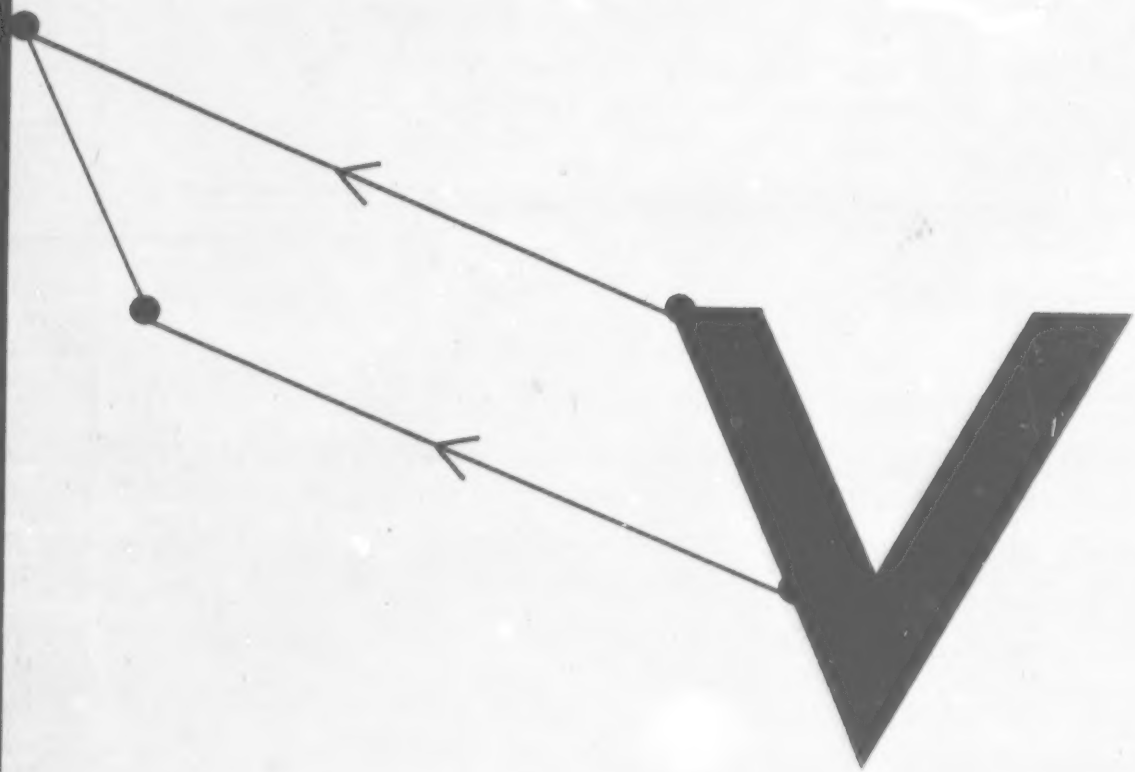
Find the image of the boomerang by the translation determined by the red couples.

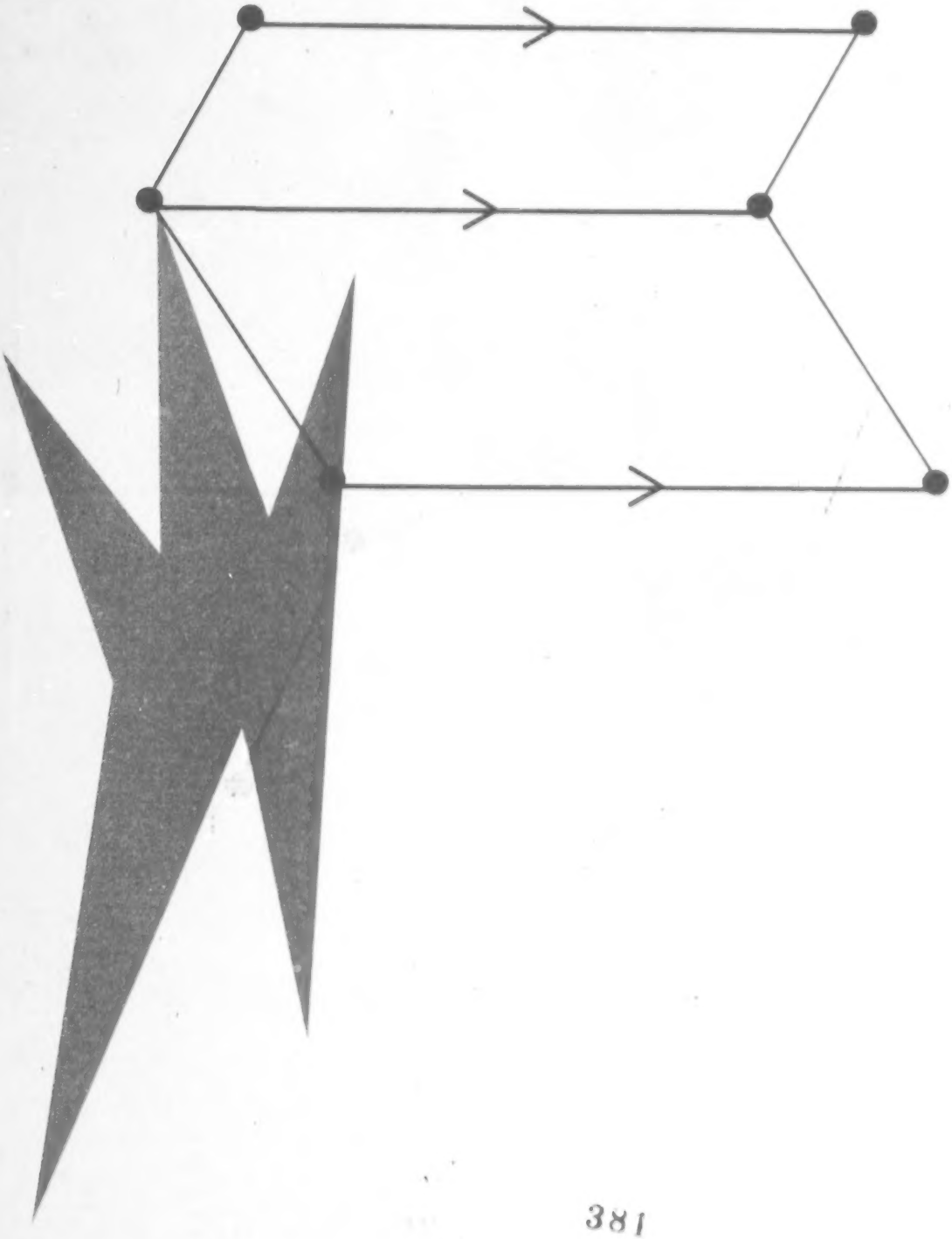
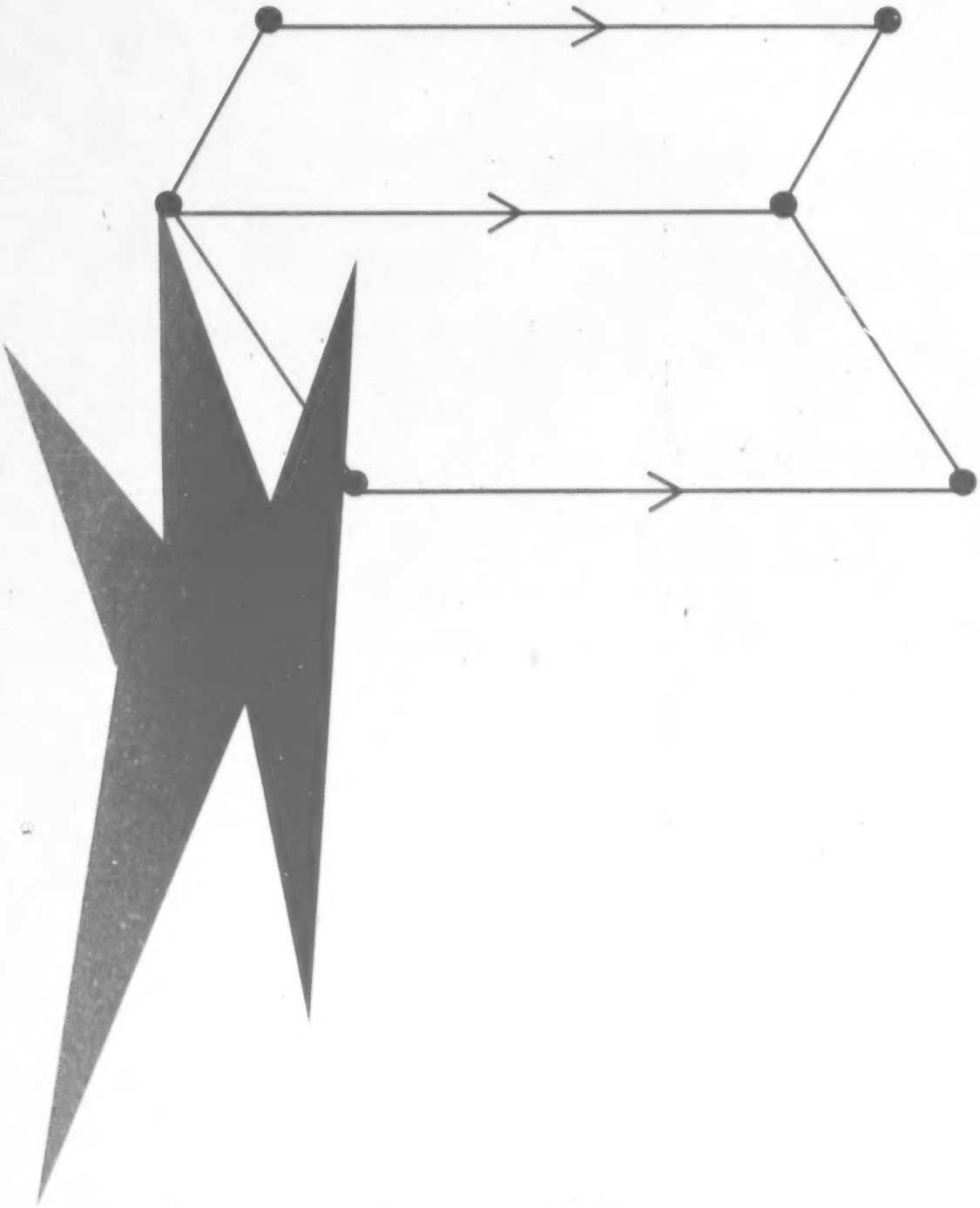




Name _____ **G25** **

Find the image of " V " by the translation determined by the red couples.



Name _____ **G26** ***

Find the image of the star by the translation determined by the red couples.